Art Gallery Theorems for Guarded Guards

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We prove two art gallery theorems in which the guards must guard one another in addition to the gallery. A set $\mathcal{G}$ of points (the guards) in a simple closed polygon (the art gallery) is a guarded guard set provided (i) every point in the polygon is visible to some point in $\mathcal{G}$; and (ii) every point in $\mathcal{G}$ is visible to some other point in $\mathcal{G}$. We prove that a polygon with n sides always has a guarded guard set of cardinality $\lfloor (3n-1)/7 \rfloor$ and that this bound is sharp ($n \geq 5$); our result corrects an erroneous formula in the literature. We also use a coloring argument to give an entirely new proof that the corresponding sharp function for orthogonal polygons is $\lfloor n/3 \rfloor$ for $n \geq 6$; this result was originally established by induction by Hernández-Peñalver.