



# Sonar Placement in Ports and Waterways

Presented by:

Amir Ghafoori, PhD Student, Rutgers University, NJ

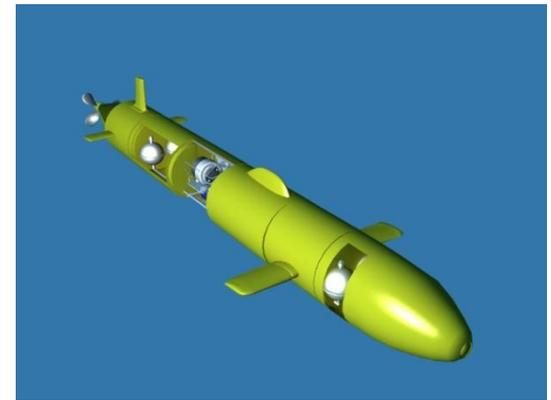
Advisor:

Tayfur Altiok, Professor, Rutgers University, NJ

CAIT-DIMACS Laboratory for Port Security

# Protect against terrorist attacks

- ▶ Divers
- ▶ AUV's
- ▶ Hull mounted objects



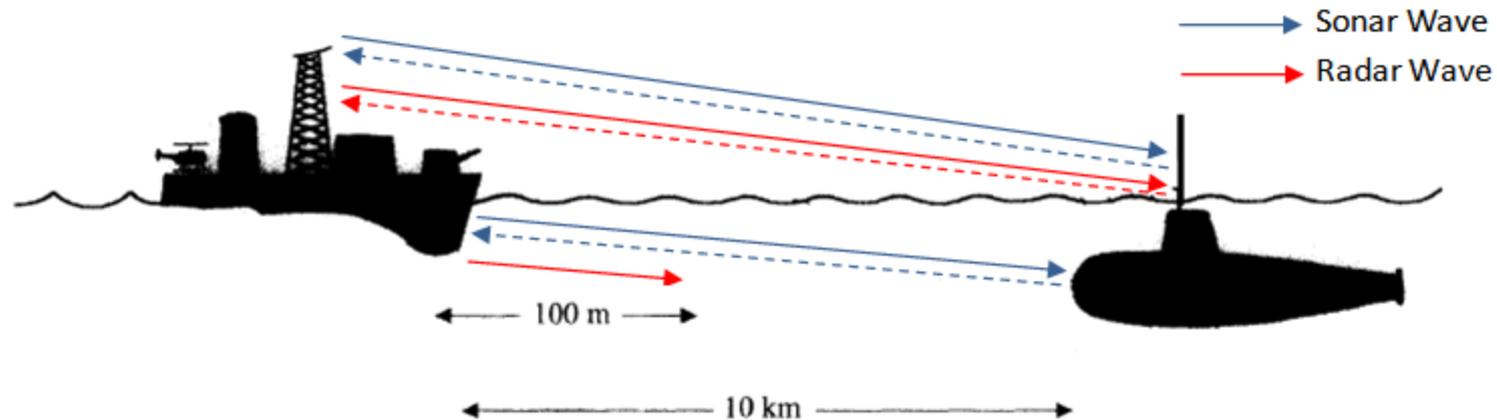
# Environment

- ▶ Infeasibility of electromagnetic sensors
- ▶ A type of sensor called SONAR (SOund NAvigation and Ranging) is used
- ▶ Sonars work based on sound waves



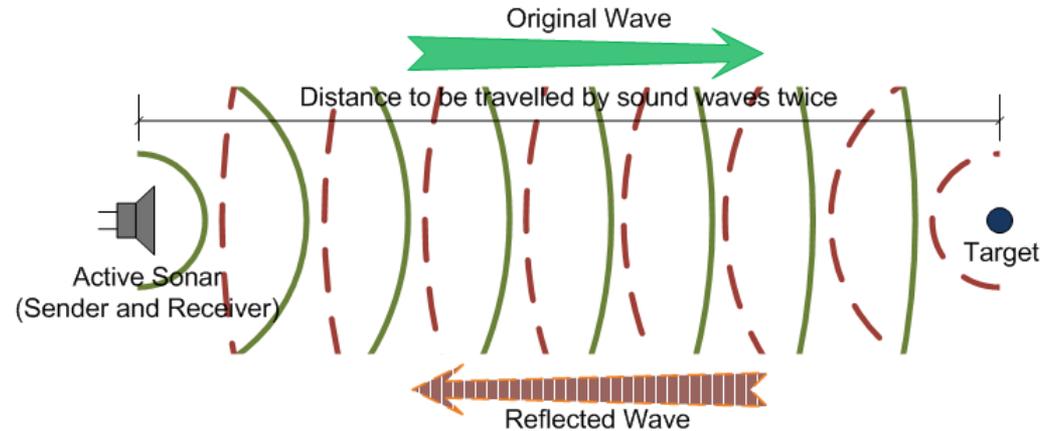
# Sonar's advantage over radar

- ▶ Electromagnetic waves get stuck in sea water
- ▶ Sound waves can travel in sea water even for tens of miles

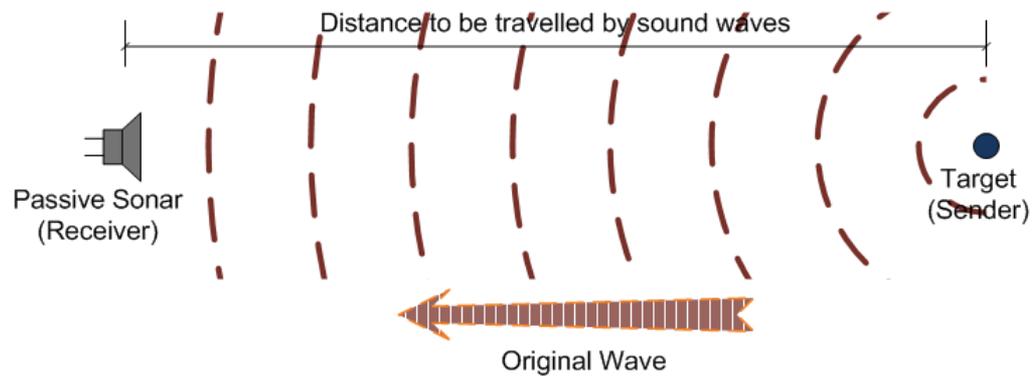


# Sonar Types

## ▶ Active



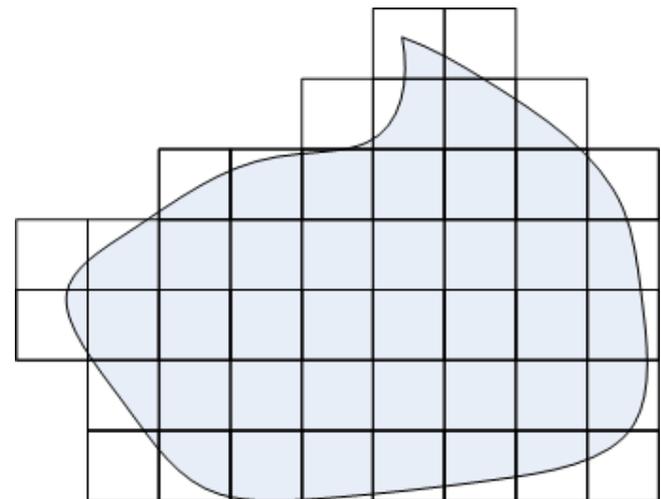
## ▶ Passive



# Model

- ▶ A Risk Minimization problem, with integer (binary) decision variables
- ▶ With:
  - Multiple coverage
  - Detection probability reduces by distance from the sonar
  - Various properties of sonars
  - Different sonar typesAre considered in the model.

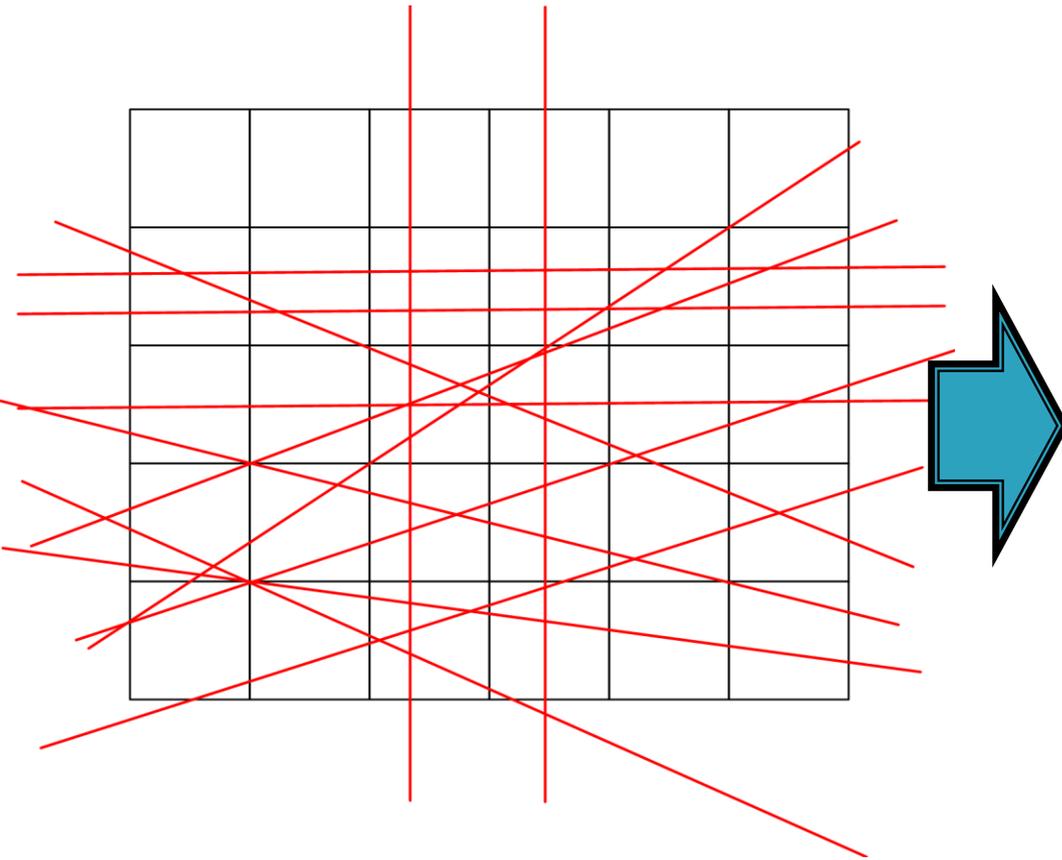
- ▶ Discretization



# Characteristic Values



# Assessing $a_{ij}$ Values



0	0	1	1	0	1
3	3	3	5	4	3
2	2	5	6	3	2
4	3	3	4	3	2
3	3	4	4	1	2

# A Simple Model!

# Notation

$a_{ij}$  = characteristic value of cell  $i, j$

$p$  = detection probability of a sonar

$c$  = budget for placing sonars

$n$  = number of cells a sonar can cover

$NC_{ij}$  = set of neighboring cells of  $i, j$  that a sonar positioned at cell  $i, j$  can cover including  $i, j$  itself

$$x_{ij} = \begin{cases} 1 & \text{if a sonar is placed at cell } i, j \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if cell } i, j \text{ is covered by a sonar} \\ 0 & \text{otherwise} \end{cases}$$

# Formulation

$$\text{Min} \sum_i \sum_j a_{ij} (1 - p \cdot y_{ij})$$



Objective  
Function

$$n \cdot x_{ij} \leq \sum_{k,l \in NC_{ij}} y_{kl} \quad (1)$$

$$y_{ij} \leq \sum_{k,l \in NC_{ij}} x_{kl} \quad (2)$$



Placement  
Constraints

$$c \sum_i \sum_j x_{ij} \leq b \quad (3)$$



Cost  
Constraint

$$x_{ij}, y_{ij} \in \{0,1\}$$



Decision  
Variables

# Explanation

$$\text{Min} \sum_i \sum_j a_{ij} (1 - p \cdot y_{ij})$$

This objective function minimizes a risk-like measure according to cell coverage and also the importance of cells ( $a_{ij}$  values)

$$R = E[C] = E[C \mid \text{Successful Attack}] \cdot P(\text{Successful Attack}) =$$

$$E[C \mid \text{Successful Attack}] \cdot P(\text{Successful Attack} \mid \text{Attack Happens}) \cdot P(\text{Attack Happens}) = C \cdot V \cdot T$$

# Main Model

- ▶ Featuring
  - Multiple detection of sonars
  - Range dependent detection probability
  - Various types of sonars

# Optimization Model

$$\text{Min } \sum_i \sum_j a_{ij} \{1 - [((1 - t_{ij}) \cdot \sum_n dp_n \cdot y_{ijn}) + t_{ij} \cdot dp_{\max}]\}$$

$$\text{St: } \quad d_{mn} \cdot x_{ijm} \leq \sum_{(k,l) \in N_{ijmn}} y_{kln} \quad \forall i, j, m, n$$

$$y_{ijn} \leq \sum_m \sum_{(k,l) \in N_{ijmn}} x_{klm} \quad \forall i, j, n$$

Placement  
Constraints

$$\sum_i \sum_j \sum_m c_m \cdot x_{ijm} \leq b$$

Cost  
Constraint

$$\sum_n y_{ijn} - 1 \leq M \cdot t_{ij} \quad \forall i, j$$

$$M(1 - t_{ij}) + \sum_n y_{ijn} \geq 2 \quad \forall i, j$$

Multiple  
Coverage  
Constraints

$$x_{ijm}, y_{ijn}, t_{ij} \in \{0, 1\}$$

Decision  
Variables

# Decision Variables

$$x_{ijm} = \begin{cases} 1 & \text{if a sonar of type } m \text{ is placed in cell } (i, j) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ijn} = \begin{cases} 1 & \text{if cell } (i, j) \text{ is covered by coverage type } n \\ 0 & \text{otherwise} \end{cases}$$

$$t_{ij} = \begin{cases} 1 & \text{if cell } (i, j) \text{ is covered by more than one sonar} \\ 0 & \text{otherwise} \end{cases}$$

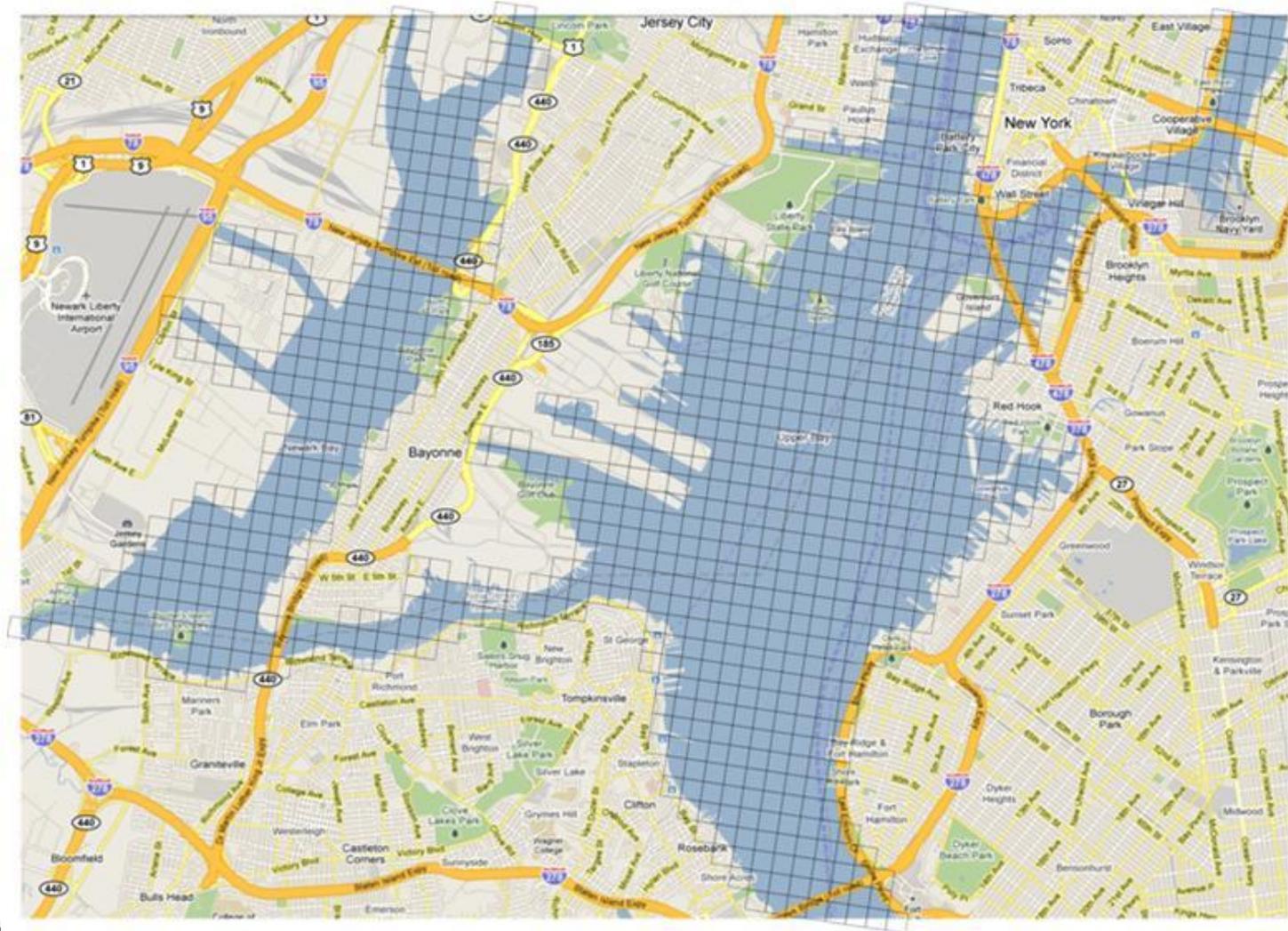
# Test Case

# New York Harbor

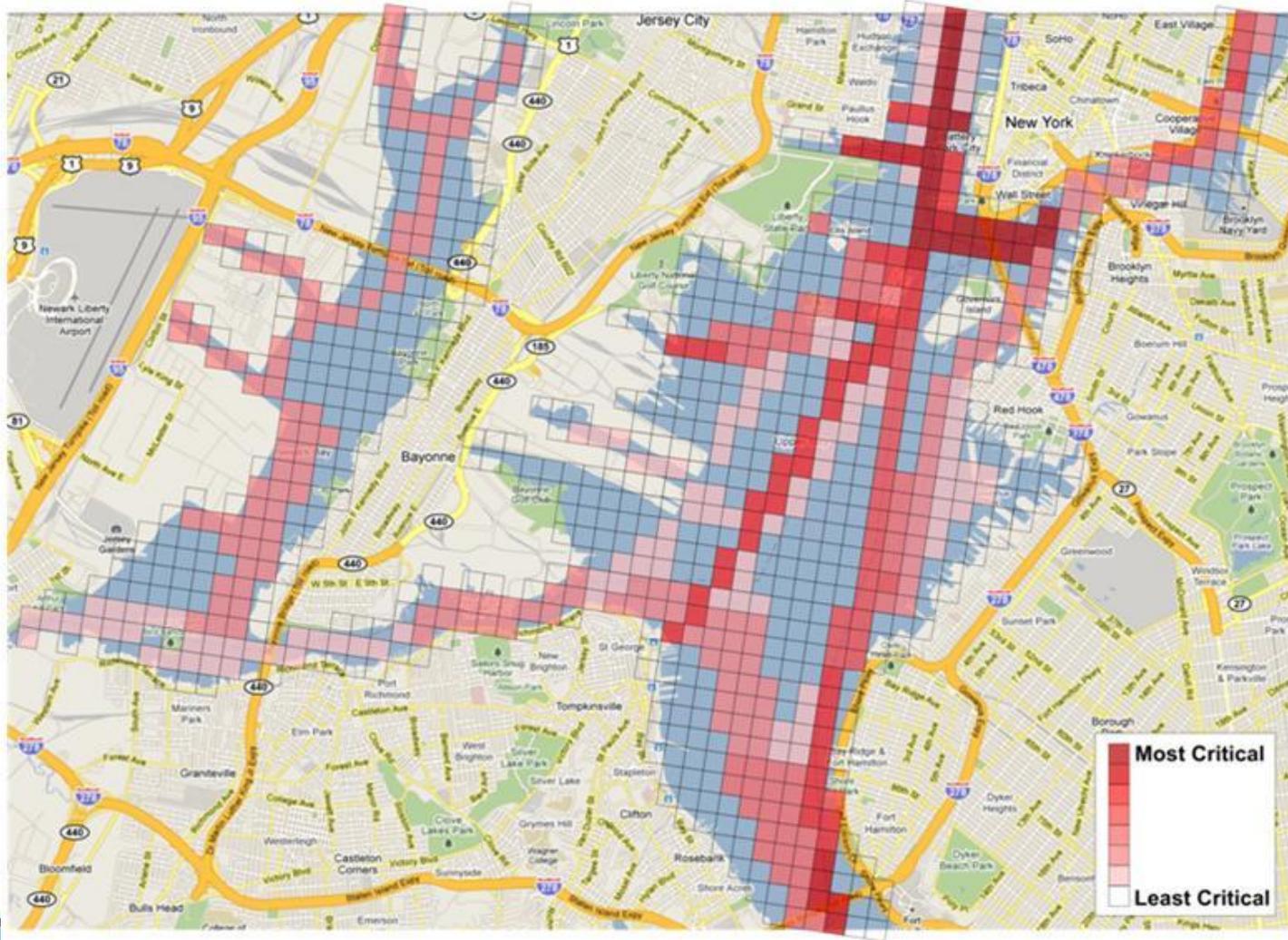
(as an example)



# Grid



# Defining Criticality level of Cells







**Thank you!**