Delay-Constrained Unicast: Improved upper bounds

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PRINCETON UNIVERSITY

Joint work with

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Sreeram Kannan

Pramod Viswanath

DIMACS workshop on Network Coding, 17 December 2015

Delay-constrained unicast [Wang-Chen '14]

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Single flow with delay constraint D

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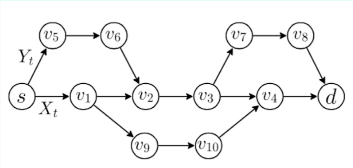

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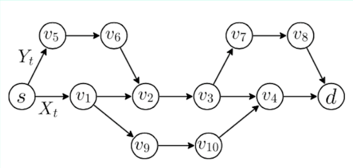

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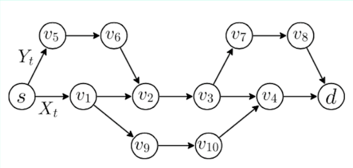

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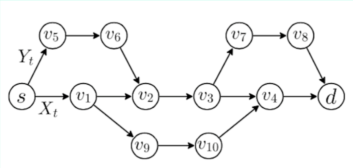

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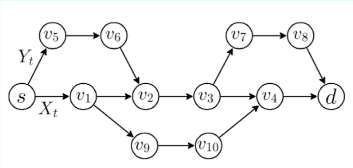

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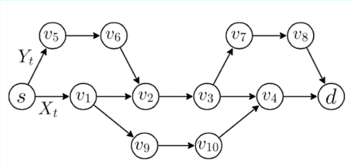

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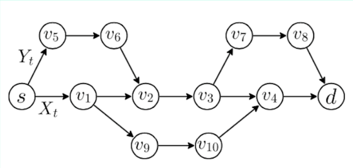

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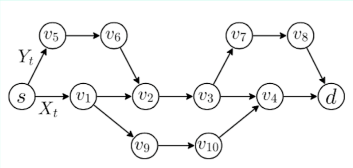

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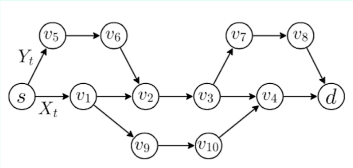

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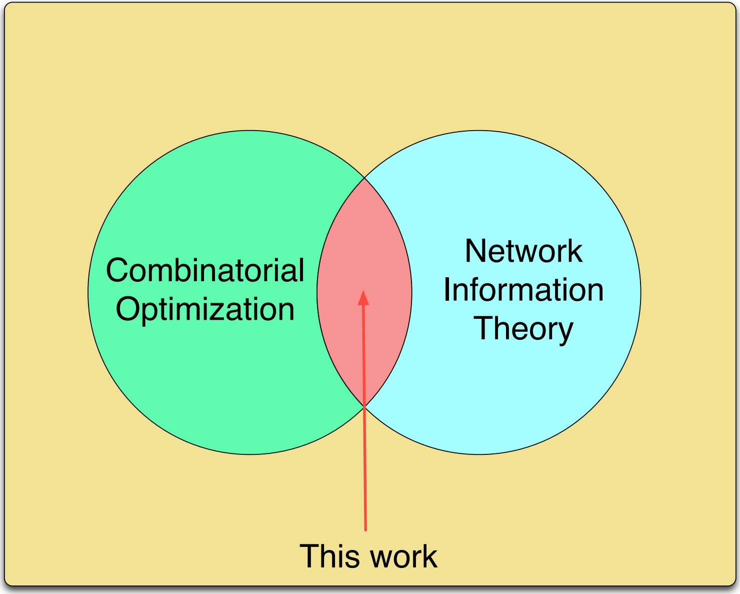
We improve over this


Combinatorial Optimization
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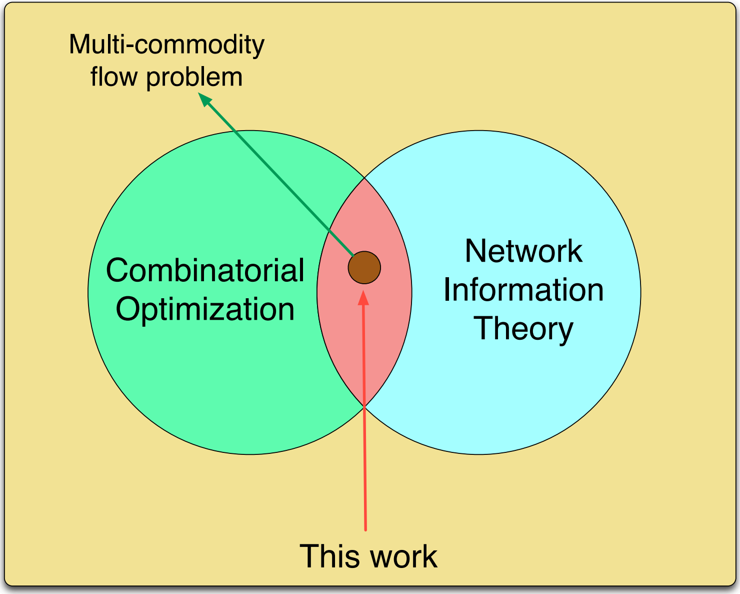
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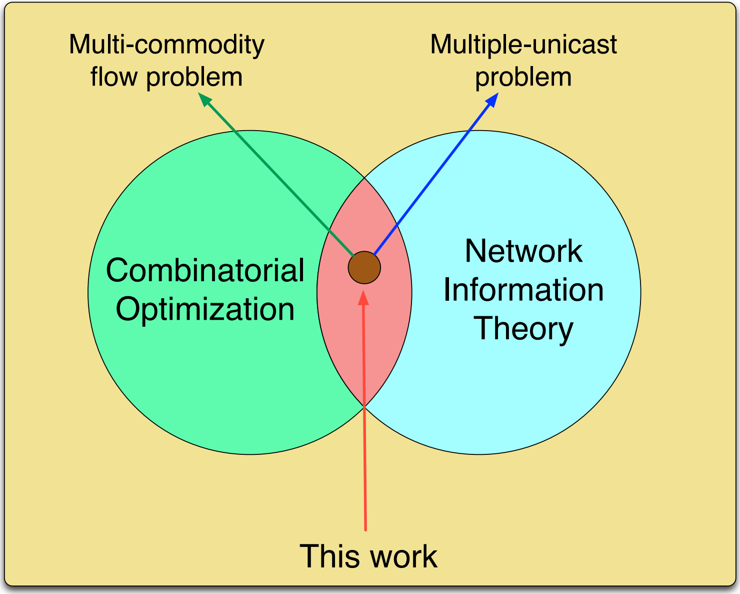
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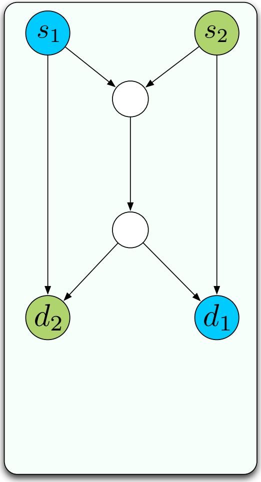

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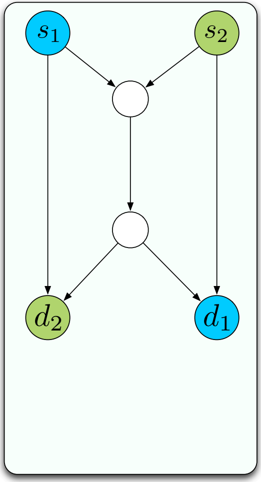

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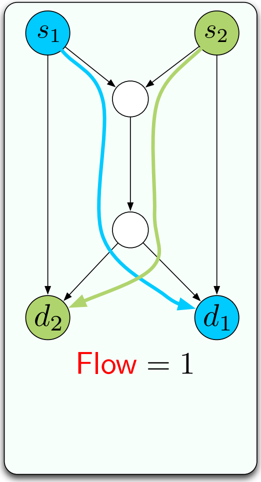

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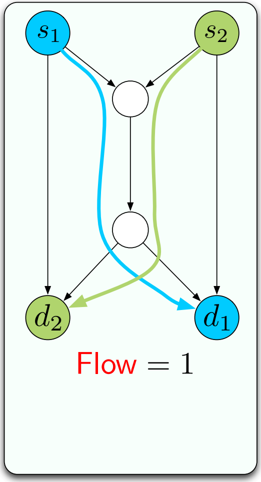

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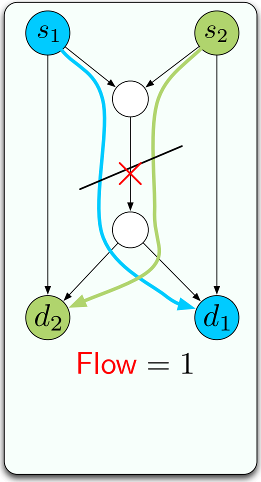

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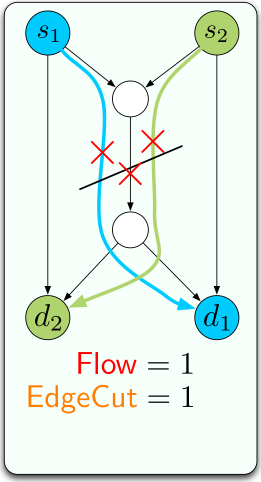

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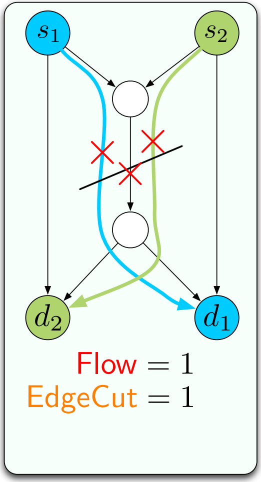

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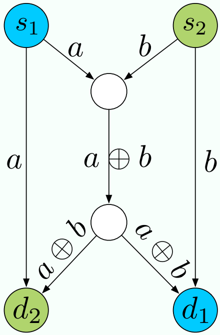

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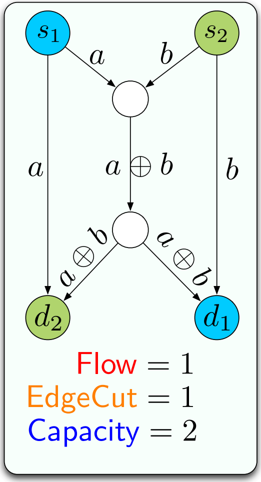

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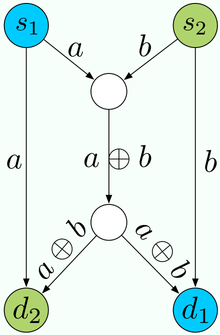
EdgeCut $\neq$ Cutset bound



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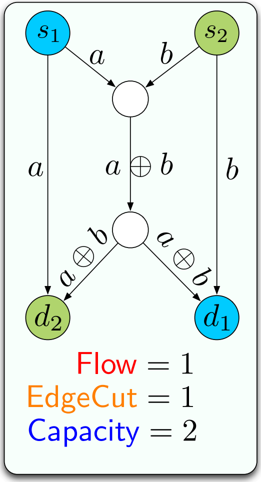

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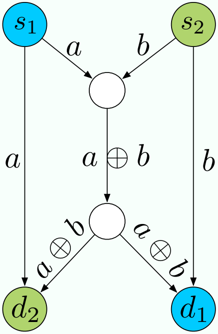
Capacity: Maximum <u>information</u> flow

EdgeCut $\neq$ Cutset bound

Capacity $\leq$ Cutset bound

However, we may have EdgeCut $<$ Capacity



Flow $= 1$
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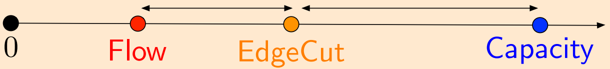

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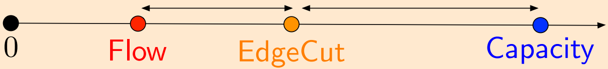

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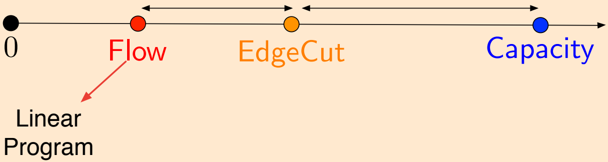

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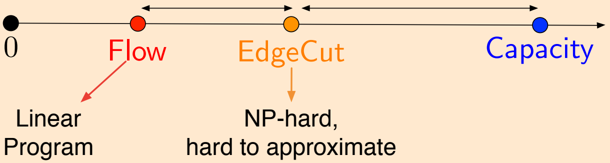

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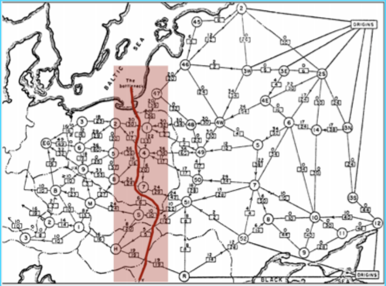

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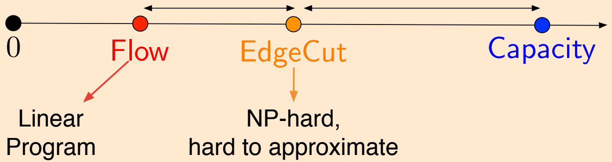

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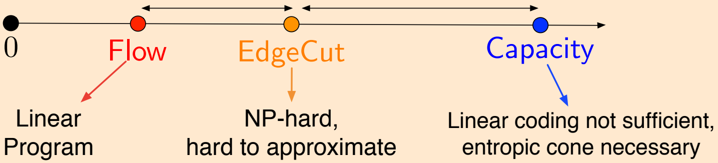

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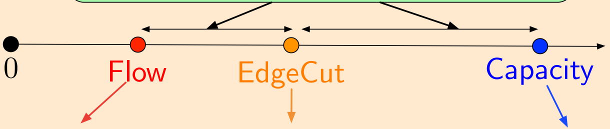
Can be multiplicative factor k apart



Linear Program

NP-hard, hard to approximate

Linear coding not sufficient, entropic cone necessary

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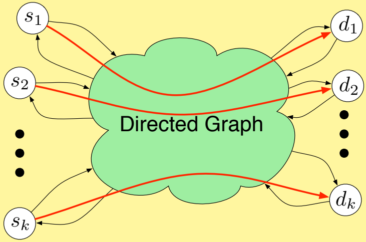

Multiple-unicast: flow from s_i to d_i for all i

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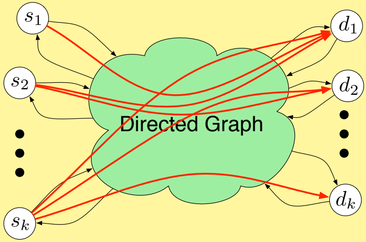

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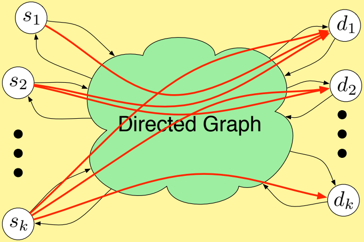

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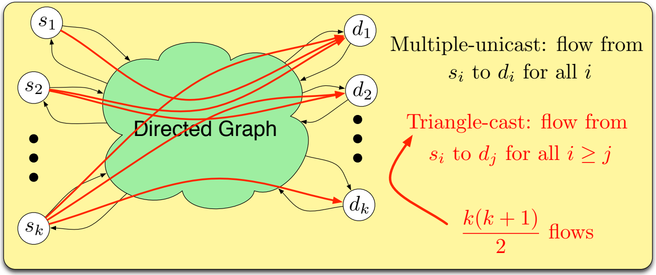
$\frac{k(k+1)}{2}$ flows

Triangle-cast



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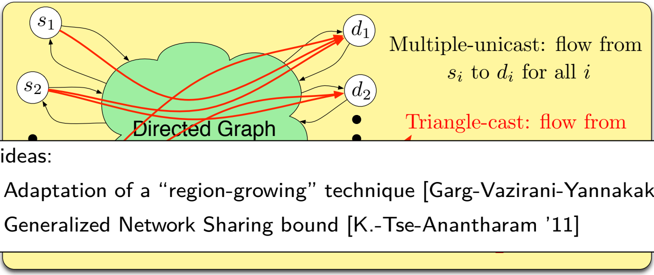

Main ideas:

- Adaptation of a "region-growing" technique [Garg-Vazirani-Yannakakis '96]
- Generalized Network Sharing bound [K.-Tse-Anantharam '11]

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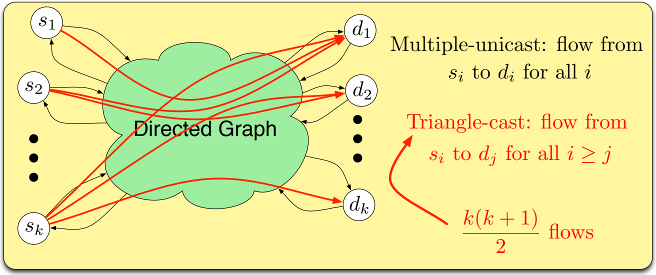

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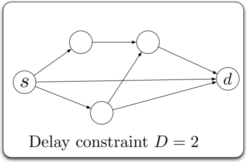
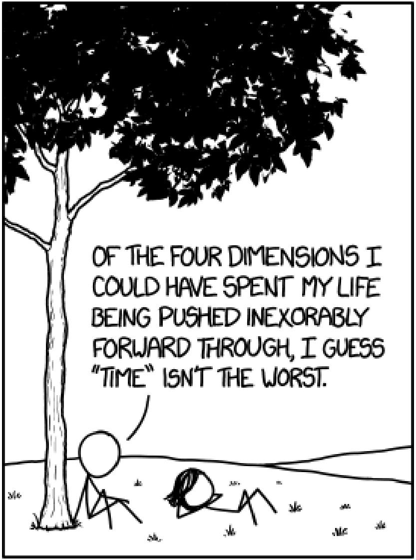
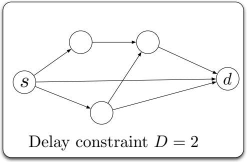


Delay constrained network



Reproduced from xkcd.com



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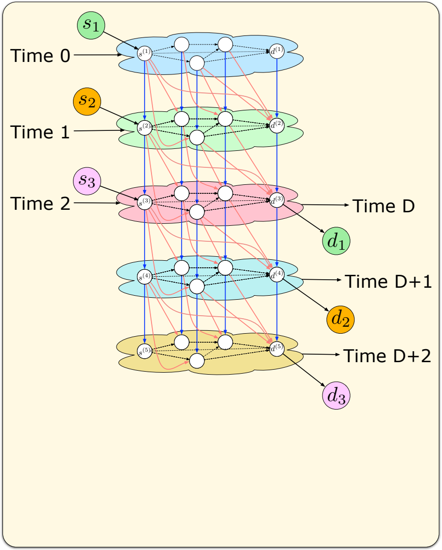

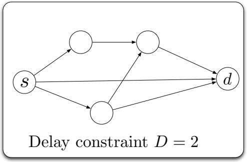

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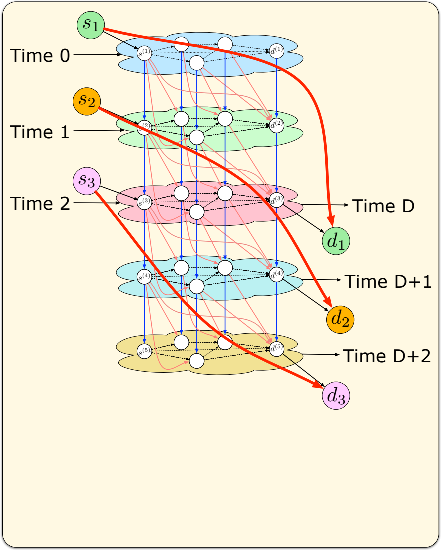

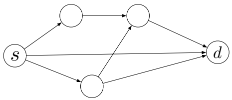

Delay constraint $D = 2$

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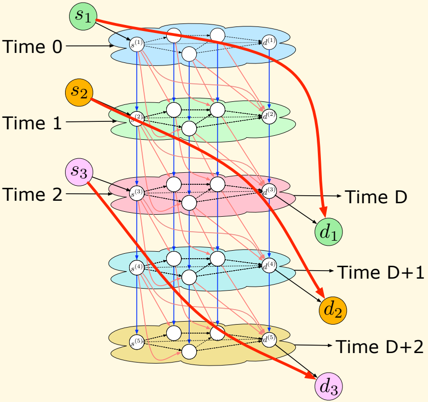
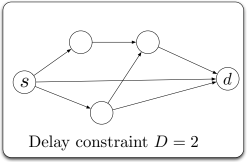
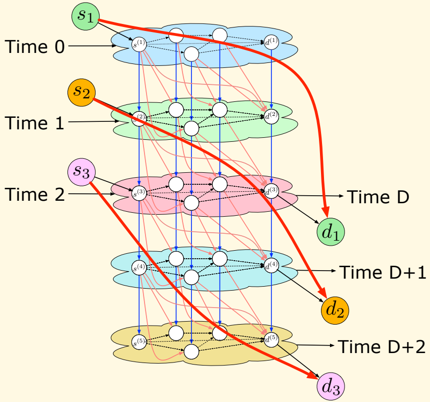


Multiple-unicast

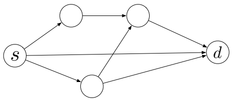


Delay constrained network



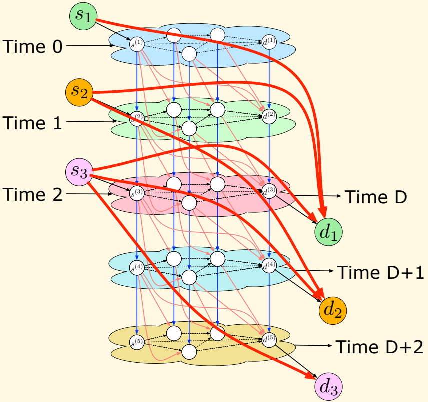
Multiple-unicast

Information delivered earlier is ok!



Delay constraint $D = 2$

Delay constrained network



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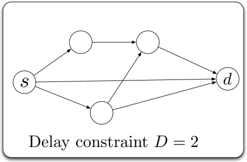

Delay constrained network

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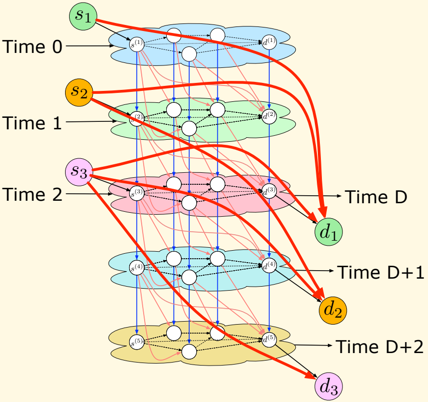
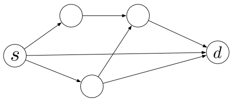


Multiple-unicast

Information delivered earlier is ok!

Hence, triangle-cast!



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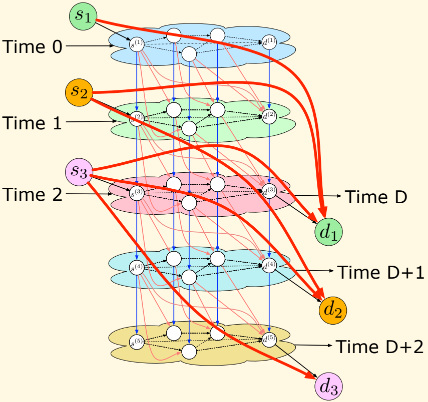

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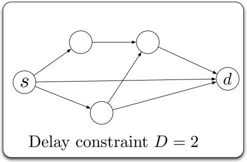


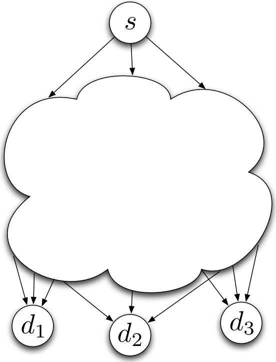
Delay constrained network

Main Result 2

$$\frac{\text{Capacity}}{\text{Flow}} \leq 8 \log_e(D+1)$$

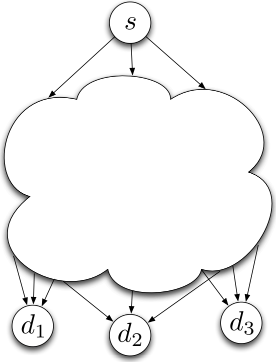
$D =$ Delay

Open 1: "Multicast"



Multicast: same information to all destinations

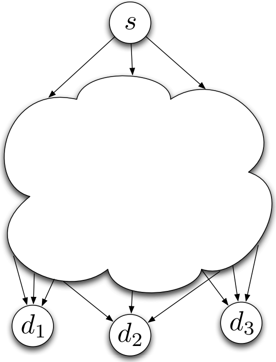
Open 1: "Multicast"



Multicast: same information to all destinations

What happens with delay constraint?

Open 1: "Multicast"



Multicast: same information to all destinations

What happens with delay constraint?

- Practical constraint
- Intra-flow coding
- Coding strategies? - Random coding does not work

Open 2: "Triangle-cast gap"

Theorem (this work)

For k-triangle-cast,

$$\frac{\text{EdgeCut}}{4\log_e(k+1)} \leq \text{Flow} \leq \text{Capacity} \leq \text{EdgeCut}$$

Open 2: "Triangle-cast gap"

Theorem (this work)

For k-triangle-cast,

$$\frac{\text{EdgeCut}}{4\log_e(k+1)} \leq \text{Flow} \leq \text{Capacity} \leq \text{EdgeCut}$$

Conjecture

For k-triangle-cast,

$$\frac{\text{EdgeCut}}{2} \leq \text{Flow} \leq \text{Capacity} \leq \text{EdgeCut}$$

Hence, for delay-constrained unicast,

$$\frac{\text{Capacity}}{\text{Flow}} \leq 2$$

Open 3: "Symmetry Principle"

Principle

Under suitable symmetry in traffic pattern, Flow, EdgeCut, Capacity are all not "too far" apart.

Open 3: "Symmetry Principle"

F : Flow
EC : EdgeCut
C : Capacity

Open 3: "Symmetry Principle"

F : Flow
EC : EdgeCut
C : Capacity

Bidirected Networks	$\frac{\text{EC}}{\Theta(\log k)} \leq \text{F} \leq \text{EC}$ [Leighton-Rao '88] [Linial-London-Rabinovich '94]	$\text{F} \leq \text{C} \leq \text{EC}$ [K.-Viswanath '12]
Symmetric Demands	$\frac{\text{EC}}{\Theta(\log^3 k)} \leq \text{F} \leq \text{EC}$ [Klein-Plotkin-Rao-Tardos '93]	$\text{F} \leq \text{C} \leq \text{EC}$ [K.-Viswanath '12]
Group-cast	$\frac{\text{EC}}{2} \leq \text{F} \leq \text{EC}$ [Naor-Zosin '01]	$\text{F} \leq \text{C} \leq 2 \times \text{EC}$ [K.-Viswanath '12]
Triangle-cast	$\frac{\text{EC}}{\Theta(\log k)} \leq \text{F} \leq \text{EC}$ [This work]	$\text{F} \leq \text{C} \leq \text{EC}$ [This work]

Open 3: "Symmetry Principle"

F : Flow
EC : EdgeCut
C : Capacity

Bidirected Networks	$\frac{EC}{\Theta(\log k)} \leq F \leq EC$ [Leighton-Rao '88] [Linial-London-Rabinovich '94]	$F \leq C \leq EC$ [K.-Viswanath '12]
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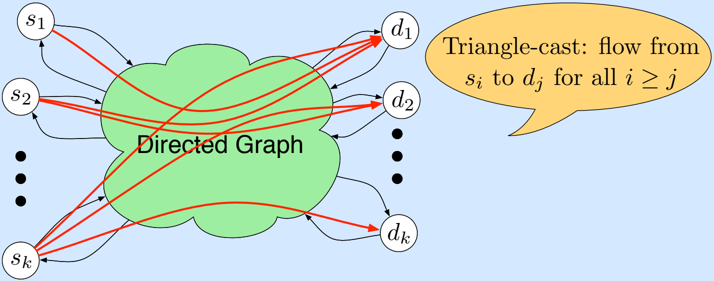
Group-cast	$\frac{EC}{2} \leq F \leq EC$ [Naor-Zosin '01]	$F \leq C \leq 2 \times EC$ [K.-Viswanath '12]
Triangle-cast	$\frac{EC}{\Theta(\log k)} \leq F \leq EC$ [This work]	$F \leq C \leq EC$ [This work]
???		

Conclusion



Showed a $4 \log_e(k+1)$ Flow – EdgeCut – Capacity approximation guarantee

$$\Downarrow$$

Delay-constrained unicast (delay D) has $\frac{\text{Capacity}}{\text{Flow}} \leq 8 \log_e(D+1)$