Real-time Computation of Data Depth Using the Graphics Pipeline

Suresh Venkatasubramanian

AT&T Labs–Research

The Interplay Between Analysis and Visualization

- Most methods for computing data depth solve the problem, and then visualize the answers.
- Much of data analysis is exploratory and interactive.
- Not only do we need fast solutions, we need ways of interacting with (possibly very large) data.

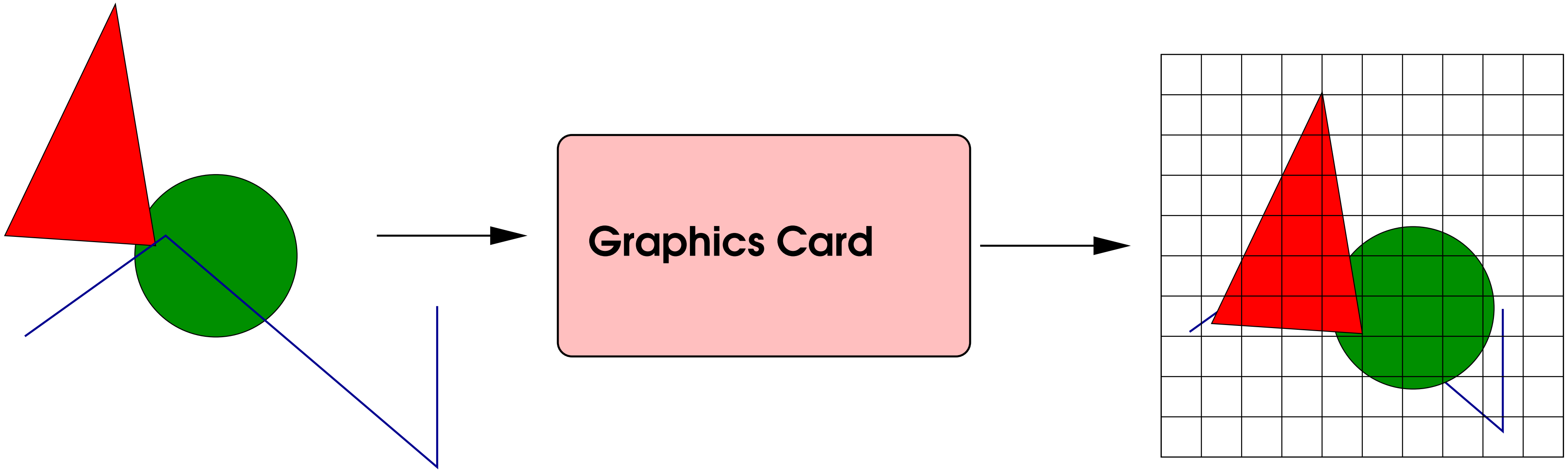
Can we combine analysis and visualization?

- Modern video cards have immense untapped computing potential.
- There is a growing trend in graphics and scientific computing to treat the video card as a fast co-processor.

AT&T

Graphics Cards Can Compute !

A graphics card takes a stream of objects (points, lines, triangles), and renders them on a screen.



Each pixel in the screen can be viewed as a small processing unit.

glBlend	$a = a \oplus b$
z-test	$a = \min(a, b)$

The Pipeline And Data Analysis, or Who Cares ?

- The interactive nature of data analysis makes speed a crucial consideration.
- Visualization is a key component: the use of graphics cards is natural.
- Demonstrable performance gain in areas like scientific computing.
- Serious efforts are underway to make the computations robust.
- The graphics card as a *streaming co-processor* is becoming common in diverse areas (graphics,robotics,numerical analysis, physical simulation, geometry).

Overview Of This Talk

A brief overview of the graphics pipeline

- How do we write programs for the graphics pipeline ?
- The architecture of a card.

Computing various data depth measures in hardware

- A simple algorithm for location depth.
- Implementation in hardware.
- Error Analysis and Performance
- Extensions to simplicial depth, Oja depth, **colored** location depth, and other depth measures.

Joint work with Shankar Krishnan (AT&T) and Nabil Mustafa (Duke)

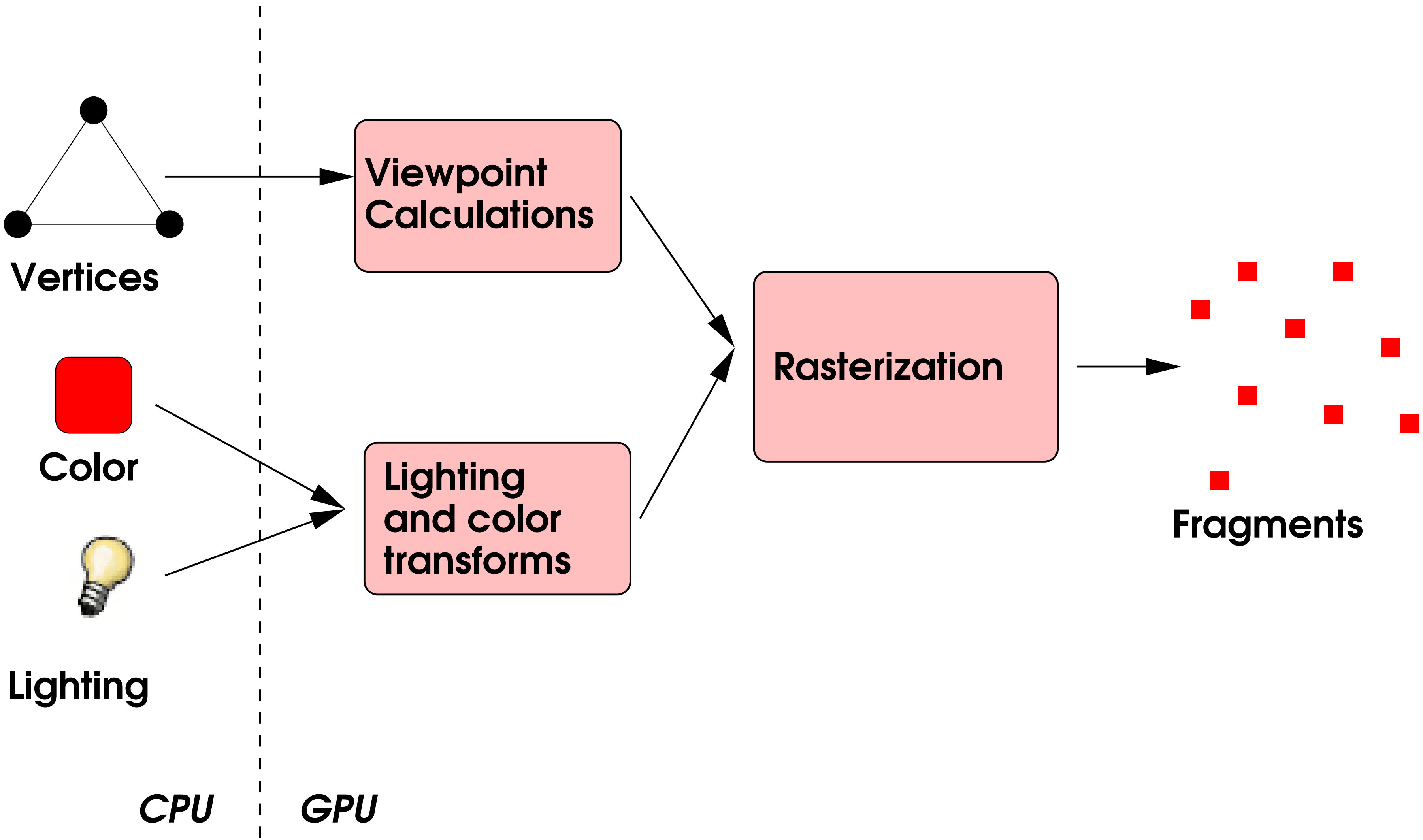
The Graphics Pipeline

AT&T

An Example OpenGL Program

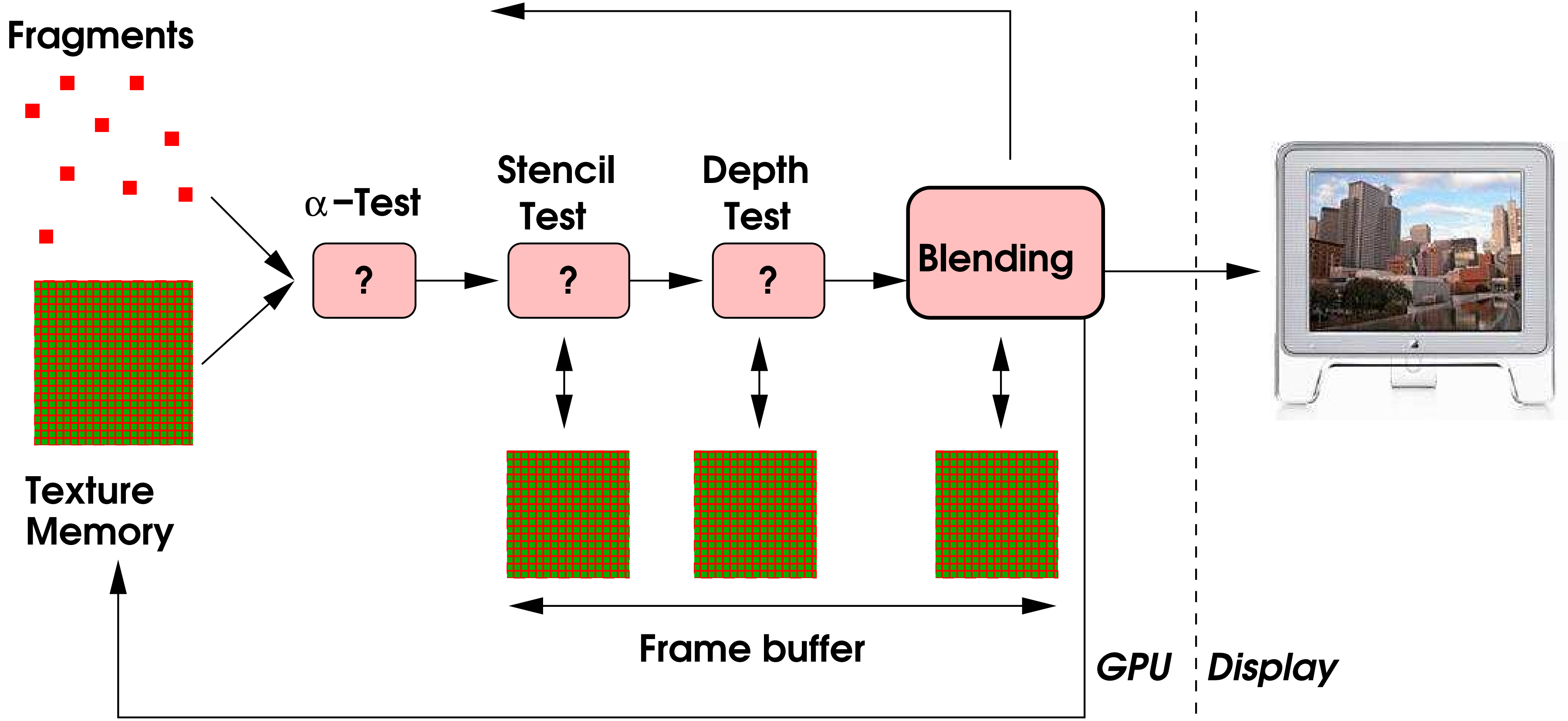
```
#include <gl.h>
...
glLight(..) // Set lighting
glOrtho(..)// Set viewpoint

// Now draw objects
glColor(1,0,0);
glBegin(GL_TRIANGLES)
glVertex(x1,y1,z1)
...
glEnd()

gcc triangle.cc -lGL
```

Processing Objects in the GPU: Step 1



The Fixed-Function Pipeline

Processing fragments in the GPU: Step 2



The Fixed-Function Pipeline

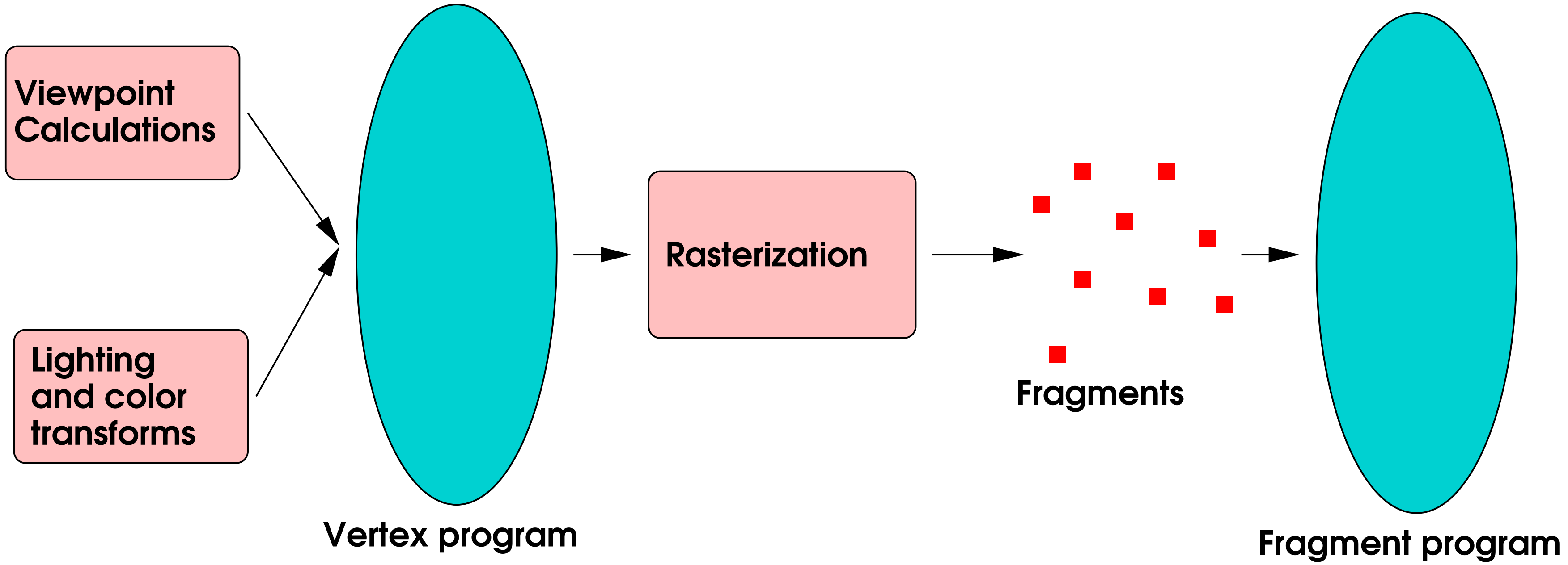
So where's the computation ?

- Stencil test
  if (buffer.stencil = K) continue
  else drop fragment.
- Depth test
  if (frag.depth < buffer.depth) continue
  else drop fragment.
- Blending operations
  buffer.color = buffer.color *op* fragment.color

– General arithmetic and boolean function for blending.
– General comparison functions.
– Convolution and histogramming operators.

Each pixel executes the same program in "parallel"

Programable Pipelines



- Vertex program executes on each vertex.
- Fragment program executes on each fragment.

Why is it so fast?

- The processor is highly optimized for *streaming* operations
- On a per-unit area basis, far more computational (ALU) units than a standard CPU.
- Because of FIFO nature of computation, almost non-existent memory latency.
- Immense *spatial parallelism*: each pixel can be thought of as a tiny parallel processor (all executing the same program).

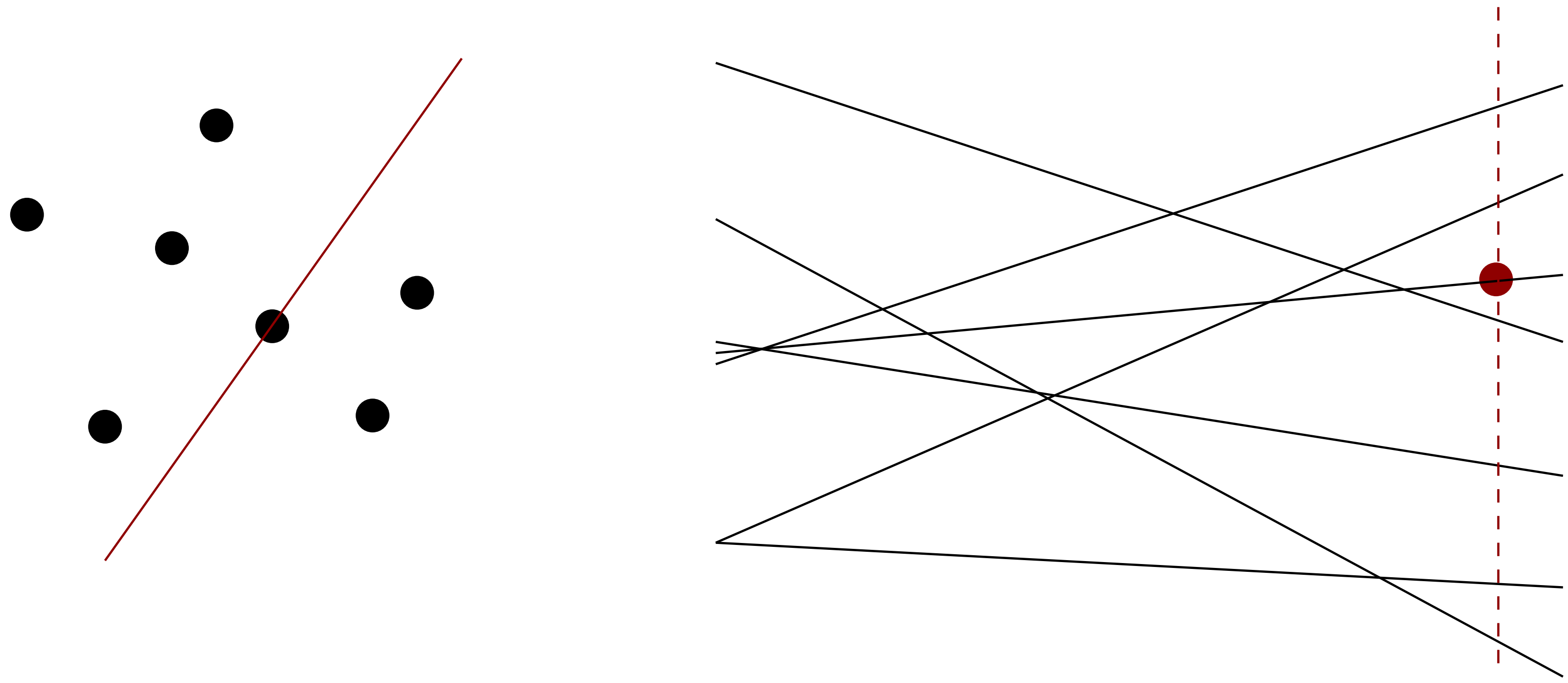
Cost Model:

- Each rendering pass is a "unit-cost" operation.
- Reading data back into main memory is expensive.
- Objective is to *minimize the number of passes*.
- Akin to standard notions of stream computations.

In each pass, only a fixed set of operations can be performed

Data Depth Computation

Halfspace Depth: Primal and Dual

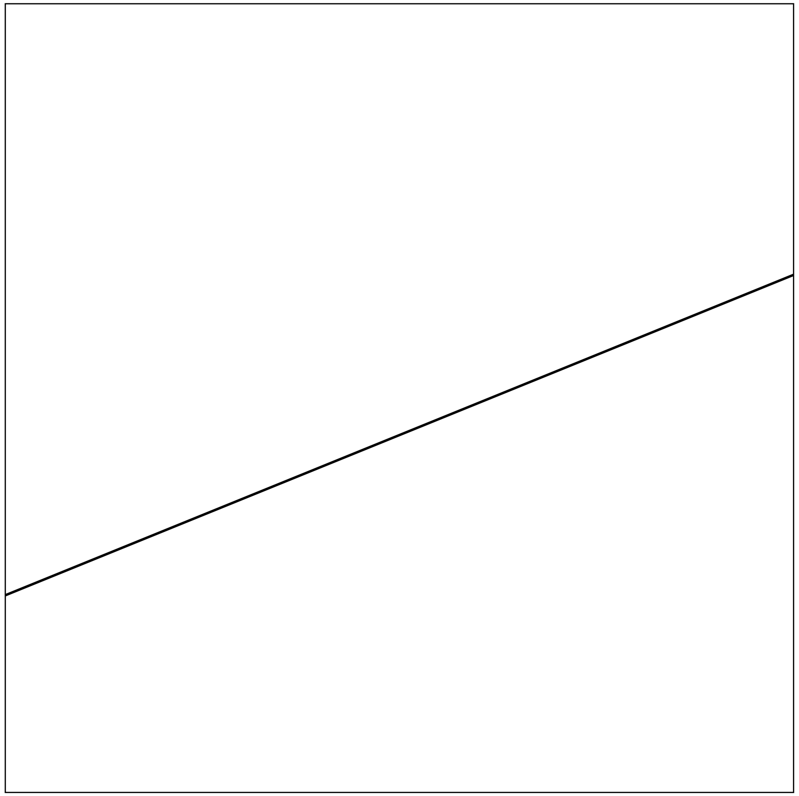


Depth of point in primal $\equiv$ Minimum depth of line in dual

Template For Hardware-Based Approach

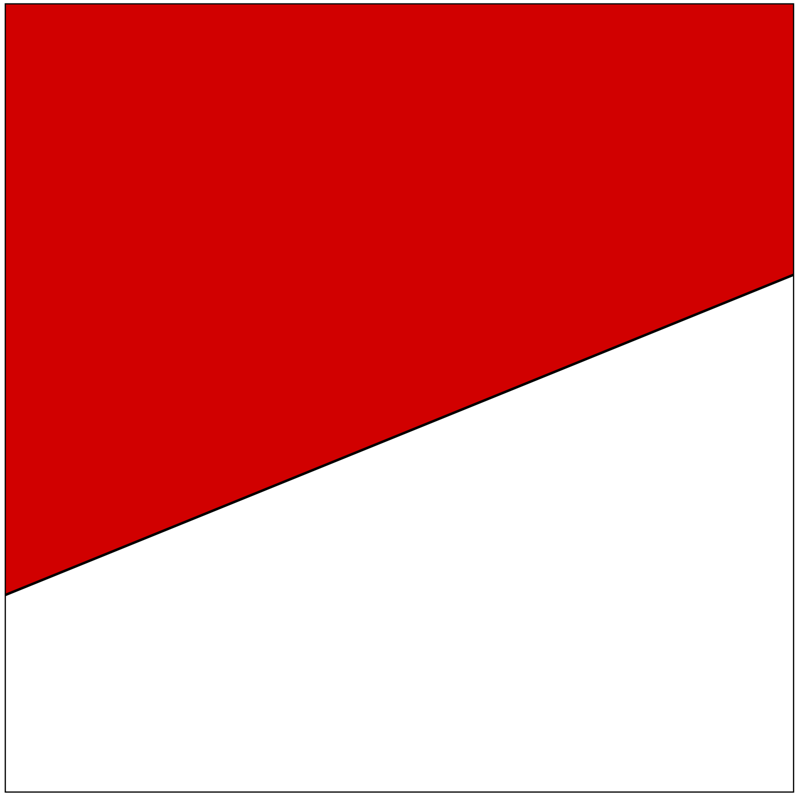
1. Construct dual arrangement. For each point in the dual, determine its depth.
2. For each point on a line in the dual, draw it in the primal plane with an associated value equal to its depth
3. At each point in primal, retain the smallest value encountered.

Step 1: Computing Dual Depth



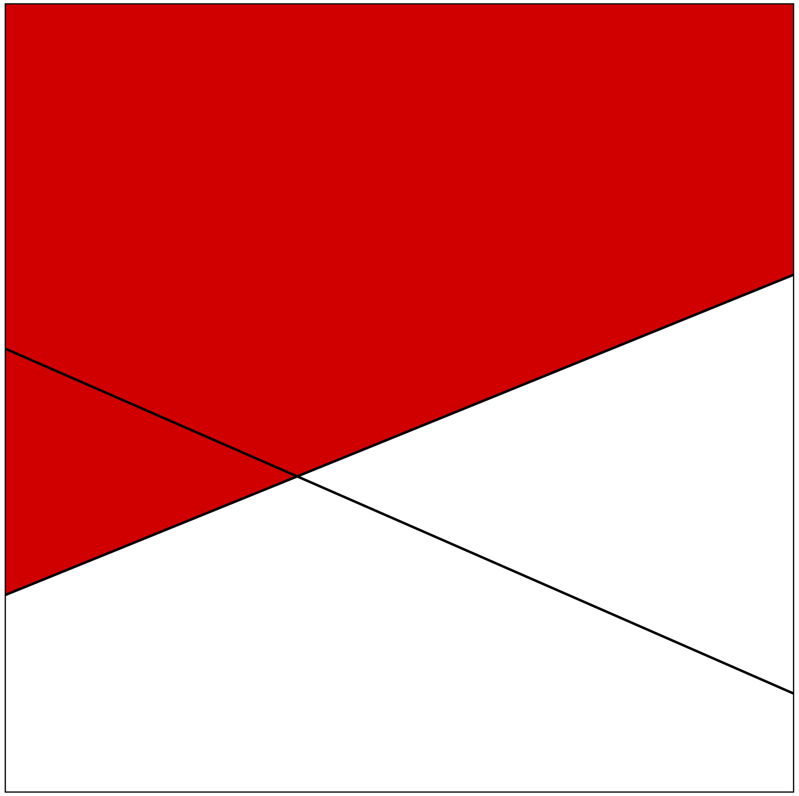
- Draw trapezoid for each line.
-
-
-

Step 1: Computing Dual Depth



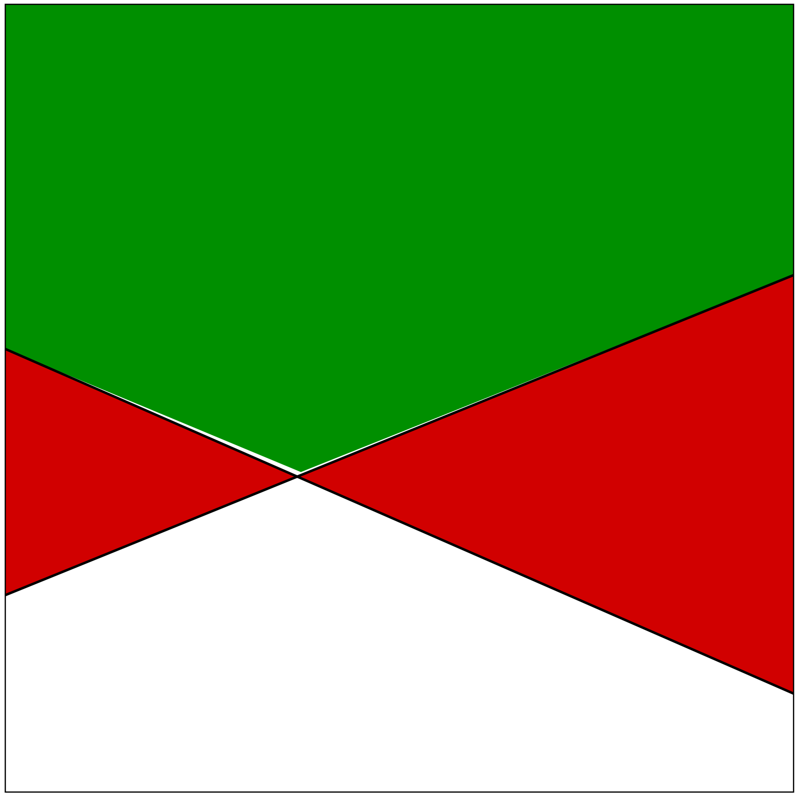
- Draw trapezoid for each line.
- Increment counter at each touched pixel.
-
-

Step 1: Computing Dual Depth



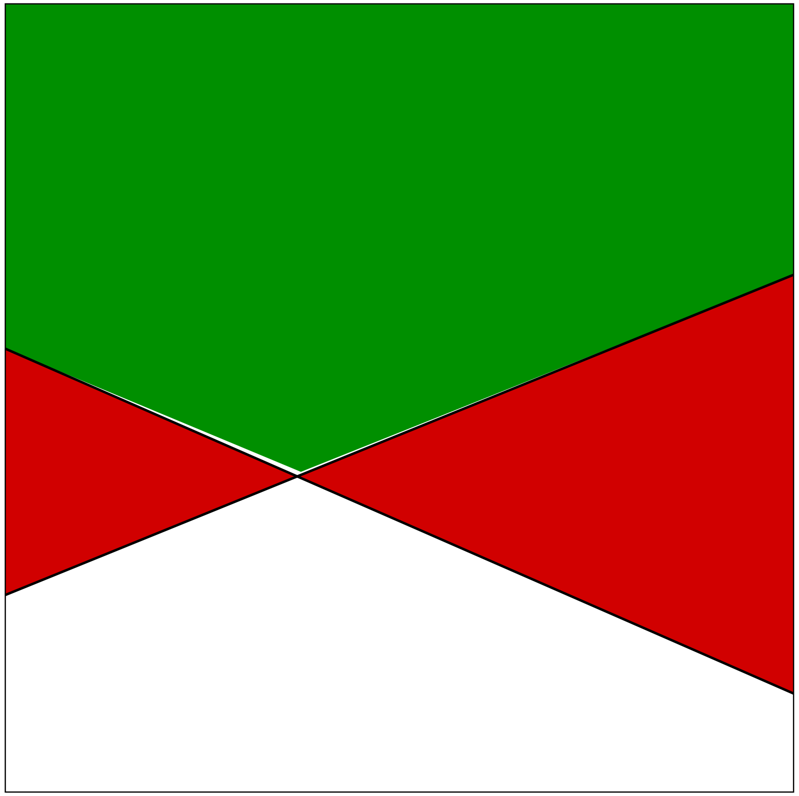
- Draw trapezoid for each line.
- Increment counter at each touched pixel.
- Draw next line.
-

Step 1: Computing Dual Depth



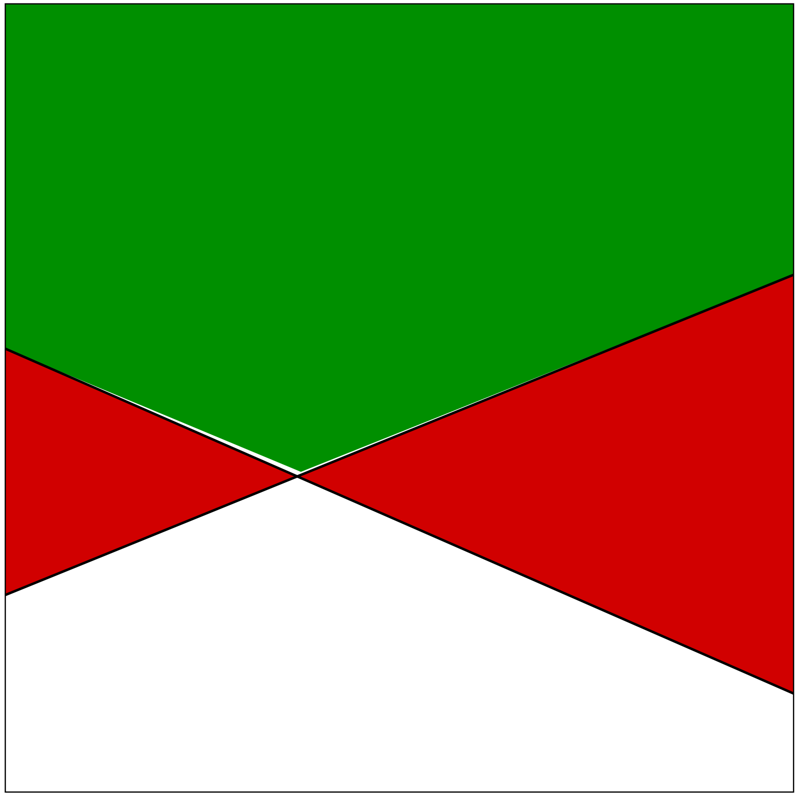
- Draw trapezoid for each line.
- Increment counter at each touched pixel.
- Draw next line.
- Increment counter as before.

Step 1: Computing Dual Depth



- Draw trapezoid for each line.
- Increment counter at each touched pixel.
- Draw next line.
- Increment counter as before.
- Repeat for all lines.

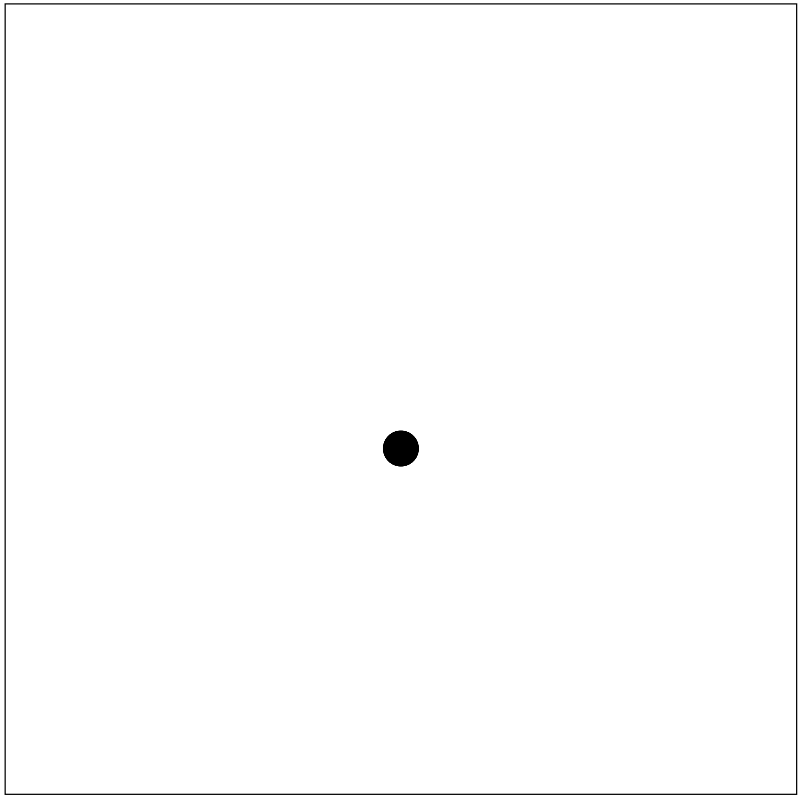
Step 1: Computing Dual Depth



- Draw trapezoid for each line.
- Increment counter at each touched pixel.
- Draw next line.
- Increment counter as before.
- Repeat for all lines.

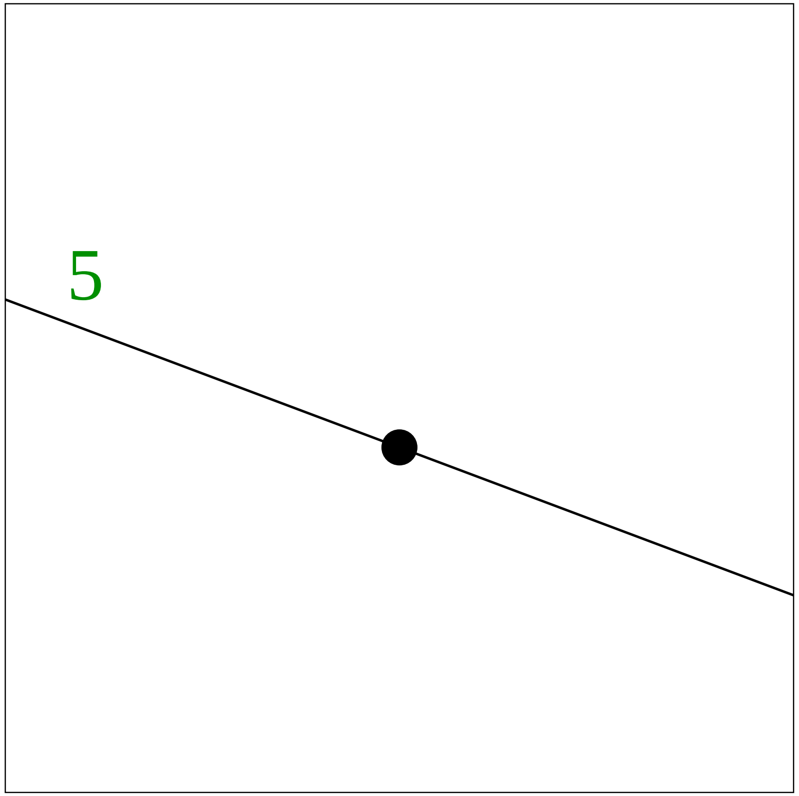
> At end of Step 1, all pixels in dual have correct depth

Step 2: Drawing In The Primal Plane



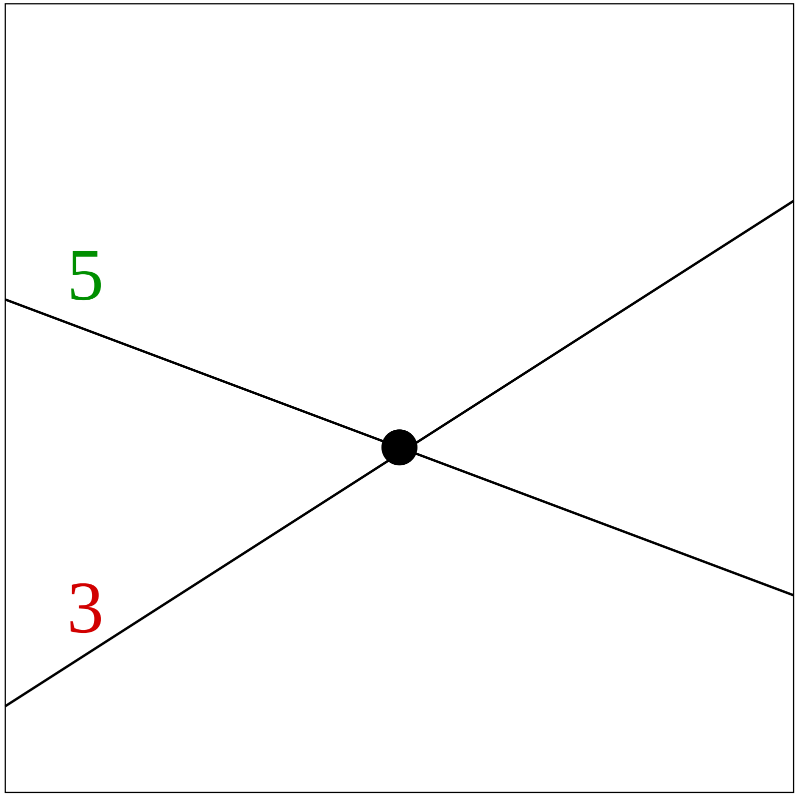
- For all points lying on dual lines...
-
-

Step 2: Drawing In The Primal Plane



- For all points lying on dual lines...
- Draw primal line with dual depth value.
-

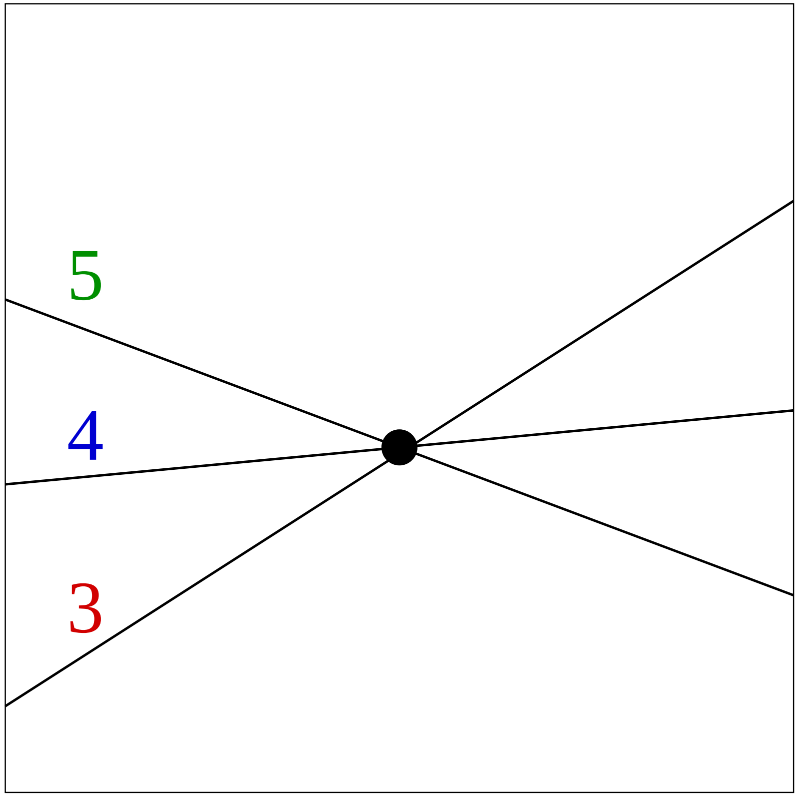
Step 2: Drawing In The Primal Plane



- For all points lying on dual lines...
- Draw primal line with dual depth value.
- Repeat...

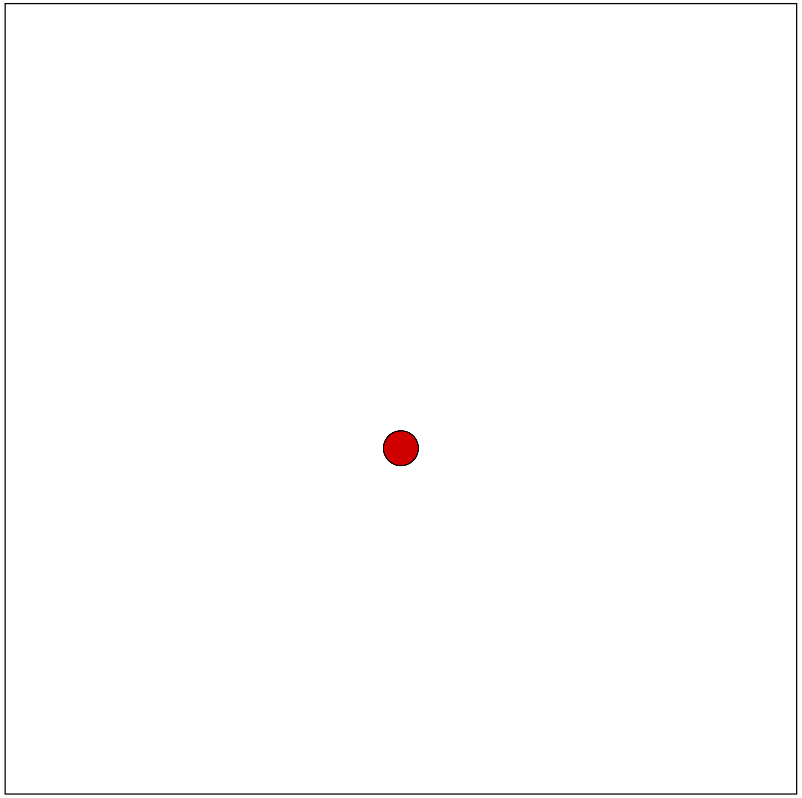
AT&T

Step 2: Drawing In The Primal Plane



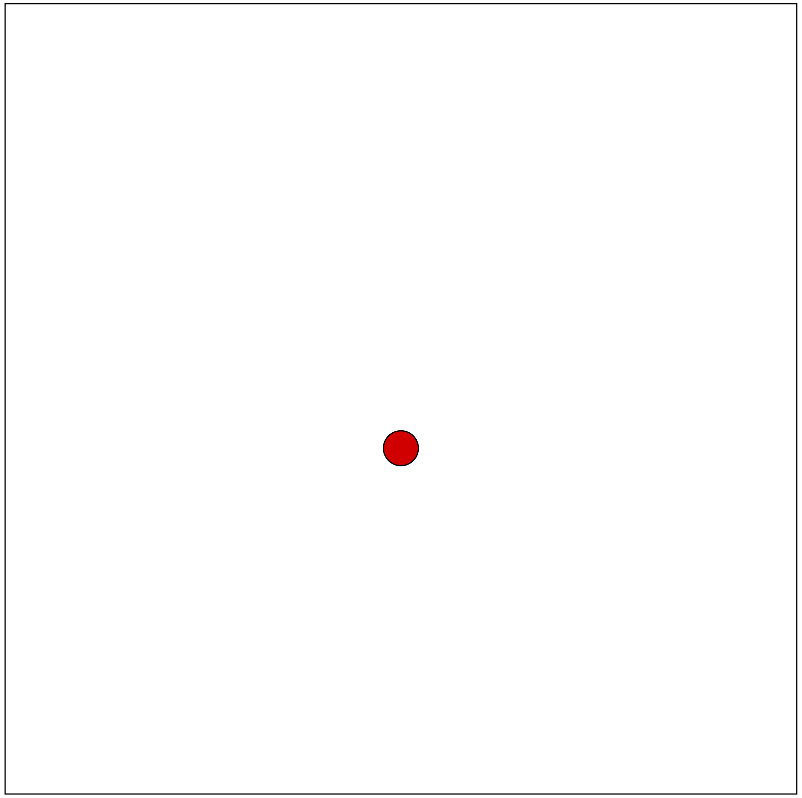
- For all points lying on dual lines...
- Draw primal line with dual depth value.
- Repeat...

Step 2: Drawing In The Primal Plane



- For all points lying on dual lines...
- Draw primal line with dual depth value.
- Repeat...
- Update pixel with minimum value seen.

Step 2: Drawing In The Primal Plane



- For all points lying on dual lines...
- Draw primal line with dual depth value.
- Repeat...
- Update pixel with minimum value seen.

At end of Step 2, all pixels in primal have correct depth

Bounded Duals

The screen has bounded size ! (typically $[-1,1]^2$)

If two points are almost above each other in the primal, the dual point is near ∞.

$\boxed{\text{Solution: use multiple duals.}}$

Definition. *A point is* bounded *if it lies in the range* $[-1,1] \times [-2,2]$.

Theorem. *There exists two dual mappings $\mathcal{D}_1, \mathcal{D}_2$ such that each intersection point in the dual arrangement is bounded in one of them.*

Proof Sketch: Each dual covers a different portion of the space of directions $\mathcal{S}^1$.
$\square$

Pixelization Error

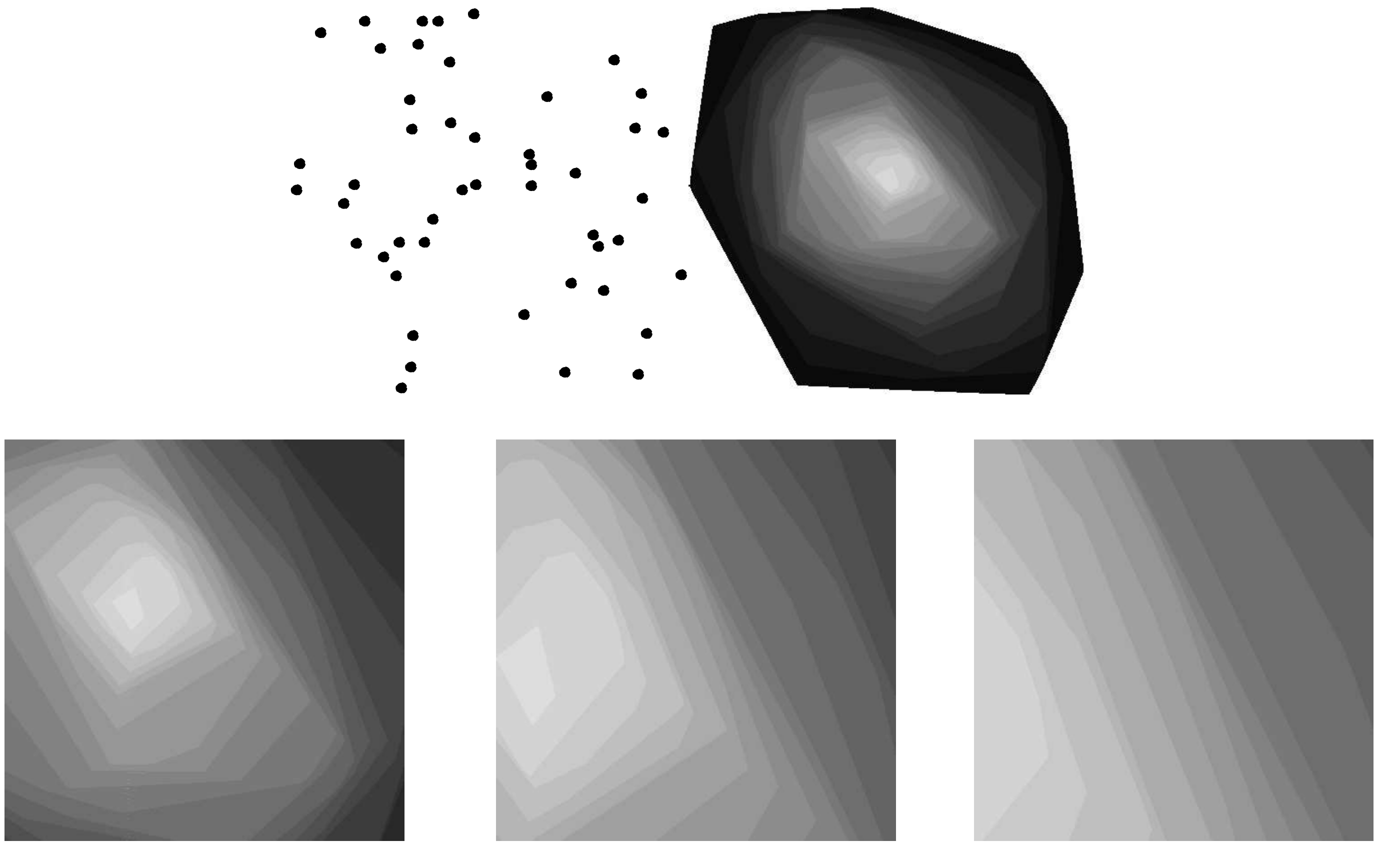
The screen has bounded resolution !. No exact solution is possible.

A Grid Algorithm:
For a **given** point set P, determine grid resolution W needed to compute an answer correctly.

- In general, the desired grid resolution is a simple function of the input point set.
- The higher the grid resolution, the slower the running time.

Levels of Detail

Because of the relative speed of computation, we can compute a fast approximate answer, and refine the answer by *zooming* into regions of interest.



Running Time

- Step 1 can be performed in two passes (one for each dual).
- One readback is required to obtain the dual depth values.
- Step 2 can also be performed in one pass. However, W^2 objects are rendered (which could be much larger than n).

Size	Running time (s)
50	0.6
100	0.9
500	1.9
1000	2.5
5000	6.3
10000	11.1

Running Time

- Step 1 can be performed in two passes (one for each dual).
- One readback is required to obtain the dual depth values.
- Step 2 can also be performed in one pass. However, W^2 objects are rendered (which could be much larger than n).

Size	Running time (s)
50	0.6
100	0.9
500	1.9
1000	2.5 (1.9)
5000	6.3 (3.2)
10000	11.1 (4.5)

Movie

Other Depth Measures

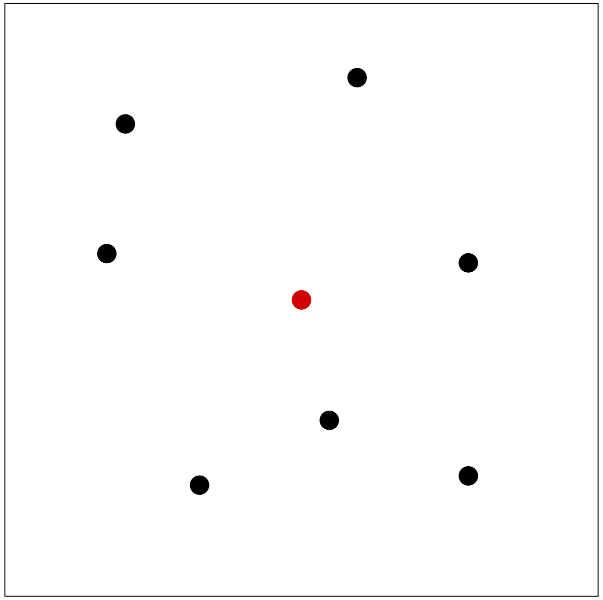
Can We Build Upon This ?

Various algorithm modules can be implemented in hardware:

- Envelope calculations.
- Dual mappings.
- Distance fields
  - Voronoi Diagrams
  - Power Diagrams
  - General Metrics
- Median (and k-selection in general)
  - Can be used to extract levels from an arrangement.

Simplicial Depth

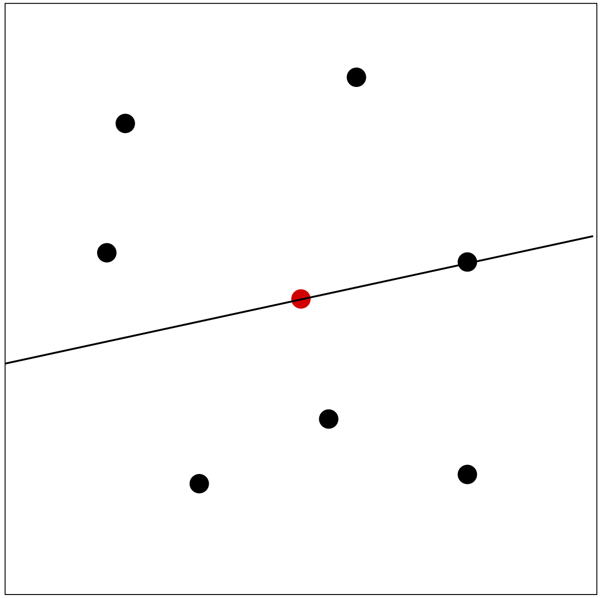
Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
-
-
-

Simplicial Depth

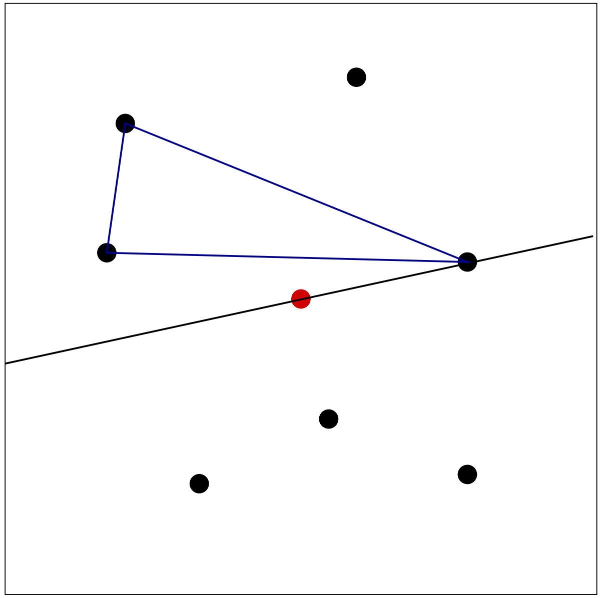
Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
- Take horizontal line ℓ through p and rotate anticlockwise till it hits a point q
-
-

Simplicial Depth

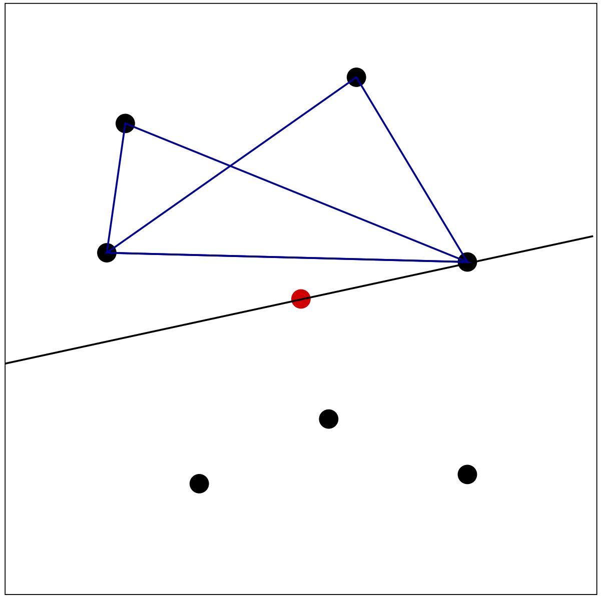
Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
- Take horizontal line ℓ through p and rotate anticlockwise till it hits a point q
- All pairs of points on either side of ℓ define simplices *not* containing p.
-

Simplicial Depth

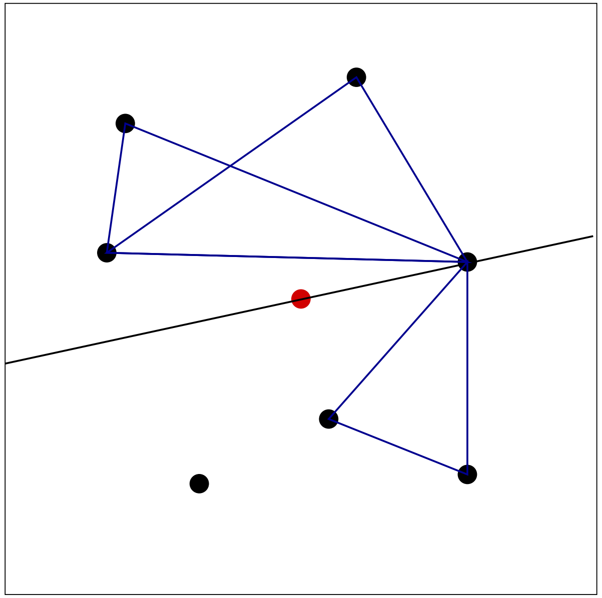
Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
- Take horizontal line ℓ through p and rotate anticlockwise till it hits a point q
- All pairs of points on either side of ℓ define simplices *not* containing p.
-

Simplicial Depth

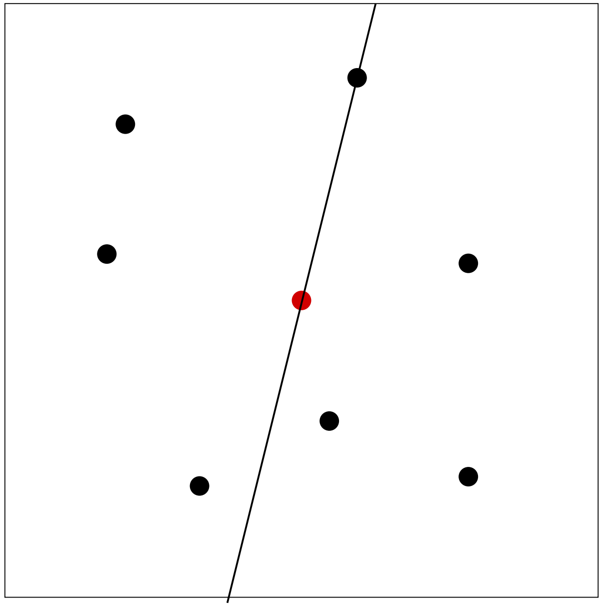
Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
- Take horizontal line ℓ through p and rotate anticlockwise till it hits a point q
- All pairs of points on either side of ℓ define simplices *not* containing p.
-

Simplicial Depth

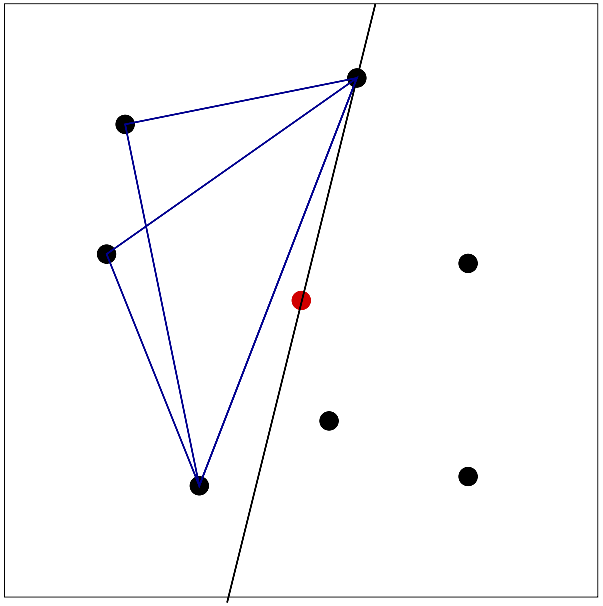
Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
- Take horizontal line ℓ through p and rotate anticlockwise till it hits a point q
- All pairs of points on either side of ℓ define simplices *not* containing p.
- Repeat for all lines

Simplicial Depth

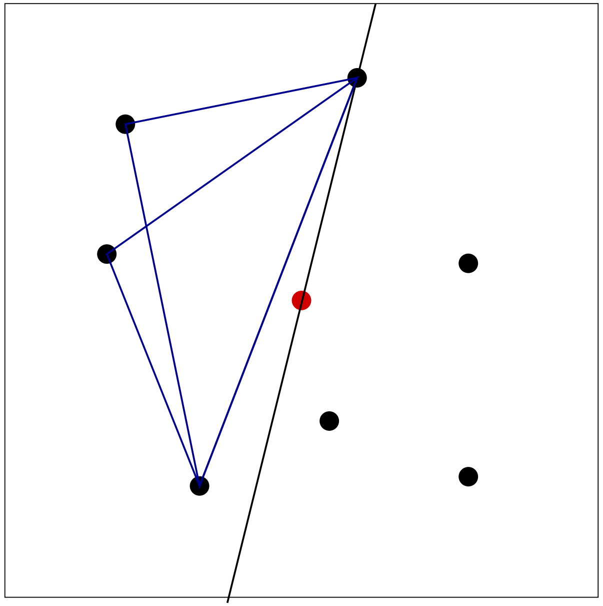
Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
- Take horizontal line ℓ through p and rotate anticlockwise till it hits a point q
- All pairs of points on either side of ℓ define simplices *not* containing p.
- Repeat for all lines

Simplicial Depth

Count number of simplices *not* containing p and subtract from $\binom{n}{3}$. **[RR96]**



- Sort points radially around p.
- Take horizontal line ℓ through p and rotate anticlockwise till it hits a point q
- All pairs of points on either side of ℓ define simplices *not* containing p.
- Repeat for all lines

Number of simplices one on side of ℓ can be computed from number of points on one side of ℓ.

Halfspace depth computation can be used to compute simplicial depth

Oja Depth

Definition (Oja Depth). *Given a point set P, the Oja depth of a point q is the sum of the volumes of all simplices of $P \cup \{q\}$ that contain q as a vertex.*

Contribution to the depth of q by the pair p, p' is precisely

$$d(q, l(p, p') \cdot d(p, p')/2$$

Thus the depth of a point q can be written as

$$\text{depth}(q) = \sum_{\ell \in \mathcal{L}} w_\ell \cdot d(q, \ell)$$

This defines a weighted *distance field*, where each object ℓ has weight w_ℓ, and the influence of ℓ is proportional to the distance from it.

All such distance fields can be computed in the graphics pipeline very efficiently.

AT&T

Other Measures

- Line of best fit
- LMS estimator.
- Best fit circle
- Colored halfspace depth
  - Each point is colored, and the depth of a point is expressed in terms of the number of *unique colors*.

Conclusions

- Graphics cards provide a natural fast platform for many kinds of geometric computations.
- For visualization- and interaction-heavy problems, this is a viable approach.
- When viewed from the perspective of streaming envelope computations, different problems can be solved using similar methods.

Future Directions:

- Other depth measures ? More sophisticated approaches that exploit the full power of the pipeline ?
- Underlying computational questions: What makes certain problems *streamable* ?