Strategic Analysis for Prediction Markets

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Prediction Market Games

- Agent has
  - Beliefs (about uncertain proposition, other agents)
  - Preferences (for money, other outcomes)
  - Opportunity to trade in a prediction market
- Defines a game
  - Severely incomplete and imperfect info, dynamic revelation, huge strategy space, …
- What **will** or **should** it do?

Empirical Evidence

- Observations
  - Data from prediction markets "in the field": IEM, historical PE markets, FX, HSX, NewsFutures, Costa Rican bookies,…
- Experimentation
  - Controlled laboratory settings: CalTech, PSU, HP, Humboldt, Frankfurt, Maastricht, GMU…

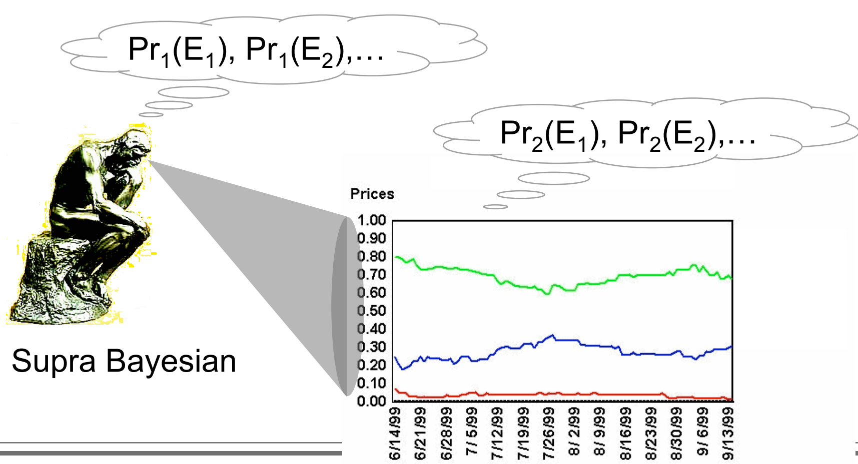
Theory as a Guide

- Assume (e.g.): common prior, common knowledge of rationality
- Q: How should a rational agent trade on asymmetric information?
- A: Don't (Milgrom & Stokey, 1982)
- To get trading (*in theory*), need some irrationality, reasons beyond info asymmetry, subsidies, …

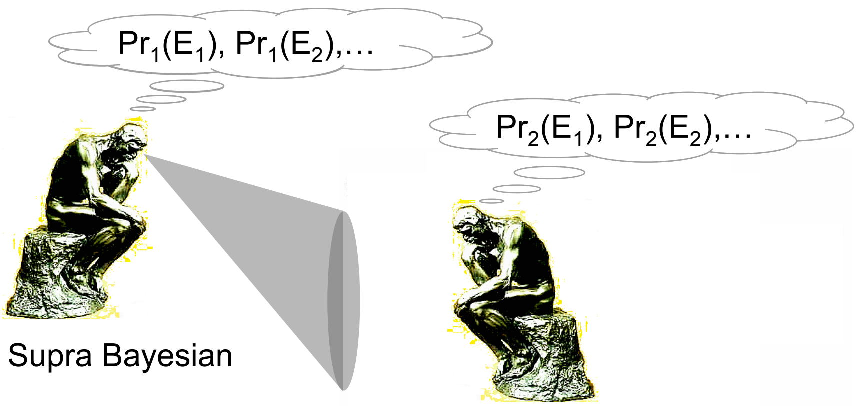
Irrational Model #1

- Agents ignore information signal in prices
  - Also competitive (price takers)
  - E.g.: Manski, Wolfers & Zitzewitz, Pennock & Wellman
- Price aggregation depends on form of utility function
  - GLU: (weighted) arithmetic mean (LinOP)
  - CARA: (weighted) geometric mean (LogOP)

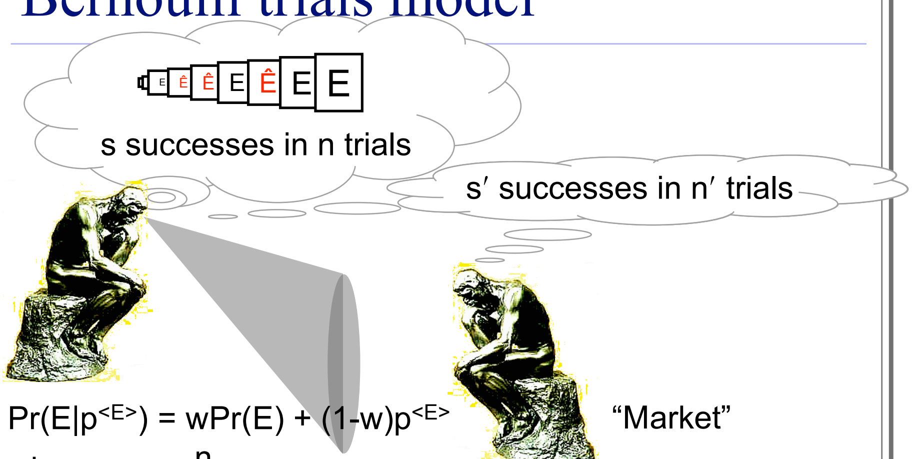
Learning from prices



Learning from prices



Bernoulli trials model



$Pr(E|p^{<E>}) = wPr(E) + (1\text{-}w)p^{<E>}$

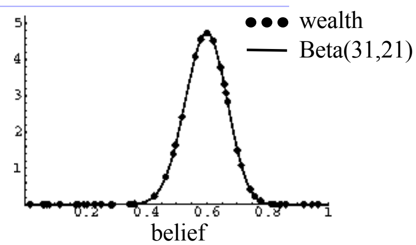
where $w \equiv \frac{n}{n+n'}$

Equilibrium with Learning

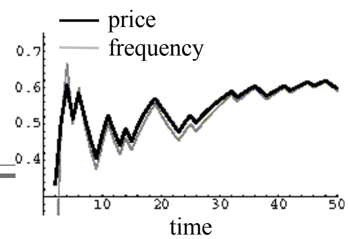
- Weighted average update
  $Pr(E|p^{<E>}) = wPr(E) + (1\text{-}w)p^{<E>}$
  and agents with GLU
  $\Rightarrow$ still LinOP prices
  confidence-based wts
- Geometric average update
  $Pr(E|p^{<E>}) \propto Pr(E)^{w}(p^{<E>})^{(1\text{-}w)}$
  and agents with CARA
  $\Rightarrow$ still LogOP prices

Market Adaptation

- Single security
- Multiperiod market
- Agents with GLU
- Fixed beliefs



successes:30 trials:50



Agent-Based Simulations

- Tell us what happens when agents follow particular policy in particular environment
- Still does not tell us what agents **will** or **should** do
- Sometimes augmented with evolutionary dynamics to search for stable populations

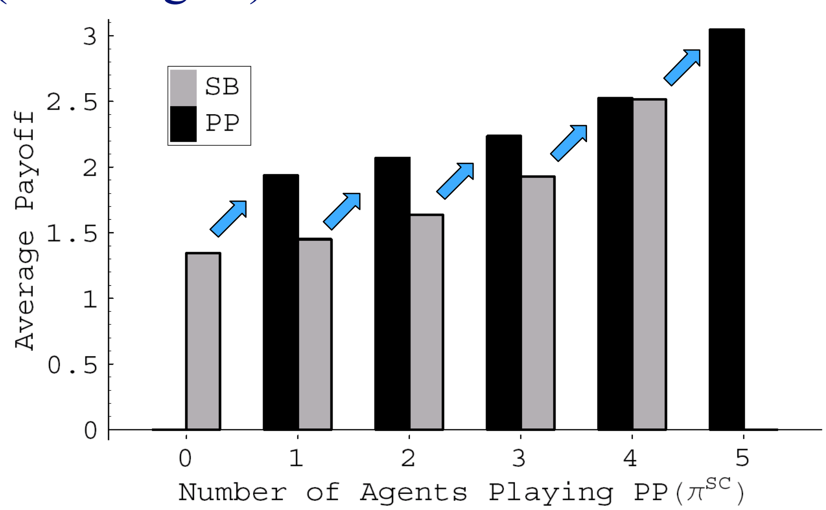
Alternative: Empirical Game-Theoretic Analysis

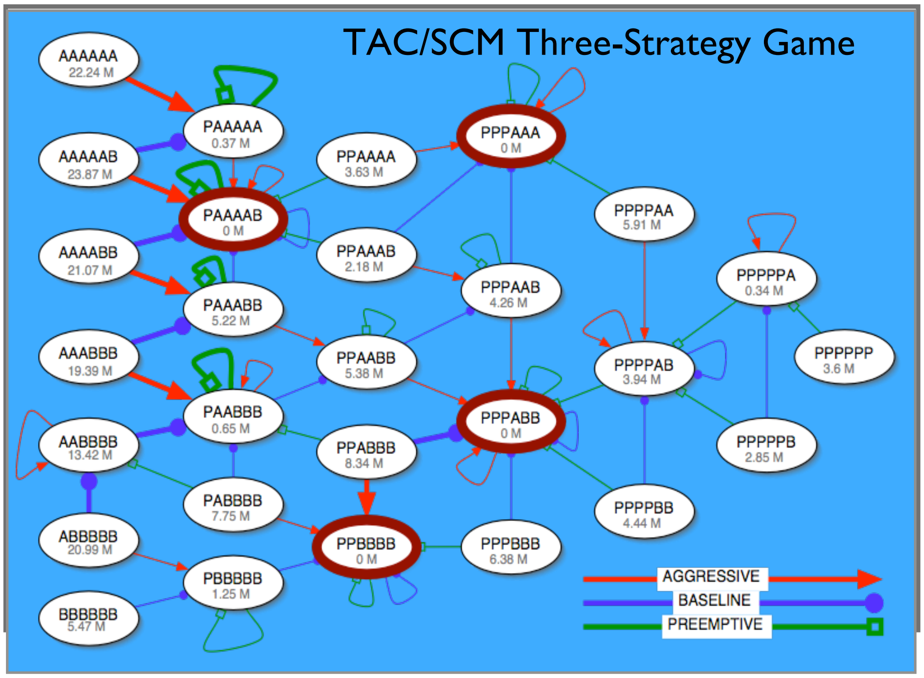
- Explore restricted set of strategies
- Estimate empirical game from simulated profiles
- Analyze using standard game-theoretic concepts and tools

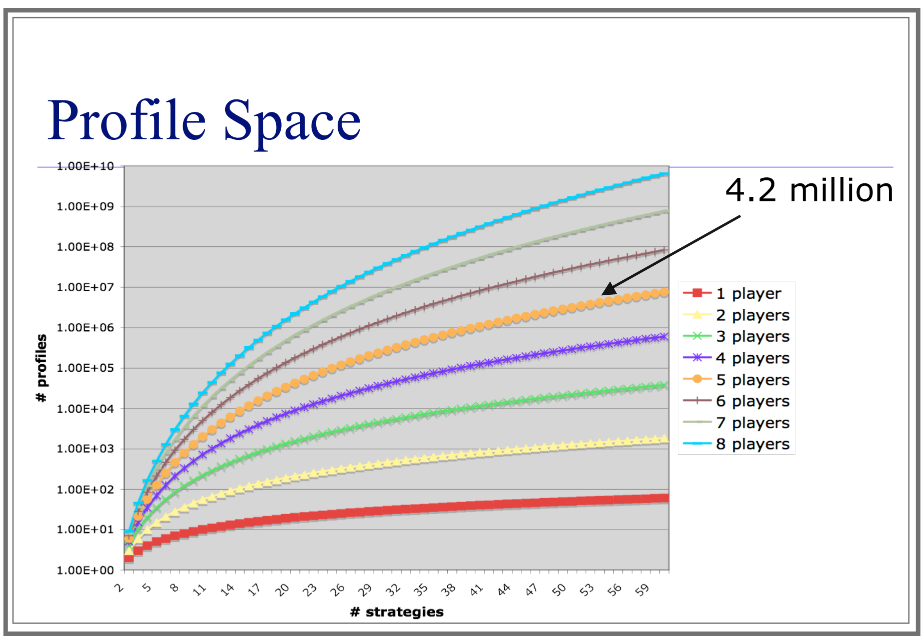
Example: Simultaneous Ascending Auctions (SAAs)

- Ubiquitous
  - Explicit design: FCC, others…
  - Implicit due to independence of operation
- Large gap in strategic understanding
- Simulation studies using portfolio of old and new strategy proposals

Example Payoff Matrix (2 strategies)




TAC/SCM Three-Strategy Game
AAAAAA 22.24 M
AAAAAB 23.87 M
AAAABB 21.07 M
AAABBB 19.39 M
AABBBB 13.42 M
ABBBBB 20.99 M
BBBBBB 5.47 M
PAAAAA 0.37 M
PAAAAB 0 M
PAAABB 5.22 M
PAABBB 0.65 M
PABBBB 7.75 M
PBBBBB 1.25 M
PPAAAA 3.63 M
PPAAAB 2.18 M
PPAABB 5.38 M
PPABBB 8.34 M
PPBBBB 0 M
PPPAAA 0 M
PPPAAB 4.26 M
PPPABB 0 M
PPPBBB 6.38 M
PPPPAA 5.91 M
PPPPAB 3.94 M
PPPPBB 4.44 M
PPPPPA 0.34 M
PPPPPB 2.85 M
PPPPPP 3.6 M
AGGRESSIVE
BASELINE
PREEMPTIVE


Profile Space
4.2 million
profiles
1.00E+10
1.00E+09
1.00E+08
1.00E+07
1.00E+06
1.00E+05
1.00E+04
1.00E+03
1.00E+02
1.00E+01
1.00E+00
strategies
1 player
2 players
3 players
4 players
5 players
6 players
7 players
8 players

Experiments on U(5,5)

- Evaluated 4916 strategy profiles
- 200K–200M samples each (avg. 10M)

- Key Result
  - Profile with all 5 PP(F^{SC}) is pure-strategy NE
  - Payoff = 4.51
  - Can verify by examining only 53 profiles (this one plus 52 *deviations*).

Other Equilibria?

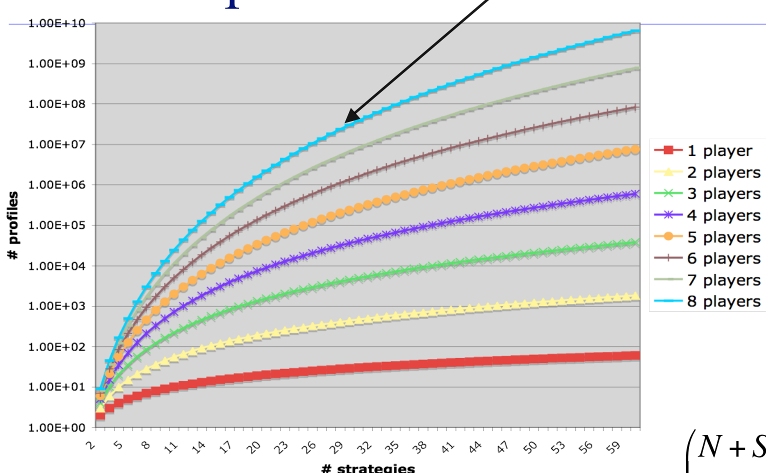
- Not among 4915 other pure profiles tested
  - Can rule out all at $\varepsilon > 0.13$
  - Can rule out 4913 at $\varepsilon > 0.25$
- Not within *cliques*
  - Clique: strategy set for which we evaluated all profile combinations
  - PP(F^{SC}): only strategy surviving iterated elimination of dominated strategies in clique subgames
- Not among 46 pairs

Searching for Walverine…

- Michigan's TAC Travel-Shopping agent
- Parametrized strategy space
  - Flight delay parameters
  - Entertainment trading policy
  - Hotel bid shading…
- Restrict attention to a discrete set of *S* strategies (parameter settings).

Profile Space

49 million



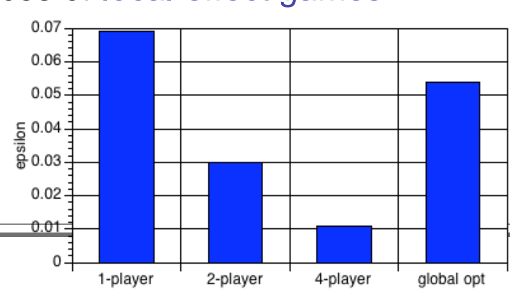
$$\binom{N+S-1}{N}$$

Reduced Games

- Let each "player" control two TAC agents
- Transformed to 4-player game
  - Less fidelity
  - More tractable
  - (*S* = 31, only 46,376 profiles)
- 2-player: 496 profiles
- 1-player: 31 profiles

Why Trust Reduced-Game Results?

- Claim: Equilibria in reduced game likely to be relatively stable in full game
- Evidence:
  - Random instances of local-effect games
  - 2-strategy
  - 8-player



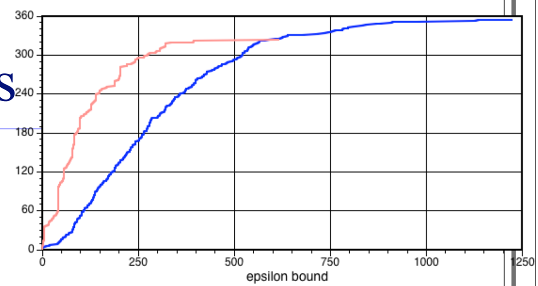
Searching *N*-Player TAC Classic (*S*=31)

N	Profiles	Explored	Expl %	samples/ profile
4	46,376	1452	3.1	17.4
2	496	354	71.4	27.2
1	31	31	100.0	98.5

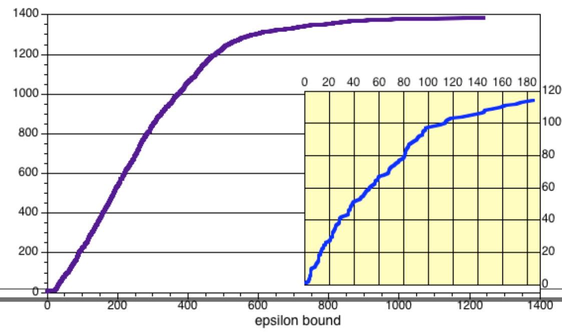
Analyzing (Partial) Reduced Games

- *N*=1 (31 profiles)
  - Identified unique pure-strategy NE (PSNE)
- *N*=2 (344)
  - "Confirmed" 1 PSNE, refuted 340 ($\varepsilon > 10$)
  - 3 confirmed eq. mixture pairs
  - Refuted 304 candidate mixture pairs ($\varepsilon > 10$); 292 ($\varepsilon > 20$)
- *N*=4 (1429)
  - Refuted 1423 candidate PSNE ($\varepsilon > 10$); 1421 ($\varepsilon > 20$)
  - Est. 114 candidate mixture pairs
    - Confirmed 1 ($\varepsilon < 1$)
    - refuted 99 ($\varepsilon > 10$); 83 ($\varepsilon > 20$)

Epsilon Bounds



2-player



4-player

Conclusions

- Prediction markets pose interesting open problems in trading strategy
- Empirical game analysis
  - Bridges simulation and game theory
  - Can provide conclusions given partial data
  - Provides insights about SAA, TAC games
- Any bets on what it might show for prediction markets?