# On Computing All Immobilizing Grasps of a Simple Polygon with Few Contacts

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**Abstract:** We study the output-sensitive computations of all the combinations of the edges and vertices of a simple polygon $P$ that allow a form closure grasp with less than four point contacts. More specifically, we present an $O(m^{\frac{4}{3}} \log^{\frac{1}{3}} m + K)$-time algorithm to compute all $K$ pairs of concave vertices, an $O(n^2 \log^2 n + K)$-time and $O(m^2 \log^2 m + nm^{\frac{2}{3}} \log^{\frac{4}{3}} m + K)$-time algorithm to compute all $K$ triples of one concave vertex and two edges, and two concave vertices and an edge respectively, where $n$ is the number of edges, and $m$ is the number of concave vertices of $P$. We also present an $O(n^2 \log^4 n + K)$-time algorithm that enumerates all the edge triples with a second-order immobility grasp using Czyzowicz's conditions in [4].

## 1 Introduction

Many applications such as robot hand grasping and manufacturing operations require an object to be *immobilized*, such that any motion of the object violates the rigidity of the object or the contacts.

An attractive theoretical model for immobility was formulated by Reuleaux [11] in 1876. He defines a rigid body to be in *form closure* if a set of contacts along its boundary constrains all finite and *infinitesimal* motions of the body. This notion is stronger than immobility, as for instance an equilateral triangle with a point contact in the middle of each edge is immobilized, but is not in form closure (it permits an infinitesimal rotation around its center). Form closure depends only on the position of the contacts and their normals, and is invariant with respect to the curvature of body and contacts. This is not true for immobility in general (if we replace the equilateral triangle in the example by its inscribed circle, contacts and normals remain identical, but the body is no longer immobilized). Markenscoff et al. [8] and Mishra et al. [9] independently showed that, with the exception of a circle, any two-dimensional body can be put in form closure with four frictionless point contacts, and that almost any three-dimensional body can be put in form closure with seven such contacts. We will call a configuration of frictionless point contacts that put an object in form closure a *form-closure grasp*.

We consider the problem of computing all form-closure grasps of a polygonal part with at most four frictionless point fingers. The availability of *all* grasps of a certain part allows a user–usually a machinist–to select the grasps that best meet specific additional requirements, for instance with respect to accessibility, which may vary from one operation to another. As the computation of all grasps along a given combination of edges and concave vertices can be accomplished in constant time [10, 16], the algorithmic challenge is to efficiently report all combinations of edges and vertices that yield at least one grasp.

An algorithm to compute, for a simple $n$-vertex polygon, all the edge combinations that have a form-closure grasp with four fingers was presented by van der Stappen et al. [16]. The algorithm runs in $O(n^{2+\epsilon} + K)$-time, where $K$ is the number of edge quadruples reported. Brost and Goldberg [1] studied the same problem in modular settings, where the contact positions are restricted to a grid.

In general, four point contacts are required for planar objects, but sometimes form-closure grasps with fewer contacts are possible by using contacts in concave vertices of the object. Form-closure grasps with less than four fingers were first studied by Gopalakrishnan and Goldberg [7], who gave an $O(n^2)$-time algorithm to find all the concave vertex pairs that allow a two-finger form-closure grasp. In Section 2.1, we improve this to $O(m^{\frac{4}{3}} \log^{\frac{1}{3}} m + K)$, where $m$ is the number of concave vertices of the polygon. Combinations of one concave vertex and two edges can be reported using the general algorithm by van der Stappen et al. We improve on that by presenting an $O(n^2 \log^2 n + K)$-time algorithm. Finally, we show how to report combinations of two concave vertices and one edge in time $O(m^2 \log^2 m + nm^{\frac{2}{3}} \log^{\frac{4}{3}} m + K)$-time.

In Section 3, we turn our attention away from form-closure to general immobility. Analogous to the above, we call a configuration of frictionless point contacts that immobilizes a rigid body an *immobility grasp*. Czyzowicz et al. [4] provided a necessary and sufficient geometric condition for a simple polygon without parallel edges to be immobilized by three point contacts. A more general analysis applicable to arbitrary objects was given by Rimon and Burdick [12, 13, 14], who define the term *second-order immobility*, as it not only takes position and normal, but also curvature of object, contacts, and possible motions into account.

An algorithm that reports, for a simple $n$-vertex polygon without parallel edges, all the edge triples that yield at least one immobility grasp was given by van der Stappen [16]. Its running time is $O(n^2 \log^2 n + K')$, where $K'$ is the number of triples considered according to some criterion. This criterion is a necessary, but not sufficient condition, and so the algorithm may consider triples that do not yield immobility grasps. We resolve this shortcoming by giving a truly output-sensitive algorithm with a running time of $O(n^2 \log^4 n + K)$-time, where $K$ is the number of edge triples yielding grasps satisfying Czyzowicz et al.'s necessary and sufficient conditions [4] for immobilization.