The Complexity of Lobbying in an Uncertain World (Extended Abstract)

Nicholas Mattei*, Judy Goldsmith*, Gábor Erdélyi†, Jörg Rothe†

Abstract

In this paper we propose a formal model for lobbying in a probabilistic environment. We offer two distinct models in which an actor can influence a voter’s preferences of voting for or against multiple issues when the voter’s preferences are represented in terms of the probability that they will vote as desired on each issue. We determine that the choice of evaluation and bribery criteria has significant influence on the computational complexity of the bribery problem. We provide three evaluation criteria and three bribery criteria for these models in order to explore their different effects on the resulting complexity. We give a formal complexity analysis for these problems and conclude that lobbying in a stochastic environment can be computationally complex.

Key words: Computational Social Choice, Computational Complexity

1 Introduction

In the American political system, laws are passed by elected officials who are supposed to represent their constituency. Individual entities such as citizens or corporations are not supposed to be able to determine the wording or passage of a law. However, they are allowed to make contributions to representatives, and it is common to include an indication that the contribution carries an expectation that the representative will vote a certain way on a particular issue.

Naturally, there are many factors that affect the representative’s vote on a particular issue. There are the representative’s personal beliefs about the issue, which presumably

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were part of the reason that the constituency elected them. There are also the campaign contributions, communications from constituents, communications from potential donors, and the representative’s own expectations of further contributions and political support.

It is a complicated process to reason about. Earlier work considered the problem of meting out contributions to representatives in order to pass a set of laws or influence a set of votes. However, the earlier computational complexity work on this problem made the assumption that a politician who accepts a contribution will in fact—if the contribution meets a given threshold—vote according to the wishes of the donor.

It was said that “An honest politician is one who stays bought,” but that does not take into account the ongoing pressures from personal convictions and opposing lobbyists and donors. We consider the problem of influencing a set of votes under the assumption that we can only influence the probability that the politician votes as we desire.

There are several axes along which we complicate the picture. The first is the notion of sufficiency: What does it mean to say we have donated enough to influence the vote? Does it mean that the probability that a single vote will go our way is greater than some threshold? That the probability that all the votes go our way is greater than that threshold? We discuss these and other criteria in the section on evaluation criteria.

How does one donate money to a campaign? In the United States there are several laws that influence how, when, and how much a particular person or organization can donate to a particular candidate. We examine ways in which money can be channeled into the political process in the section on bribery methods.

2 Related Work

Lobbying has been studied formally by economists, computer scientists, and special interest groups since at least 1983 [Rei83] and as an extension to formal game theory since 1944 [vNM44]. Each discipline has considered mostly disjoint aspects of the process while seeking to accomplish distinct goals with their respective formal models. Economists have formalized models and studied them as “economic games,” as were defined by von Neumann and Morgenstern [vNM44]. This analysis is focused on learning how these complex systems work and deducing optimal strategies for winning the competitions [Rei83, BKdV93, BKdV96]. This work has also focused on how to “rig” a vote and how optimally to dispense the funds among the various individuals [BKdV93]. The economists are interested in finding effective and efficient bribery schemes[BKdV93] as well as determining strategies for instances of two or more players [BKdV93, Rei83, BKdV96]. Generally, they reduce the problem of finding an effective lobbying strategy to one of finding a winning strategy for the specific type of game, usually the “binary multi-unit combinatorial reverse auction winner determination problem”
Economists have also formalized this problem for bribery systems in both the United States [Rei83] as well as the European Union [Cro02].

Over the last several years many computer scientists have begun to investigate lobbying and election problems [CFRS07, FHHR]. The main focus of this has been the study of the complexity of rigging or changing voting results in various systems [FHH06]. The proofs of complexity have been sought under the idea that the harder, computationally, a system is to tamper with, the more resistant it is to tampering [FHHR07]. In particular, Christian et al. [CFRS07] showed that “Optimal Lobbying” (OL) is complete for the complexity class \( \text{W}[2] \) \(^1\). The OL problem is a deterministic and non-weighted version of the problem that we present in this paper. Erdélyi et al. [EHRS07] extend “Optimal Lobbying” into “Optimal Weighted Lobbying” (OWL) by adding the constraint that lobbying extracts some price from the lobbies. This in turn can be seen as a special case of a “binary multi-unit combinatorial reverse auction winner determination problem” [SSGL02].

We seek to extend the models, and provide algorithms and analysis for these extended models. There are still elements of the reverse auction winner determination problem but the extensions allow the seller to express desire over the objects, thus fundamentally changing the problem in both the economic and complexity senses. This change is a result of the probabilistic modeling of the seller’s reaction to the bribery. We also show novel computational and algorithmic approaches to these new problems. In this way we add to both breadth and depth to not only the models but also the understanding of lobbying behavior.

3 Models for Probabilistic Lobbying

3.1 Initial Model

We begin with a simplistic version of the \textsc{Probabilistic Lobbying Problem} (PLP, for short), in which voters start with initial probabilities of voting for an issue. We assume that each voter has known costs for increasing their probability of voting according to “The Lobby’s” agenda by a finite amount.

The question, for this class of problems, is: Given the above information, along with an agenda and a fixed budget \( B \), can The Lobby target their bribes in order to achieve their agenda? The complexity of the problem seems to hinge on the evaluation criterion for what it means to “win a vote” or “achieve an agenda.” We discuss the possible interpretations of evaluation and bribery in later sections of this paper. First, however, we will

\(^1\)A full explanation of the more exotic complexity classes discussed in this paper will be included in the full version
formalize the problem by defining the data objects needed to represent its input 2.

- $\mathbb{Q}^{m \times n}_{[0,1]}$ denotes the set of $m \times n$ matrices over $\mathbb{Q}_{[0,1]}$ (the rational numbers in the interval $[0, 1]$).

- $P \in \mathbb{Q}^{m \times n}_{[0,1]}$ is a probability matrix (of size $m \times n$), where each entry $p_{i,j}$ of $P$ gives the probability that voter $v_i$ will vote “yes” for referendum (synonymously, for issue) $r_j$. The result of a vote can be either a “yes” (represented by 1) or a “no” (represented by 0). Thus, we can represent the result of any vote on all issues as a 0/1 vector $\mathbf{X} = (x_1, x_2, \ldots, x_n)$, which is sometimes also denoted as a string in $\{0, 1\}^n$.

We can now define the discrete price function for each voter on each issue. This function will allow us to determine the price that it costs to have voter $v_i$ raise or lower his or her probability of voting for issue $r_j$. We let $k$ be a positive, fixed integer (implicit in each problem instance), and we define the discrete price function $c_{i,j}$ for each $i$ and $j$. Intuitively, $c_{i,j}$ defines the cost of raising the $i$th voter’s probability of voting “yes” on the $j$th issue. Formally we describe it below.

- $c_{i,j}$ maps $\{0, 1, \ldots, k+1\} \rightarrow \mathbb{Z}^+$ in a nondecreasing manner, where $\{0, 1, \ldots, k+1\}$ indicates steps of size $\frac{1}{k+1}$ in the probability interval $[0, 1]$, and $c_{i,j}(p_{i,j})$ is the price of raising the probability of the $i$th voter voting “yes” on the $j$th issue to $p_{i,j} + \frac{1}{k+1}$.

- The domain of $c_{i,j}$ consists of $k+2$ elements of $\mathbb{Q}_{[0,1]}$ including 0, $p_{i,j}$, and 1. Note that $k = 0$ enforces $p_{i,j} \in \{0, 1\}$.

- The image of $c_{i,j}$ consists of $k+2$ nonnegative integers including 0.

- $c_{i,j}(p_{i,j}) = 0$ represents the starting probability of voter $v_i$ voting on issue $r_j$.

- For any two elements $a, b$ in the domain of $c_{i,j}$, if $p_{i,j} \leq a \leq b$ or $p_{i,j} \geq a \geq b$, then $c_{i,j}(a) \leq c_{i,j}(b)$. This corresponds to a guarantee of monotonicity on the prices.

We represent the discrete price function as a table $C_P$ with $c_{i,j}$ in the row entries and the columns as the probabilities. These are the same for all $c_{i,j}$. Thus the entries of $C_P$ can be thought of as “price tags” that The Lobby must pay in order to change the probabilities of voting.

The Lobby also has an integer-valued budget $B$ and an “agenda,” which we will denote as a vector $\mathbf{Z} \in \{0, 1\}^n$, where $n$ is the number of issues, containing the outcomes The

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2This model was first discussed by Reinganum [Rei83] in the continuous case and we translate it here to the discrete case. This will allow us to present algorithms for and the complexity analysis of the problem.
Lobby would like to see on the corresponding issues. Without loss of generality, The Lobby’s agenda is all “yes” votes, so the target vector is $\vec{Z} = 1^n$.

In a later section we define three ways by which The Lobby can choose to bribe voters. These methods are significant to both the complexity and algorithmic results that are presented in the Results section. We also define three methods for determining whether The Lobby has succeeded in a particular problem instance. Like the bribery methods the evaluation criteria play an important role in determining the complexity of the problems themselves.

**Example 1** We set up this problem instance by with $k = 9$, $m = 2$ (number of issues), and $n = 3$ (number of voters). We will use this as a running example for the rest of this paper. In addition to the above definitions for $k$, $m$, and $n$, we also give the following matrix for $P$. (Note that this example is normalized for an agenda of $\vec{Z} = 1^3$, which is why The Lobby has no incentive for lowering the acceptance probabilities, so those costs are omitted below.)

A $P$ matrix of:

$$P = \begin{pmatrix}
  r_1 & r_2 & r_3 \\
v_1 & 0.8 & 0.3 & 0.5 \\
v_2 & 0.4 & 0.7 & 0.4
\end{pmatrix}$$

A $C_P$ matrix of costs:

<table>
<thead>
<tr>
<th>$c_{i,j}$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1,1}$</td>
<td>0</td>
<td>100</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{1,2}$</td>
<td>0</td>
<td>70</td>
<td>100</td>
<td>140</td>
<td>310</td>
<td>520</td>
<td>600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{1,3}$</td>
<td>0</td>
<td>15</td>
<td>25</td>
<td>70</td>
<td>90</td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{2,1}$</td>
<td>0</td>
<td>30</td>
<td>40</td>
<td>70</td>
<td>120</td>
<td>200</td>
<td>270</td>
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<tr>
<td>$c_{2,2}$</td>
<td>0</td>
<td>10</td>
<td>40</td>
<td>90</td>
<td></td>
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</tr>
<tr>
<td>$c_{2,3}$</td>
<td>0</td>
<td>70</td>
<td>90</td>
<td>180</td>
<td>300</td>
<td>450</td>
<td></td>
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</tr>
</tbody>
</table>

In the section on bribery methods we establish three ways that The Lobby can influence the voters. These will referred to by $B_i$ where $1 \leq i \leq 3$. In this way we can more specifically define the ways in which The Lobby has influence over the voters. In addition to the three bribery methods described in the bribery methods section we also define three ways in which The Lobby can win a set of votes. These evaluation criteria are defined later and will be referred to as $C_j$, where $1 \leq j \leq 3$. These criteria are important because votes counted in various ways can result in distinct outcomes depending on voting and evaluation systems [MW93].

Using the above criteria we can now define the first problem that we will study in this work, $B_i-C_j$ PROBABILISTIC LOBBYING PROBLEM (or $B_i-C_j$-PLP, for short), is
defined as follows:

**Problem Definition:**
Name: $B_i-C_j$-PLP
Given: A probability matrix $P \in \mathbb{Q}^{m \times n}_{[0,1]}$ with table $C_P$ of price functions and a target vector $\vec{Z} \in \{0, 1\}^n$ of votes and budget $B$.

**Question:** Is there a way for The Lobby to influence $P$ (using the type of bribery $B_i$ and the evaluation criterion $C_j$, where $i, j \in \{1, 2, 3\}$, without exceeding budget $B$) such that the result of the votes on all issues equals $\vec{Z}$?

### 3.2 Bribery Methods

We first formalize the different bribery methods by which The Lobby can influence votes on issues. In our effort to define how money is given to a voter it is necessary to strike a balance between reality and modeling. There are a myriad of ways that individuals, groups, and governments can work to affect how a voter thinks about an issue. It is not possible to describe all these methods in this work (or, possibly, in formal mathematics). What we are trying to provide with this section is different ways that are, at the same time, feasible to model and contain some truth in a real world analog.

Previous complexity work only considered a deterministic case of this problem where money was distributed on a row-wise basis ($B_3$ below) [EHRS07, CFRS07]. While other work only looked at direct lobbying on a per-voter basis ($B_1$ below) [Rei83]. In this work we attempt to formalize all these previous methods in our model as well as a new way in which The Lobby can inject money into the system ($B_2$ below). We feel that these three methods, while not all completely realistic, comprise a general view of how lobbying can happen.

#### 3.2.1 Micro-Bribery ($B_1$)

The first method at the disposal of The Lobby for influencing voters is what we will call *micro-bribery*. We define micro-bribery to be the editing of individual elements of the $P$ matrix according to the costs in the $C_P$ matrix. This is akin to The Lobby picking not only which voter to influence but also which issue to influence that voter. This bribery criterion allows the most flexible version of bribery. One could think of this as private donations made to candidates in support of specific issues.

This method of bribery gives the most general version of the problem. The Lobby can decided which voters *and* which issues to influence individually. This can, in the United States political system, be though of as direct campaign contributions. This type of direct
contribution, or bribe, is common as The Lobby may have the utmost discretion with its funding.

### 3.2.2 Issue Bribery ($B_2$)

The second method at the disposal of The Lobby for influencing voters is issue bribery. We can see from the $P$ matrix that each column represents how the voters think about a particular issue. In this method of bribery The Lobby can pick a column of the matrix and edit it according to some amount. The money will be equally distributed among all the voters and the voter probabilities will move accordingly. So, for an amount of, say, $d$ dollars each voter receives a fraction of $d/m$ and their probability of voting “yes” changes accordingly. This can be thought of as special interest group donations. Special interest groups such as PETA focus on issues and dispense their funds across an issue rather than by voter. The bribery could be funneled through such groups.

This method of bribery can, in a lose sense, be though of as The Lobby working through special interest groups. The Lobby wants voter’s minds (probabilities) changed on certain issues but cannot give different amounts of money to individual voters. Therefore, The Lobby’s only choice is to throw a lump sum of money at an issue and change voter’s minds this way. The real world analog, loosely, can be thought of as Political Action Committee (PAC’s, in the United States). These groups focus their money, advertising, and influence over a small subset (or even single) issues.

### 3.2.3 Voter Bribery ($B_3$)

The third and final method at the disposal of The Lobby for influencing voters is voter bribery. We can see from the $P$ matrix that each row represents what individual voters think about all the issues on the docket. In this method of bribery, The Lobby picks a voter and then pays to edit the entire row at once with the funds being equally distributed over all the issues. So, for an amount of, say, $d$ dollars a fraction of $d/n$ is spent on each issue, which moves accordingly. The cost of moving the voter is generated using the $C_P$ matrix as before. This method of bribery is analogous to “buying” or pushing a single politician or voter. The Lobby donates so much money to the individual voter that he or she has no choice but to move their votes toward The Lobby’s agenda.

This method of bribery can, in a lose sense, be thought of as The Lobby “buying off” a single voter. The idea is that The Lobby donates so much money to a single candidate that they cannot help but be influenced in some manner by this money. This method has real world analogs but the distribution of funds is not entirely realistic. However, we chose this method as it is the same method employed in the OL problem [CFRS07].
3.3 Evaluation Criteria

Defining criteria for how an issue is won is the next important step in formalizing our models. Again, in our effort to define what it means to “win a vote” it is necessary to strike a balance between reality and what can realistically be modeled. Due to the stochastic nature of our problem we feel that it is reasonable to have different metrics for the determination of winning.

Previous complexity work only considered a deterministic version of this problem and therefore used, as a metric for winning, a strict majority [EHRS07, CFRS07]. We include this method in a probabilistic sense due to its previous use. We believe the other methods proposed below allow for greater fidelity of the model when considered in a probabilistic environment.

3.3.1 Strict Majority (C1)

For each issue a strict majority (> \( \frac{m}{2} \)) of the individual voters have probability at least some threshold, \( t \), of voting according to the agenda.

This corresponds to, in a deterministic case, having a majority vote. This simple voting metric is how votes are registered in the United States. This metric, again in the deterministic case, is also how winning is determined in OL [EHRS07, CFRS07]. We extend this idea for our model by allowing a parameter, \( t \), to represent how confident we are that each voter will vote according to The Lobby’s agenda. In this way we feel that this method is a valid evaluation criteria when voting is considered probabilistically.

In our running example we would say that the result of all the votes can be described as \( \vec{X} = (0, 0, 0) \) with \( t = 0.5 \) because none of the issues has a strict majority of voters over 0.5 on any issue.

3.3.2 Winning Majority (C2)

For each issue, \( r_j \), of a given \( P \), we define:

\[
\overline{p}_j = \frac{\sum_{i=0}^{m} p_{i,j}}{m}
\]

We can now evaluate the vote to say that \( r_j \) is accepted if and only if \( \overline{p}_j > t \) where \( t \) is some threshold.

This corresponds to, in the deterministic case, having a simple majority vote. This model, however, is more powerful in the probabilistic case due to the fact that it incorporates all the voter’s probabilities into a single number. We feel that this measure, while not incredibly realistic, gives a better overall picture of the voter’s preferences in our model.
This would, in our running example, with $t = 0.5$, give us a result vector of $\vec{X} = (1, 0, 0)$.

### 3.3.3 Possible Futures Majority (C₃)

Given all the possible outcomes of a vote (in the probabilistic sense) we say that The Lobby has achieved a win on an issue ($r_i$) if and only if the sum of the probabilities of the winning futures (as defined by a strict majority) is above some threshold $t$.

For each issue, $j$, we define:

$$PW_j = \sum_{s \subseteq \{1...m\}} \left( \prod_{i \in s} p_{i,j} \cdot \prod_{e \notin s} (1 - p_{e,j}) \right)$$  \hspace{1cm} (2)

We can now evaluate the vote to say that $r_j$ is accepted if and only if $PrW_j > t$. The equation above calculates the sum of the probabilities that a strict majority of voters will vote yes on the $j$th issue. A brute-force evaluation of $PrW_j$ requires the enumeration of all subsets of possible voter outcomes. This evaluation criterion can be thought of as flipping coins to determine each voter’s votes. Each voter/issue pair can be thought of as a weighted coin. Instead of having equal probability of coming up “heads” or “tails,” the probabilities correspond to the voter’s probability of voting on a certain issue.

We then want to determine the probability that the voter/issue coins land so that The Lobby receives a majority of yes votes on that issue. We then say that this issue is a win if and only if the probability is above some threshold $t$.

This method, while computationally complex, provides the best analog to the real world. Even with the massive amount of lobbying that The Lobby has performed, it is still possible that the voters will not vote as The Lobby wants. Because of this inherent uncertainty in the problem the best way to determine the outcome is to look at all possible futures.

### 3.4 Issue Weighting

Our first modification to the model will bring in the concept of issue weighting. It is reasonable to surmise that certain issues will be of more importance to The Lobby than others. For this reason we will allow The Lobby to specify weights to the issues that they deem more important. These weights will be defined for each issue.

We will specify these weights as a vector $\vec{W} \in \mathbb{Z}^n$ with size $n$ equal to the total number of issues in our problem instance. The higher the weight, the more important that
particular issue is to The Lobby. Along with the weights for each issue we are also given an objective value $O \in \mathbb{Z}^+$ which is the minimum weight The Lobby wants to see passed. In this way we allow The Lobby to define which issues are most important to its agenda.

Since this is a partial ordering, it is possible for The Lobby to have an ordering such as: $w_1 = w_2 = \cdots = w_n$. If this is the case than we can see that we are left with an instance of $B_i$-$C_j$-PLP. Now we have defined a special case of $B_i$-$C_j$-PLP, which we will refer to as the $B_i$-$C_j$-PROBABILISTIC LOBBYING PROBLEM WITH ISSUE WEIGHTING or $B_i$-$C_j$-PLP WITH ISSUE WEIGHTING for short. We can now define this problem formally below.

Problem Definition:
Name: $B_i$-$C_j$-PLP WITH ISSUE WEIGHTING
Given: A probability matrix $P \in \mathbb{Q}_{m \times n}$ with table $C_P$ of price functions and a lobby target vector $\vec{Z} \in \{0, 1\}^n$, a lobby weight vector $\vec{W} \in \mathbb{Z}^n$, an objective value $O \in \mathbb{Z}^+$, and budget $B$.
Question: Is there a way for The Lobby to influence $P$ (using the type of bribery $B$, and the evaluation criterion $C_j$, where $i, j \in \{1, 2, 3\}$, without exceeding budget $B$) such that the result of the votes on all issues equals $\vec{Z}$ and the selections from $\vec{W} \in \mathbb{Z}^n$ is $\geq O$?

4 Complexity Analysis

We now provide a formal complexity analysis of the models and combinations of evaluation and bribery criteria. Table 1 summarizes our results for $B_i$-$C_j$-PLP, $i, j \in \{1, 2, 3\}$.

4.1 Evaluation Complexity

We will start with proofs of complexity and some statements regarding the three evaluation criteria, $C_1$, $C_2$, and $C_3$. With these in hand it will be easier to show complexity classes for the other problem definitions we have discussed earlier.

Proposition 1 For a given $P$ matrix, we can evaluate whether or not it is a win in polynomial time for any evaluation criteria, $C_1$ and $C_2$.

Theorem 1 Determining if a given $P$ matrix is a win according to evaluation criterion $C_3$ is in $\mathsf{PP}$. 
Table 1: Complexity Results for $B_i$-$C_j$-PLP

<table>
<thead>
<tr>
<th>Bribery Criterion</th>
<th>Evaluation Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_1$</td>
</tr>
<tr>
<td>$B_1$</td>
<td>P</td>
</tr>
<tr>
<td>$B_2$</td>
<td>P</td>
</tr>
</tbody>
</table>

4.2 $B_i$-$C_j$-PLP Complexity

Here we provide complexity results for $B_i$-$C_j$-PLP. Some of these results have been previously published [CFRS07] and we will mention them below.

**Theorem 2** $B_1$-$C_1$-PLP and $B_1$-$C_2$-PLP are in P.

Christian et al. [CFRS07] proved that a similar problem to ours is W[2]-complete; see Downey and Fellows [DF99] for the definition of this complexity class. We state this problem here as is common in parameterized complexity:

**Problem Definition:**

**Name:** Optimal Lobbying (OL, for short)

**Given:** An $m \times n$ matrix $E$, a positive integer $k$, and a length $n$ 0/1 vector $\vec{X}$. Each row of $E$ represents an agent. Each column represents an issue in the election. The 0/1 values represent the natural inclination of the agent with respect to the issue put to a vote. The vector $\vec{X}$ represents the outcome preferred by The Lobby.

**Parameter:** $k$ (representing the number of agents to be influenced).

**Question:** Is there a choice of $k$ rows of the matrix, such that these rows can be changed such that in each column of the resulting matrix, a majority vote in that column yields the outcome targeted by The Lobby?

This problem was shown to be W[2]-complete by a reduction from $k$-DOMINATING SET (showing the lower bound) and to INDEPENDENT-$k$-DOMINATING SET (showing the upper bound). The voter/issue matrix was extended to construct a graph whereby the agenda could be achieved if and only if there existed a dominating set of size $k$ for the graph [CFRS07]. We can leverage this result by showing that OL is a special case of $B_3$-$C_1$-PLP and thus polynomial-time reduces to $B_3$-$C_1$-PLP.

**Theorem 3** Both $B_3$-$C_1$-PLP and $B_3$-$C_2$-PLP are W[2]-hard.

**Theorem 4** $B_2$-$C_1$-PLP and $B_2$-$C_2$-PLP are in P.
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<table>
<thead>
<tr>
<th>Bribery Criterion</th>
<th>Evaluation Criterion</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>NPP</td>
</tr>
<tr>
<td>B2</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>NPP</td>
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</tbody>
</table>

Table 2: Complexity Results for B_i-C_j-PLP WITH ISSUE WEIGHTING

Proof. B_1-C_3-PLP and B_2-C_3-PLP are in NPP.

Theorem 5 B_3-C_3-PLP is W[2]-hard.

4.3 B_i-C_j-PLP WITH ISSUE WEIGHTING Complexity

Table 2 summarizes our results for B_i-C_j-PLP WITH ISSUE WEIGHTING, i, j ∈ {1, 2, 3}.

Theorem 6 B_1-C_1-PLP WITH ISSUE WEIGHTING and B_1-C_2-PLP WITH ISSUE WEIGHTING are in P.

Theorem 7 For each j ∈ {1, 2, 3}, B_3-C_j-PLP WITH ISSUE WEIGHTING is W[2]-hard.

Theorem 8 B_2-C_1-PLP WITH ISSUE WEIGHTING and B_2-C_2-PLP WITH ISSUE WEIGHTING are in P.

Theorem 9 B_1-C_3-PLP WITH ISSUE WEIGHTING and B_2-C_3-PLP WITH ISSUE WEIGHTING are in NPP.

5 Conclusions and Future Work

This paper introduces a novel perturbation to an existing problem and some beginning analysis for these problems. The formal models presented in this work will allow for further investigation into the theory of bribery in elections. The complexity results within serve as a starting point for the investigation of these problems and other that are currently being worked on by those in the computational social choice community.

These results and models are significant as they will serve as a starting point for future work. In most real systems bribery would have two competing actors, rarely is only one
actor attempting to bribe voters. For this reason we see extending this work into a two-
player game and evaluating the complexity in that model.

The political system of the United States maintains rules and regulations regarding
the methods used for donating money in elections. We feel that since this is a regulated
system it can be modeled effectively and lead to insights in not only the area of social
choice but government and politics as well. We will also work to model this system as
effectively as possible in future work as well.

References


