Social Choice Inspired Ordinal Measurement

Alexis Tsoukiàs

LAMSADE - CNRS, Université Paris-Dauphine

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Outline

What is the problem?

Suppose a device can bee in 4 states:

- BUS: Business as usual
- BAW: Be Aware
- CTC: Call The Cavalry
- RHA: Rush Away

monitoring 100 sensors providing binary information

Can we go the hard way?

There are 2^{100} possible combinations

There is no way we can produce an exhaustive association of each combination to each state.

What if we had 4 sensors providing an analogical signal?

Computationally the problem remains very hard, the combinations becoming infinite.

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Notation

- $A = \{a_1, \cdots a_j, \cdots a_n\}$ objects;
- $C = \{c_1, \cdots a_i, \cdots a_m\}$ classes; $c_i \triangleright c_{i+1}$;
- $X = X^1 \times X^2 \times \cdots X^n$ an attribute space;
- $\bar{a}_j = \langle a_j^1 \cdots a_j^n \rangle \ \in X$
- $\bar{c}_i = \langle c_i^1 \cdots c_i^n \rangle \ inX$

Option 1

$$a_j \in c_i \quad \Leftrightarrow \quad \bar{a}_i \sim \bar{c}_i$$

$\sim$ being a symmetric and reflexive binary relation (with an indifference or similarity meaning).

In this case $\bar{c}_i$ is a "prototype"

Option 2

$$a_j \in c_i \quad \Leftrightarrow \quad \bar{a}_i \succ \bar{c}_i$$

$\succ$ being an asymmetric and irreflexive binary relation (with a strict indifference or dissimilarity meaning).

In this case $\bar{c}_i$ is the "the minimum frontier" separating c_i from c_{i+1} and $\bar{c}_m = \langle \text{inf}(X^1), \cdots \text{inf}(X^n) \rangle$

The Borda path: counting values

$$x \succeq y \Leftrightarrow \sum_j r_j(x) \geq \sum_j r_j(y)$$

What do we need to know?

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What do we need to know?

the primitives: $\succeq_j \subseteq A \times A$
Differences of preferences:
- $(xy)_1 \succcurlyeq (zw)_1$
- $(xy)_1 \succcurlyeq (zw)_2$

The Condorcet path: counting preferences

$$x \succeq y \;\Leftrightarrow\; H_{xy} \geq H_{yx}$$

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the primitives: $\succeq_j \subseteq A \times A$
An ordering relation on $2^{\succeq_j}$

Positive and Negative reasons

$$a_j \succeq c_i \quad \Leftrightarrow \quad \frac{\sum w_{j\pm}}{\sum w_j} \geq \gamma \ \wedge \quad \neg \ v(c_i, a_j)$$

- w_j relative importance of each attribute ("weighted majority");
- $J_\pm \ = \ \{X^j : \ a_j^j \succeq c_i^j\}$;
- γ a threshold;
- $v(c_i, a_j) \ \Leftrightarrow \ \exists X^j : \ c_i^j \ggg a_j^j$;

Remarks

1. From a computational point of vue this is much easier: For each a_j we need at most m comparisons (m being the number of categories) which implies at most $n \times m$ comparisons in order to classify a whole set of n objects.
2. It is much more complicated to learn the various parameters such as w_j, γ, $\bar{c}_i$ etc...

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Conclusions

1. A reasonable way to perform ordinal measurement.
2. Nice axiomatisations.
3. Open preference learning problems