DIMACS Security & Cryptography Crash Course – Day 1
Hashing

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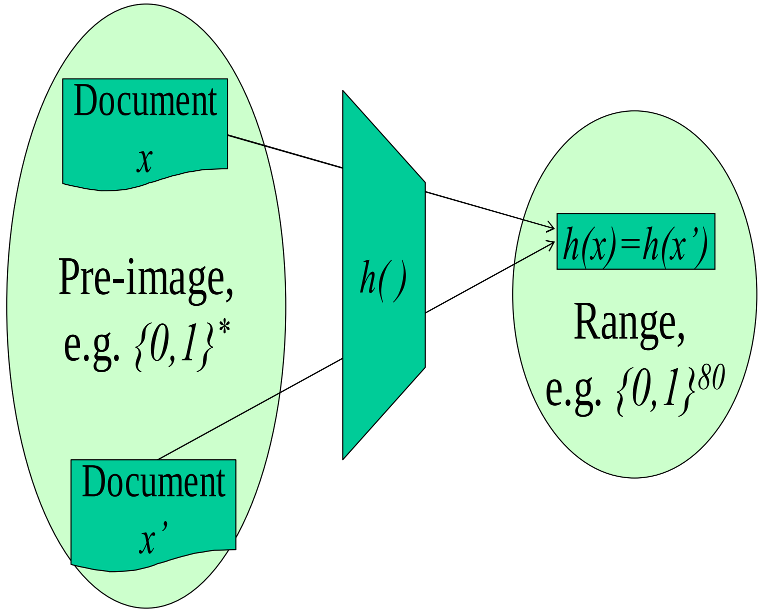
Outline

- Crypto-Hash properties
- Using and Collecting Randomness
- Randomness of Hash
- Confidentiality of Hash
- One-way functions
- Random Oracle
- Integrity & Collision Resistance
  - Collision Resistant Hash Functions (CRHF)
  - Design of CRHF
  - Merkle-Damgard construction
  - Standard hash functions
  - Conclusions

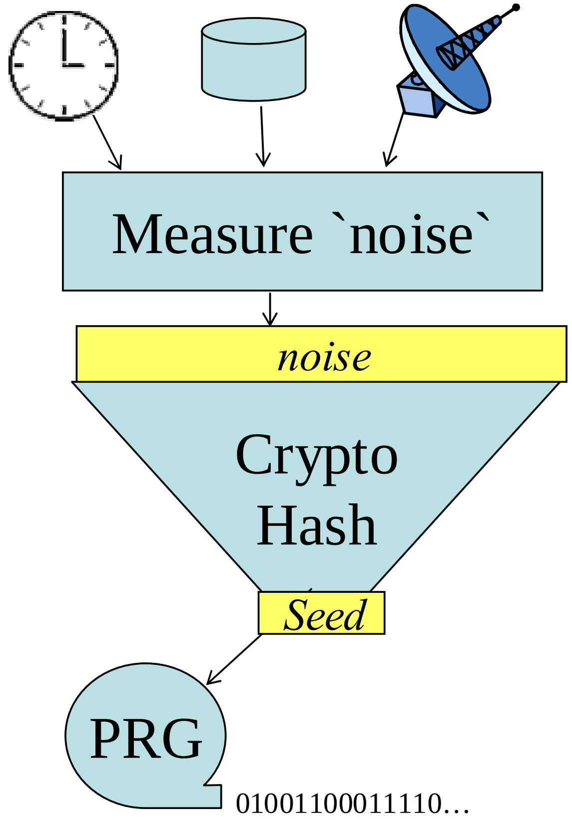
Crypto-Hash Functions - `Wish List`



- Compression
  - Unbounded/Long input
  - Short (finite) output
- Confidentiality
  - Can't find x from $h(x)$
- Collision-resistance
  - `Strong`: can't find x,x' s.t. $h(x)=h(x')$
  - `Weak`: given x, can't find $x' \neq x$ s.t. $h(x)=h(x')$
- Randomness: uniform output distribution

Collecting Randomness

- Use available sources with some randomness
  - Different `unpredictable, unobservable` events
- Extract random *seed* (n bits)
  - In practice: usually using `cryptographic hash function`
- Use PRG to generate sufficient random bits
- Certainly Ok if hash was a random function…



Random Oracle Methodology

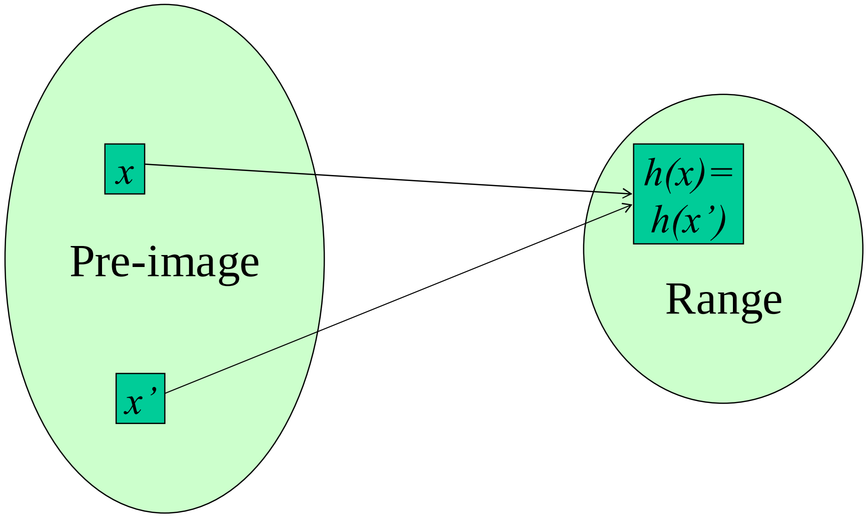
- Analyze as if hash $h()$ is a *random function*
  - Of course an invalid assumption as $h()$ is fixed!
  - Whenever $h()$ is used, we call oracle for the random function (black box containing random function)
- Good for screening insecure solutions
- Security under random oracle implies security to many (not all!!) attacks
- Not a complete proof of security, but a good argument/evidence of security.

Confidentiality of Hash

- Hash has no secret key
  - Cannot use to send secret message
- But hash should hide input
  - Cannot learn input given output (`one way function`)
- *f* is OWF (One Way Function) if:
  - *f* is computed by some PPT algorithm,
  - yet for any PPT alg. *A*: $P_A(n)=Prob\{f(A(f(x)))=f(x) : x\in_R\{0,1\}^n\}\approx_p 0$
- PPT: Probabilistic Polynomial Time algorithm
  - Time complexity < *p(n)* for some polynomial *p( )*
- $P_A(n)\approx_p 0$:
  - Every polynomial *p(n)*, exists some l_{min} s.t. if $n>l_{min}$ and $x\in_R\{0,1\}^n$ then $P_A(n)<1/p(n)$.
- Asymptotic definition; says nothing about any fixed input length
- Worse – maybe *f* exposes partial info on input?
- Most works use `random oracle` to simplify security analysis

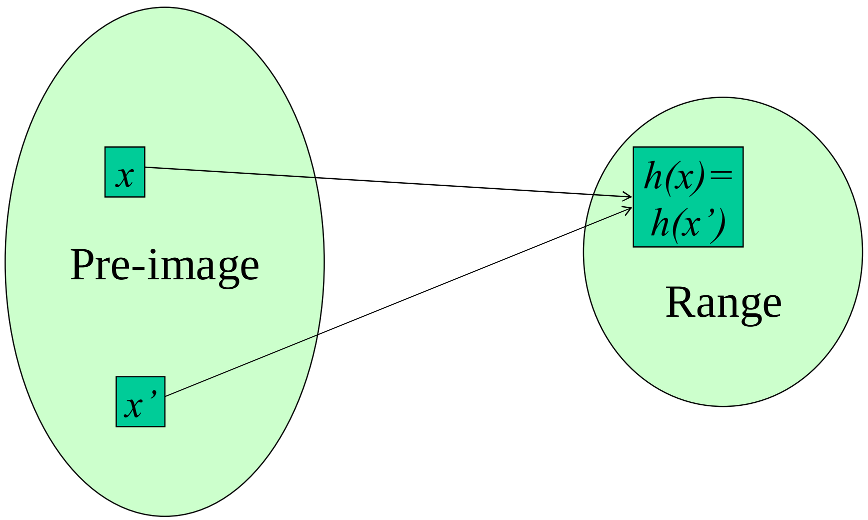
Collision Resistance

- *Simplified (Strong) Collision Resistance Assumption:* assume that it is hard (infeasible) to find a collision, i.e. $<x,x'>$ such that $x \neq x'$ yet $h(x)=h(x')$.
- Natural definition, but problematic:
  - h is fixed
  - Adversary can simply output a specific collision in it.
  - Possible fix: (public) key
- Holds for a random function (oracle)



Weak CRHF

- *Weakly* Collision Resistant Hash Function: it is hard to find a collision with a specific (random) *x*.
- A function *h* is a *Weakly CRHF* if:
  - for every length $l \geq n$,
  - given $x \in_R \{0,1\}^l$,
  - it is infeasible to find $x' \neq x$ s.t. $f(x')=f(x)$.
- Property also called *2nd pre-image resistance.*



Applying Weakly CRHF

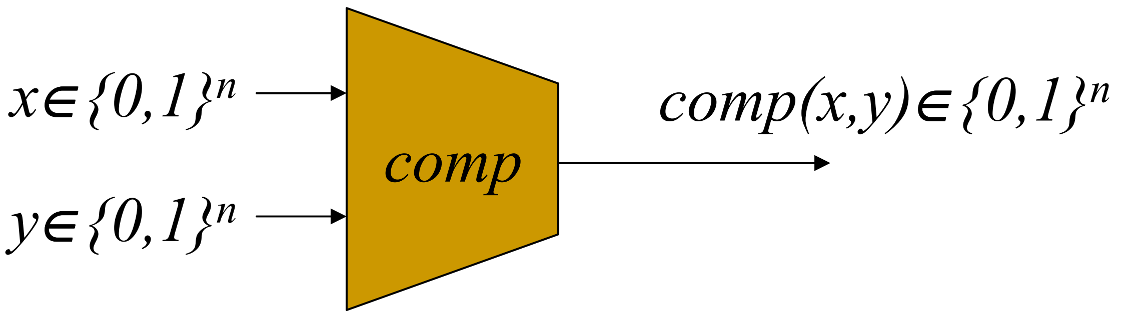
- *Weakly Collision Resistant Hash Function:* it is hard to find a collision with a specific (random) *x*.
- Uniformly distributed input (*not* chosen by Adversary!)
- Alice sends message to Bob, and signs its hash
  - Bob knows that Alice sent the message
    - Only if the message is uniformly distributed!
  - Can Bob prove Alice sent (signed) the message?

Weakly CRHF may be too weak...

- Sending signed agreement:
  - Alice reaches agreement with Bob
  - Alice signs hash of agreement
  - Bob can verify Alice signed the agreement
- But: agreement *not* uniformly distributed!
  - Maybe Bob/Alice chose it to have collision?
- Solutions:
  - Signer ensures contract is `randomized` (possibly use hash with random public key)
    - Or: keyless hash with `Simplified (Strong) Collision Resistance Assumption`
  - Signer responsible for any properly signed version

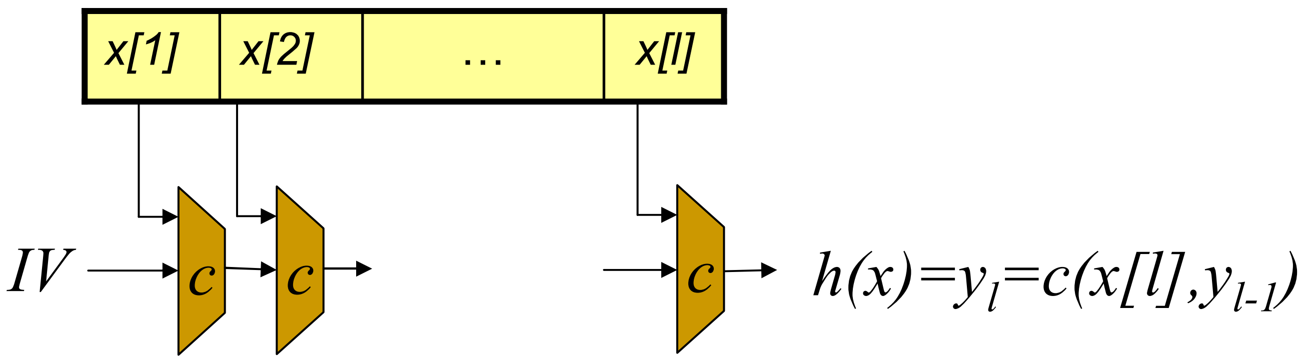
Designing CRHF

- Problem: Variable Input Length (VIL)
  - Hard to design and test (by cryptanalysis)
  - Idea: build VIL CRHF from FIL CRHF
  - FIL CRHF are also called *compression function*: $comp : \{0,1\}^{2n} \rightarrow \{0,1\}^n$



Constructing VIL CRHF from FIL CRHF

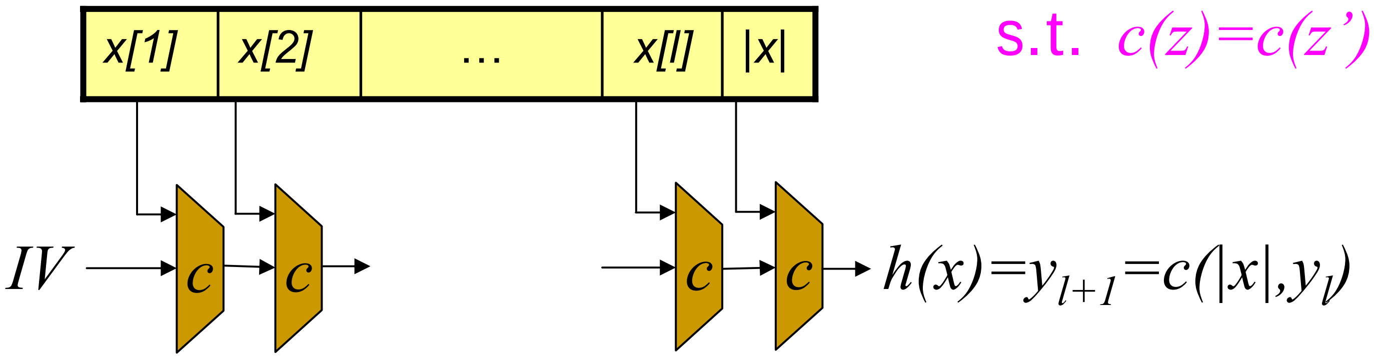
- Idea: use iterative process, compressing block by block
- Let the input x be l blocks of n bits
  - Pad the last block if necessary
- Let $y_0=IV$ be some fixed/random n bits (IV=Initialization Value)
- For $i=1,..l$, let $y_i=c(x[i],y_{i-1})$
- Output $h(x)=y_{l+1}$
- Prefix attack: Pick prefix p and random $IV=v$. Let $z=h_v(p)$ with $IV=v$. Then for any x holds: $h_z(x)=h_v(p||x)$.



Merkle-Damgard FIL→VIL Hash

- Build *h* from *compression function*: $c : \{0,1\}^{2n} \rightarrow \{0,1\}^n$
- Let the input x be l blocks of n bits
  - Pad the last block if necessary
  - Add extra block, $x[l+1]=|x|$
- Let $y_0=IV$ be some fixed n bits (IV=Initialization Value)
- For $i=1,..l+1$, let $y_i=c(x[i],y_{i-1})$
- Output $h(x)=y_{l+1}$

Claim: given $h(x)=h(x')$, for $x \neq x'$, we can find $z \neq z'$ s.t. $c(z)=c(z')$.



Standard hash functions

- Several hash standards are widely-used standards
  - Allowing security by evidence of failed cryptanalysis
  - Many efficient, free/inexpensive, interoperable implementations
  - All existing standards are for unkeyed hash functions:
    - MD5 (MD = Message Digest)
    - SHA-1 (SHA = Secure Hash Algorithm)
    - RIPEMD
- Stated Goals:
  - Collision-Resistance: `strong CRHF` and `weak CRHF`
  - Confidentiality: one-way function
- All are very efficient, e.g. cf. to encryption
- All use Merkle-Damgard iterative construction +…

Conclusion

- Crypto-Hash functions are useful for
  - Providing short `digest` of long documents
  - Extracting randomness
  - Confidentiality: hiding pre-image (original document)
  - Integrity: detecting changes
  - Proving knowledge of pre-image
- Be careful in definition/assumption used
  - One-way property may expose some (of the) input
  - Random oracle analysis – simple argument of security
  - Prefer cryptanalysis-tolerant constructions

Extras…

Finding Collisions – Birthday Paradox

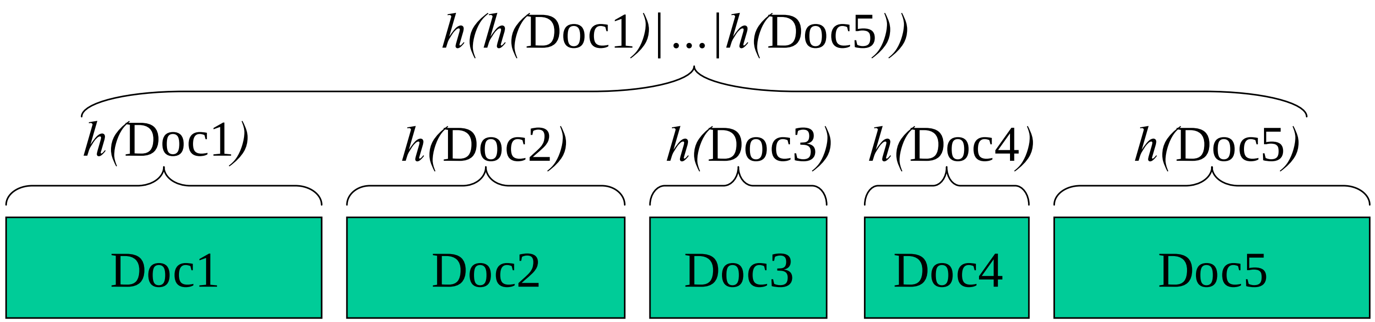
- Compute hashes of $2*2^{n/2}$ random values
- With probability > ½, there will be a collision
- Why? - `birthday paradox`(Proof omitted)
  - Intuition: probability of a collision to given x is roughly $1/2^n$; but we allow *any* collision
- Conclusion: for collision resistance we need *double* the `effective key length`
- In practice: searching 2^{64} values required one month with 10M$ machine in 1994 [OW94]
  - Expected cost today: less than 100,000$
- → Consider weaker notions

Security of MD Construction

- Theorem: if *comp* is collision-resistant, then h is collision resistant.
- Proof: we use collision in h to find collision in *comp*. Suppose $h(x)=h(x')$ for $x \neq x'$.
  - Denote $l=|x|$; note $x[i+1]=l$. Hence $h(x)=comp(l|| y_l)= comp(l'|| y'_{l'})$. Hence assume $l=l'$ and $y_l=y'_l$ (or collision in *comp*).
  - Recursively for $j=l$ to 1, we have $y_j=y'_j$, i.e. $comp(x[j]||y_{j-1})=comp(x'[j]||y'_{j-1})$.
    Hence $x[j]=x'[j]$ and $y_{j-1}=y'_{j-1}$. But $x \neq x'$! $\blacksquare$

Alternative - Hash Trees

- To hash a long document or many docs…
  - Hash each document (or part)
  - Hash all hashes (possibly recursively)
  - Can use compression function(s) (with finite input)
- Less efficient than MD when validating all inputs
- Requires to keep state (logarithmic in document size)
- Advantages when validating only some inputs:
  - Efficiency: validate only what you need
  - Reuse: some recipients may not need all docs
  - Privacy: some docs may not be shared with all

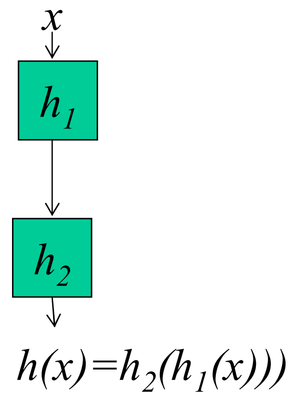


Hash with multiple properties

- We saw multiple goals/definitions for crypto-hash functions:
  - Confidentiality properties, e.g. OWHF
  - Randomness properties, e.g. *t*-resilient PR hash
  - Collision resistance properties: weak CRHF, *t*-resilient
- Goals:
  - Hash satisfying multiple goals
    - To have standard, `general-purpose` crypto-hash

Cryptanalysis-tolerance: Cascade

- Construct h by composing candidates: $h_1, h_2, \ldots$
- Cascade composition: $h(x)= h_1(h_2(x))$.
- Clearly fails for `very weak` h_1, h_2
- Example: $h_1(x)=0 \rightarrow h(x)=h_2(0)$
- Assume $h_1, h_2 : \{0,1\}^* \rightarrow \{0,1\}^L$ are *regular:*
  - For every $l>L$, $y,y' \in \{0,1\}^L$, the number of pre-images of length l of y and y' is (almost) equal
- Cascading of regular functions ensures cryptanalysis-tolerance for confidentiality:
  - If one of h_0, h_1 is one-way function, then h is one-way
- But… any collision of h_2 is a collision of h



Parallel Composition

- Parallel Composition: $h(x)= h_1(x) \| h_2(x)$
- Claim: collision for h → collisions for <u>both</u> h_1 and h_2
- Proof: suppose $h(x)=h(x')$, i.e. $h_1(x) \| h_2(x) = h_1(x') \| h_2(x')$. Hence $h_1(x) = h_1(x')$, $h_2(x) = h_2(x')$. ■
- ➔ If either h_1 or h_2 is a (weak / t-resilient) CRHF, then h is a (weak / t-resilient) CRHF.
- But parallel composition is *bad* for confidentiality
  - x \`more exposed\`
  - E.g. if h_1 not OWHF than h is not OWHF…
- We often require *hash with multiple properties*

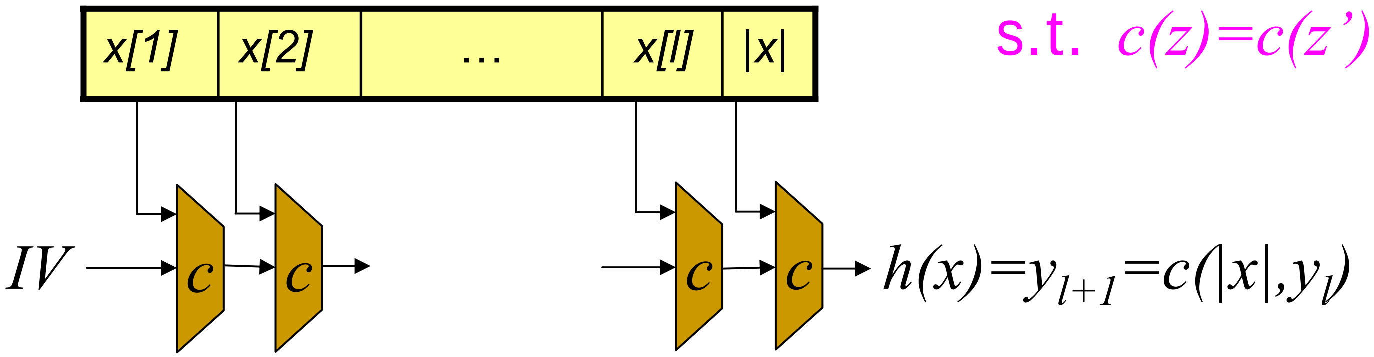
`Hybrid` composition…

- Cascade $h(x)=h_1(h_2(x))$: easier to find collisions…
- Parallel $h(x)=h_1(x)||h_2(x)$: easier to find pre-image
- What about *cascading with input*: $h(x)= h_1(x || h_2(x))$?
  - A pre-image of $h()$ provides a pre-image of h_1
  - Collision in $h()$ implies collision in h_1
  - Assuming only few collisions in h_1, say $h_1(x||y)=h_1(x'||y')$ … Requires $y'=h_2(x')$, $y=h_2(x)$
- This construction offers some confidentiality and some collision-resistance properties…
- Used in `standard` hash functions MD5, SHA-1…

Merkle-Damgard + Partial Regularity

- MD construction: Build *h* from *compression function*: $c : \{0,1\}^{2n} \rightarrow \{0,1\}^n$
- Let the input x be l blocks of n bits
- Let $y_0 = IV$ be some fixed n bits (IV=Initialization Value)
- Partial regularity: if IV is uniformly-distributed, then so is $h(x)$
- How? For $i=1,..l+1$, let $y_i = y_{i-1} + c(x[i], y_{i-1})$
- Output $h(x)=y_{l+1}$

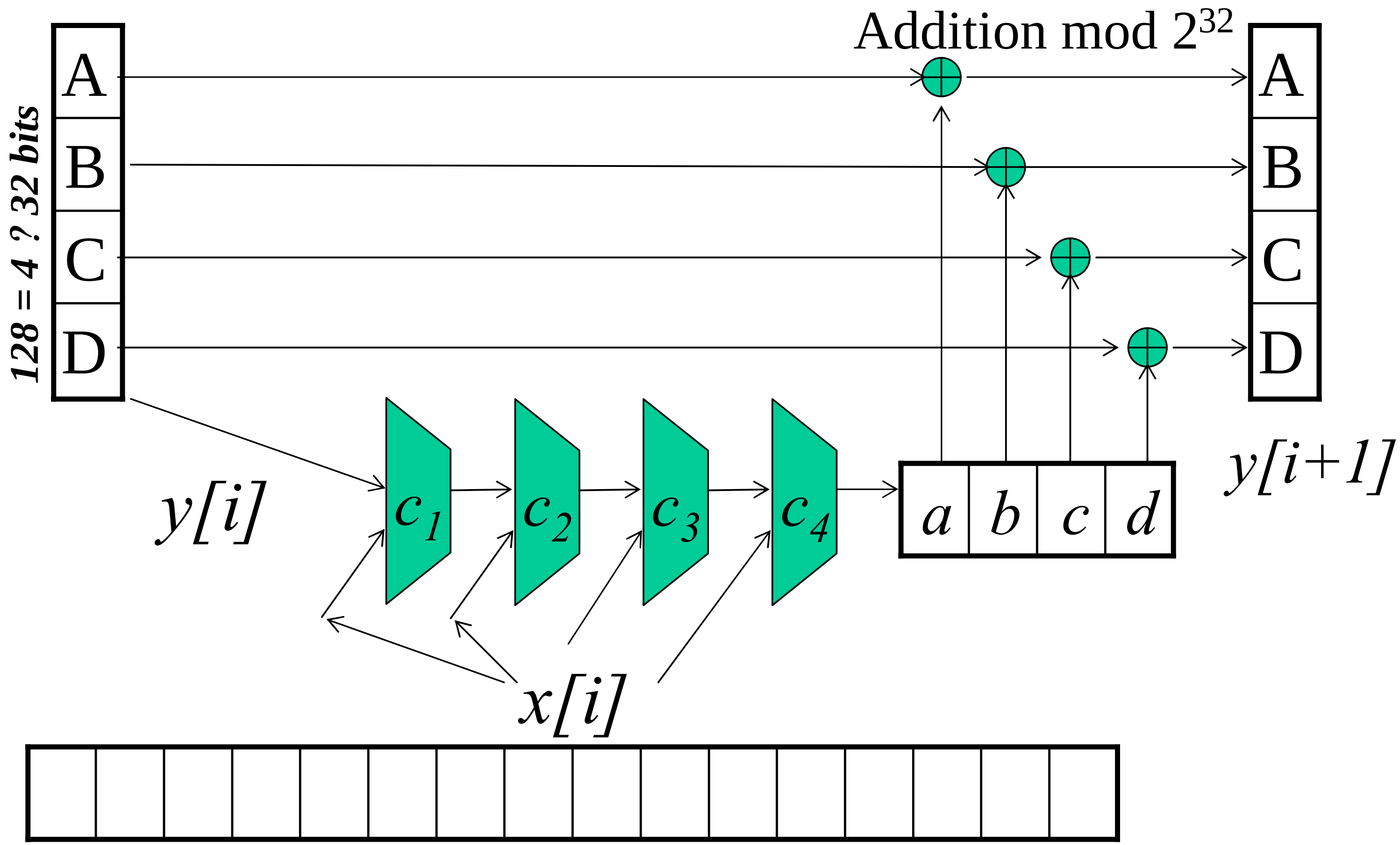
Claim: given $h(x)=h(x')$, for $x \neq x'$, we can find $z \neq z'$ s.t. $c(z)=c(z')$.



MD5

- Developed by RSA Inc.
- Output is 128 bit
  - Collisions can be found with 2^{64} time and storage
  - Believed feasible (with about 100,000$ equipment for 1 month)
- Collisions found in the compression function
  - But only in the chaining value – so not a collision for MD5 (yet)
- Still widely used, but being `phased out`
- About twice faster than RIPE-MD, SHA-1
- Compression function: Cascade of four 128b+512b→128b compression functions

MD5: Compressing block *i*



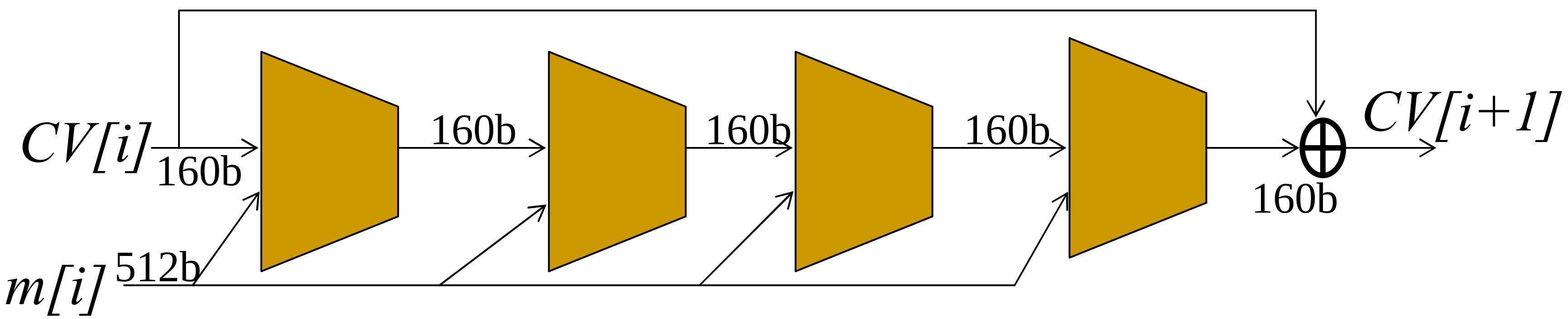
x[i] is 16 words (32 bits each) → 512b

MD5 Compression Functions

- All four functions $c_1,\ldots c_4$ have same structure
- Break 128b `chaining value` $Y[i]$ to four 32-bit words: A, B, C, D
- Each function has 16 rounds $r=1..16,\ldots 64$
- Single round computation:
  - $A_{r+1}=D_r,\ C_{r+1}=B_r,\ D_{r+1}=C_r$
  - $B_{r+1}=B_r+<<_{s[r]}\ (A_r+g(B_r,C_r,D_r)+x[i][r]+T[i])$
  - $T[i]=int(2^{32}\ abs(sin(i)))$
  - $<<_s$ is circular left shift by s; $s[r]$ is a fixed table
- No theory behind design, no analytical proof

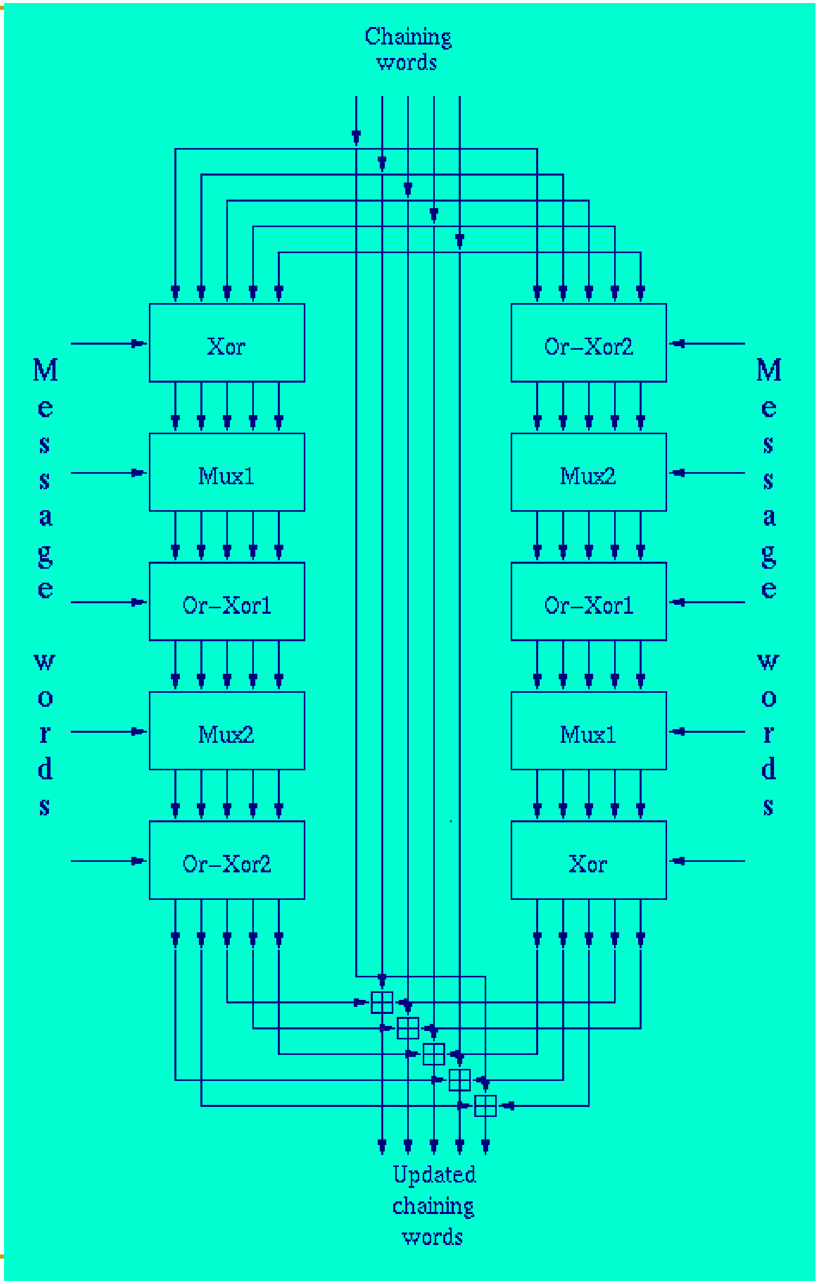
SHA-1 (Secure Hash Algorithm)

- Developed by NIST, published as FIPS 180-1
- Output is 160 bit
  - New versions: 256b, 384b and 512b proposed
- Widely used; `closed` design process, criteria
- Very similar design to MD5
  - 160b chaining block
  - Chaining value added (mod 2^{32}) to output of compression



RipeMD-160

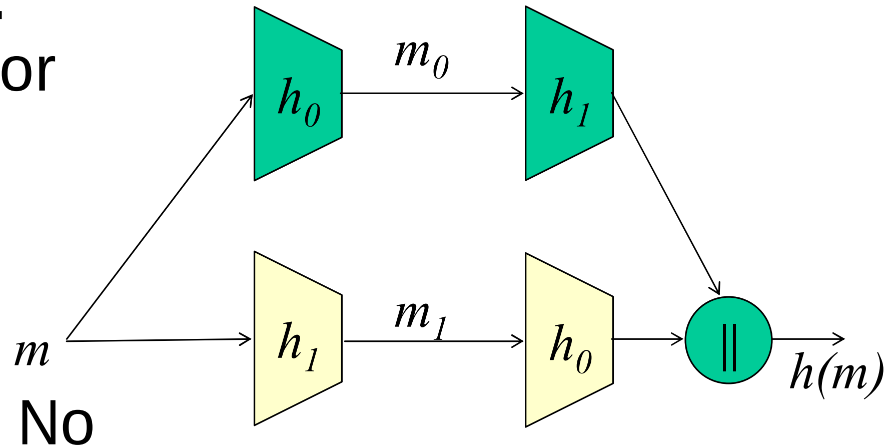
- Developed by EU RICE project
- Open design process, criteria
- Variants: 128, 160, 256 or 320 bits
- RIPEMD-160 most common
- Compression function:
  - Is RipeMD OWF, assuming one/few blocks are OWF?
  - Same for collision-resistance



Towards Cryptanalysis-tolerant Hash

- Goal: provably cryptanalysis-tolerant hash
- 1st idea: combine parallel and serial compositions:

Confidentiality: Ok for regular functions (cascade).



Collision-resistance: No
Select some $m \neq m'$.
Select h_0 s.t.:

$h_0(m)=h_0(m')$

$h_o(h_1(m))=h_0(h_1(m'))$

The *E* Cryptanalysis-tolerant Composition

- Goal: provably cryptanalysis-tolerant hash
- 2nd idea: combine *three* functions: $E[h_0, h_1, h_2]$

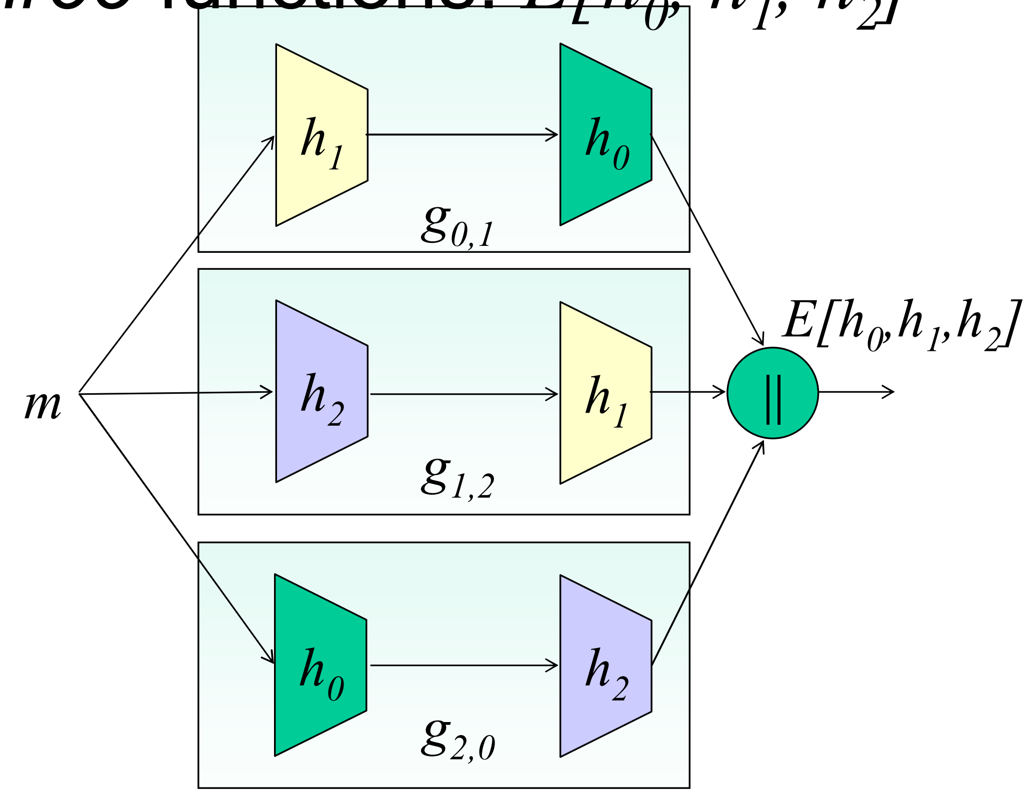
• Confidentiality: Ok

• Collision-resistance: Ok

Why? Collision of *E* → $h_o(h_1(m))=h_0(h_1(m'))$ → Collision of either h_o or h_1

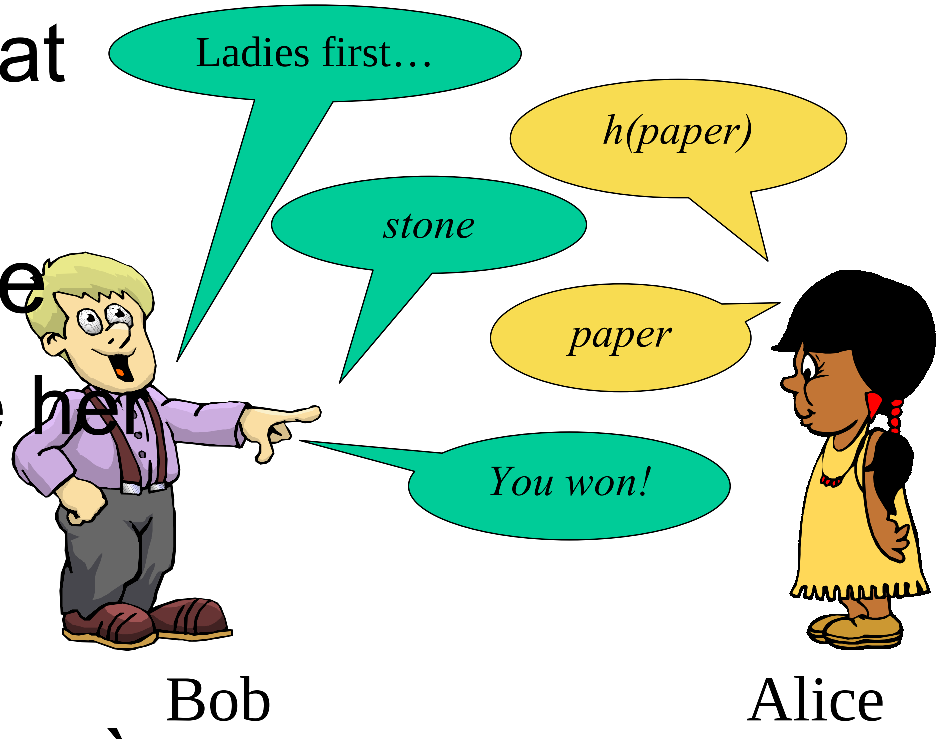
• Assuming h_0, h_1, h_2 are *all* regular functions

• Can we avoid this assumption? ... see paper

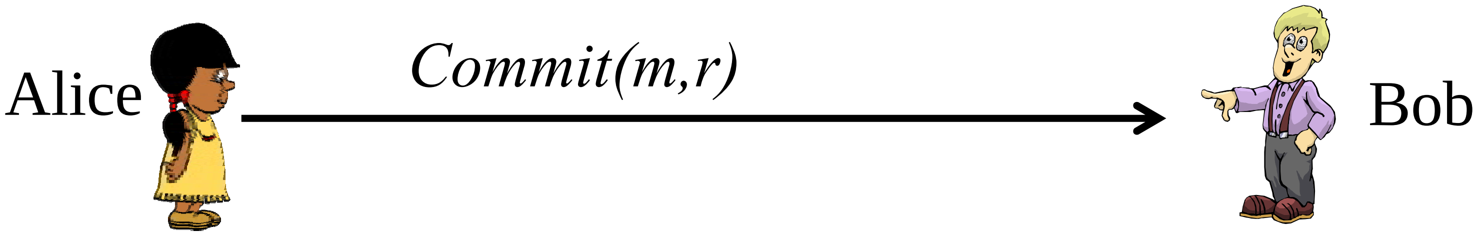


Recall `paper, stone, scissors`

- Confidentiality
  - Bob can't know what Alice chose
- Collision-resistance
  - Alice can't `change her hand`
- Randomness
  - *h*(*x*) appears `random`
  - If *h*(*x*) is deterministic, confidentiality



Commitment Schemes



- Commitment ≈ Collision resistance + privacy
- Three functions: *Commit, Decommit, Validate*
  - *Commit, Decommit* have two inputs: message, random
  - $Validate(m,Commit(m,r),Decommit(m,r))=True$
- Security properties
  - Confidentiality: $Commit(m,r)$ reveals nothing about m
  - Collision-resistance: infeasible to find m, m', d, d', c s.t. $Validate(m,c,d)=Validate(m',c,d')=True$
- Unfortunately this is impossible…

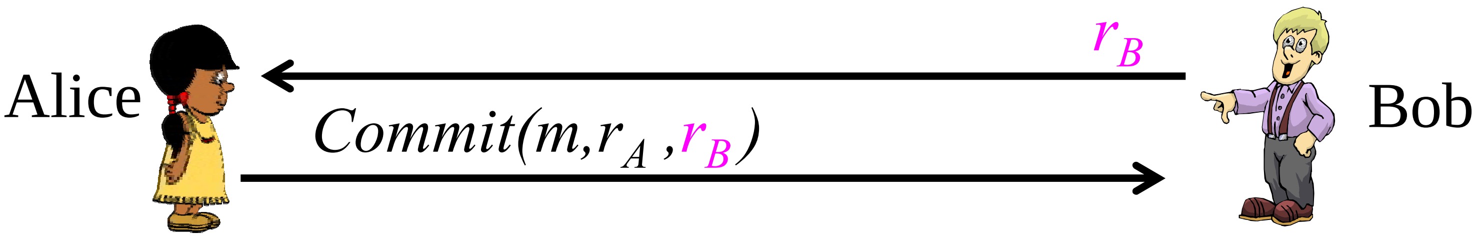
Randomness Required for Collision Resistance

- Collision-resistance: infeasible to find m, m', d, d', c s.t. $Validate(m,c,d)=Validate(m',c,d')=True$
- But: for any *Commit* function there exist collisions: $<m,r>, <m',r'>$ s.t. $c=Commit(m,r)=Commit(m',r')$
- So maybe Alice knows such collision?
  - And then: $Validate(m,c,d)=Validate(m',c,d')=True$ where $d=Decommit(m,r)$, $d'=Decommit(m',r')$
- Solutions:
  - Use *keyed commit function* with random (public) key
  - Or: ensure input to commitment is randomized
  - Recipient confirms proper randomization
- Still need random r for each new commitment!

Keyed Commitment Schemes

- Keyed functions: *Commit, Decommit, Validate*
- $Commit_k$, $Decommit_k$ have inputs: key k, message, random
- $Validate_k(m,Commit_k(m,r),Decommit_k(m,r))=True$
- Confidentiality: $Commit_k(m,r)$ reveals nothing on m
- Collision-resistance: no adversary *ADV*, given random k, can efficiently find m, m', d, d', c s.t. $Validate_k(m,c,d)=Validate_k(m',c,d')=True$
- Recipient confirms k is random, not chosen by *ADV*!
- If recipient adds randomness, we can avoid key!

Interactive Commitment Schemes



- Receiver (Bob) selects random input r_B
- Three functions: *Commit, Decommit, Validate*
  - *Commit, Decommit* have *three* inputs: message, r_A , r_B
  - $Validate(r_B,m,Commit(m,r_A,r_B),Decommit(m,r_A,r_B))=True$
- Security properties
  - Confidentiality: $Commit(m,r_A,r_B)$ reveals nothing about m
  - Collision-resistance: no adversary *ADV*, given random r_B, can efficiently find m,r_A, m', d' s.t. $Validate_k(r_B,m',Commit_k(m, r_A,r_B),d')=True$

`Paper, stone, scissors` using Interactive Commitment Scheme

Decommit is often trivial

Commitment from Hashing

- `Standard` construction in practice:
  - $Commit(m,r_A,r_B)=h(m||r_A||r_B)$
  - $Decommit(m,r_A,r_B)=r_A$
  - $Validate(r_B,m,c,d)=TRUE$ *if* $c=Commit(m,d,r_B)$
- Justified by:
  - Random oracle analysis, or ??? (ongoing work)
- Other provable-secure constructions require weaker *h*
  - But are more complex, not used in practice
  - Only keyed versions
  - Much theory work, e.g. zero-knowledge proofs,…

Application: Secure Government Bid

- Goals:
  - Receive `sealed bids` until deadline
  - Open all bids, select the best after deadline
- Concerns:
  - Leakage of info about bids to other bidders
  - Changing of bid after deadline
- Solution:
  - Publish RFP with randomizer *r*
  - Bidders send *h(bid, r, r')*
  - At deadline, government publishes all commitments to bids
  - Then participants publish *bid* and *r'*