How I decided to ask Eduardo to be my thesis adviser

Madalena Chaves

SontagFest, DIMACS, 23-25 May2011

Math 640 Topics in control theory

- When I arrived at Rutgers (around 1997), I didn't know the field of Control Theory, but decided to follow this Math 640 topics course

- The course was taught by someone called Eduardo Sontag, who was (and still is) the most wonderful and enthusiastic professor I ever met

- His enthusiasm was contagious, he gave lots of intuition and instantly captivated you to love the topics he was teaching

- There were exercises, of course, one of them involving the controllability of the shopping cart

Exercise 4.3.16 Consider a model for the "shopping cart" shown in Figure 4.2 ("knife-edge" or "unicycle" are other names for this example). The state is given by the orientation θ, together with the coordinates x_1, x_2 of the midpoint between the back wheels.

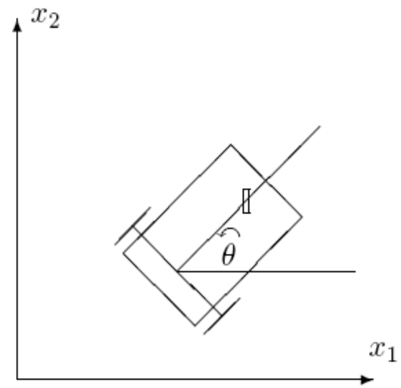


Figure 4.2: *Shopping cart.*

The front wheel is a castor, free to rotate. There is a non-slipping constraint on movement: the velocity $(\dot{x}_1, \dot{x}_2)'$ must be parallel to the vector $(\cos\theta, \sin\theta)'$. This leads to the following equations:

$$\begin{aligned} \dot{x}_1 &= u_1 \cos\theta \\ \dot{x}_2 &= u_1 \sin\theta \\ \dot{\theta} &= u_2 \end{aligned}$$

where we may view u_1 as a "drive" command and u_2 as a steering control (in practice, we implement these controls by means of differential forces on the two back corners of the cart). We view the system as having state space $\mathbb{R}^3$ (a more accurate state space would be the manifold $\mathbb{R}^2 \times \mathbb{S}^1$).
(a) Show that the system is completely controllable.
(b) Consider these new variables: $z_1 := \theta$, $z_2 := x_1 \cos\theta + x_2 \sin\theta$, $z_3 := x_1 \sin\theta - x_2 \cos\theta$, $v_1 := u_2$, and $v_2 := u_1 - u_2 z_3$. (Such a change of variables is called a "feedback transformation".) Write the system in these variables, as $\dot{z} = \widetilde{f}(z, v)$. Note that this is one of the systems Σ_i in Exercise 4.3.14. Explain why controllability can then be deduced from what you already concluded in that previous exercise. □

Exercises

- It was wonderful to see how control theory really worked and could be applied to things from satellites to everyday items: you computed some matrices, did some calculations, and in the end proved that the shopping cart was controllable

- I was so happy that I couldn't resist writing, at the end of my exercise,
  "something I'm glad to know, the next time I go
  to the supermarket!"

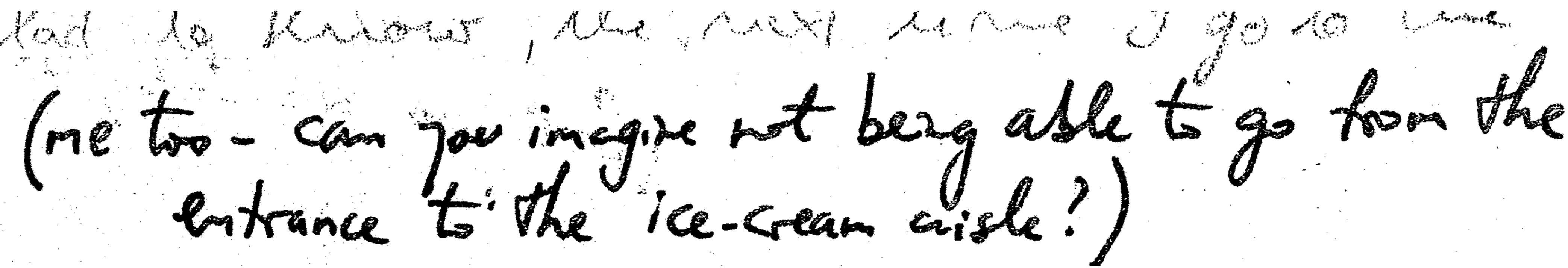
- And, of course, Eduardo always corrected and commented his student's exercises

$\Longrightarrow$ ARC holds at every $x^0 \in R^3$.

Therefore, by corollary 01.28, the shopping cart is completely controllable.

(something I'm glad to know, the next time I go to the supermarket!) (me too - can you imagine not being able to go from the entrance to the ice-cream aisle?)

Zoom in


(me too - can you imagine not being able to go from the entrance to the ice-cream aisle?)

Conclusion

- A wonderful teacher, enthusiastic, full of ideas, and always encouraging to his students

- ... and in addition he also liked ice-cream!

- So that's how I definitely decided that I would ask Eduardo Sontag to be my thesis adviser, since ice-cream is one of my favorite desserts

- I'm really happy that he accepted, and it has been a great honor and privilege to work, collaborate, and discuss with Eduardo

- Corollary: to celebrate my PhD thesis defense we all went for dinner, and then ice cream at Thomas Sweet in New Brunswick!