On Ramsey number of sparse uniform hypergraphs

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For a k-uniform hypergraph G, the Ramsey number $R(G,G)$ is the minimum positive integer N such that in every 2-coloring of edges of the complete k-uniform hypergraph K_N^k, there is a monochromatic copy of G. Say that a family $\mathcal{F}$ of k-uniform hypergraphs is $f(n)$-Ramsey if there is a positive C such that $R(G,G) \leq C\,f(n)$ for every $G \in \mathcal{F}$ with $|V(G)| = n$.

Burr and Erdós conjectured that for every d, the families $\mathcal{M}_d$ of graphs with maximum degree d and $\mathcal{D}_d$ of d-degenerate graphs are n-Ramsey. Recall that a graph is d-degenerate if each its subgraph has a vertex of degree at most d. Chvátal, Rödl, Szemerédi and Trotter proved the first conjecture.

The second conjecture is open. However, Kostochka and Rödl proved recently that $\mathcal{D}_d$ is n^2-Ramsey and then Kostochka and Sudakov proved that for every $\epsilon > 0$ and every positive integer d, the family $\mathcal{D}_d$ is $n^{1+\epsilon}$-Ramsey.

In this talk, we prove that for every $\epsilon > 0$ and for every fixed k and d, the family $\mathcal{D}_d^k$ of k-uniform hypergraphs with maximum degree at most d is $n^{1+\epsilon}$-Ramsey.