Routing in Cost-shared Networks: Equilibria and Dynamics

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(joint works with Rupert Freeman and Sam Haney; Shuchi Chawla, Seffi Naor, Mohit Singh, and Seeun Umboh) set of **agents** want to route traffic from their respective source to sink vertices

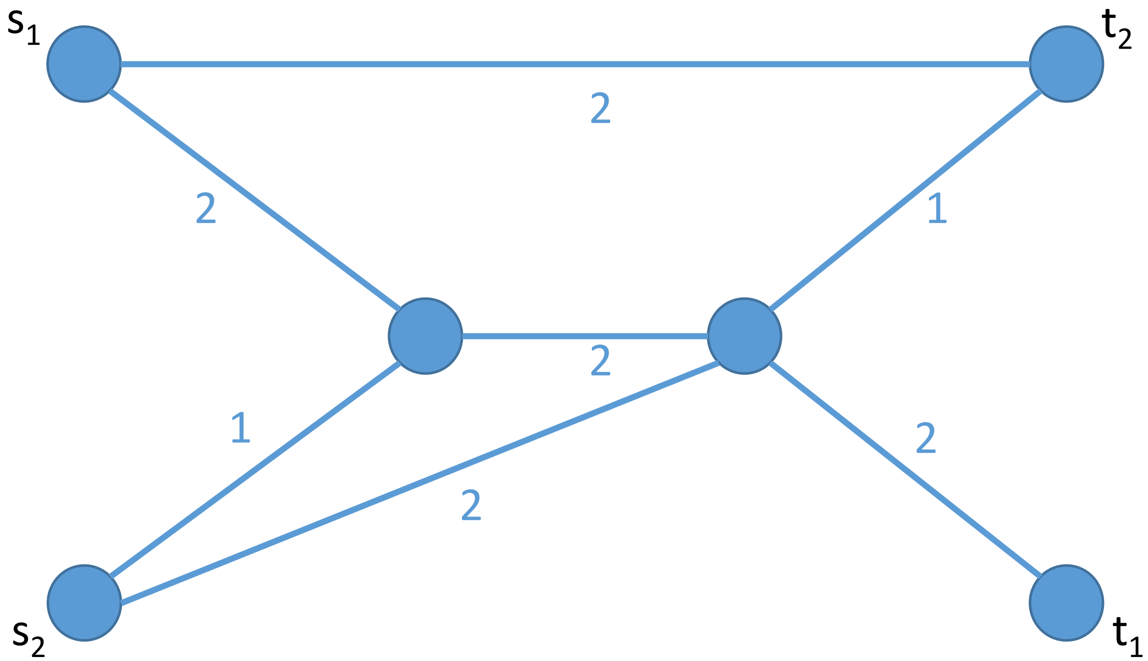
each edge used in routing has a **fixed cost** that is **shared equally** by agents using the edge

minimize **sum of cost of edges** used in routing (Steiner forest)

However …

agents are strategic!

(want to minimize their own cost)



s_1
t_2
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2
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2
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2
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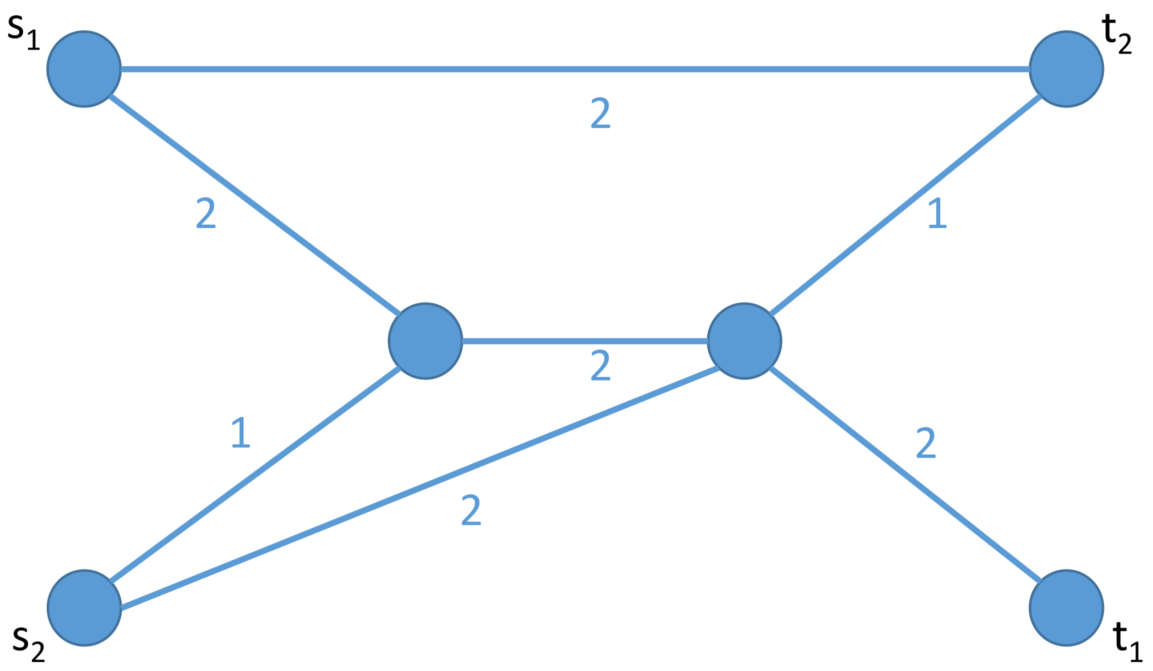
s_2
t_1


s1
t2
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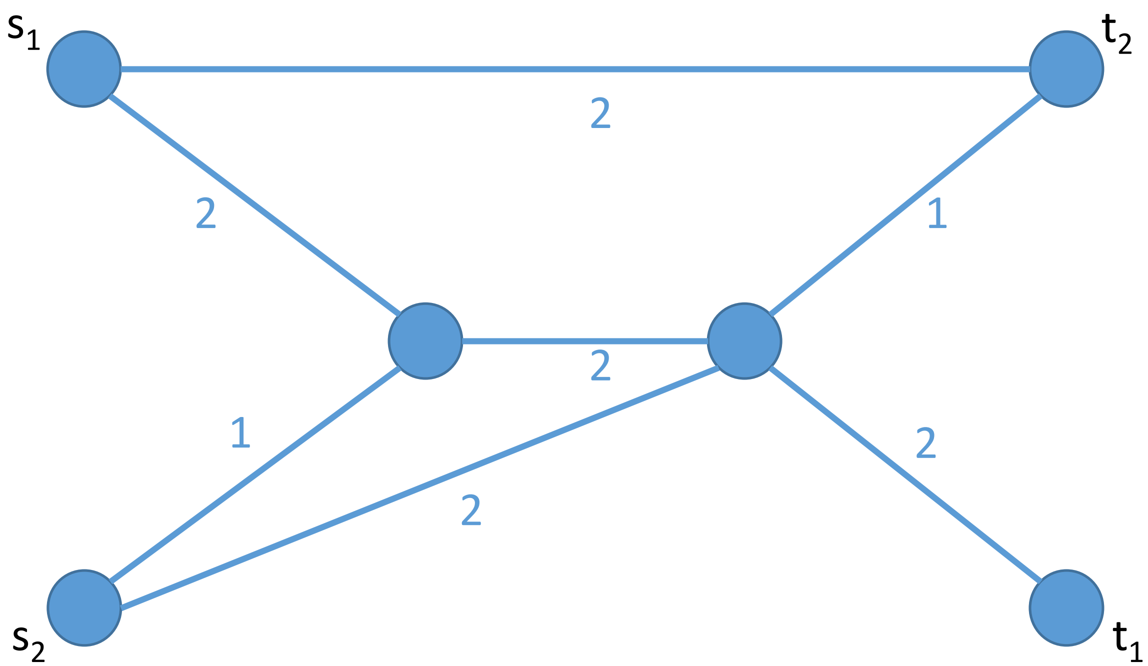
s2
t1


s1
t2
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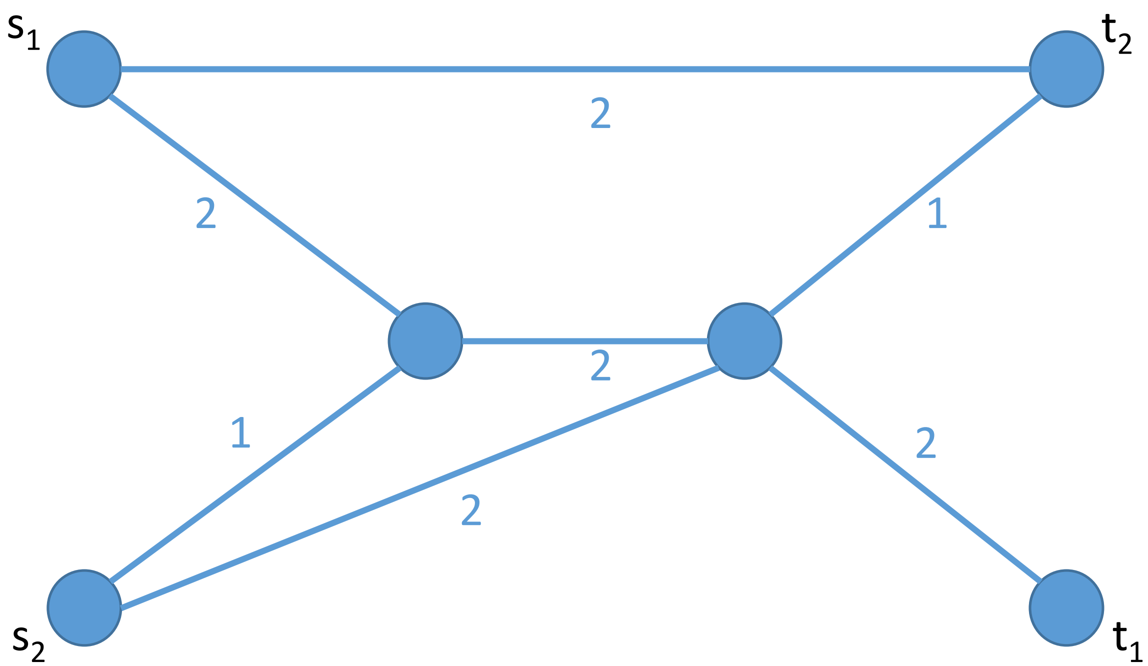
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t_2
2
2
1
2
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2
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s_2
t_1

This is (just) a game!

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equilibrium: no agent has a less expensive routing path

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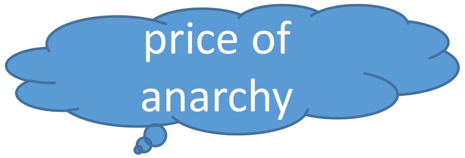
how suboptimal can an equilibrium be?
(and what can the controller do about it?)

unfortunately, **very** suboptimal



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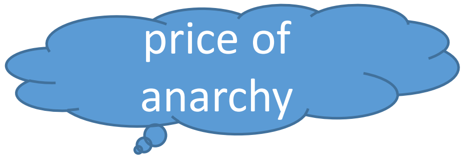

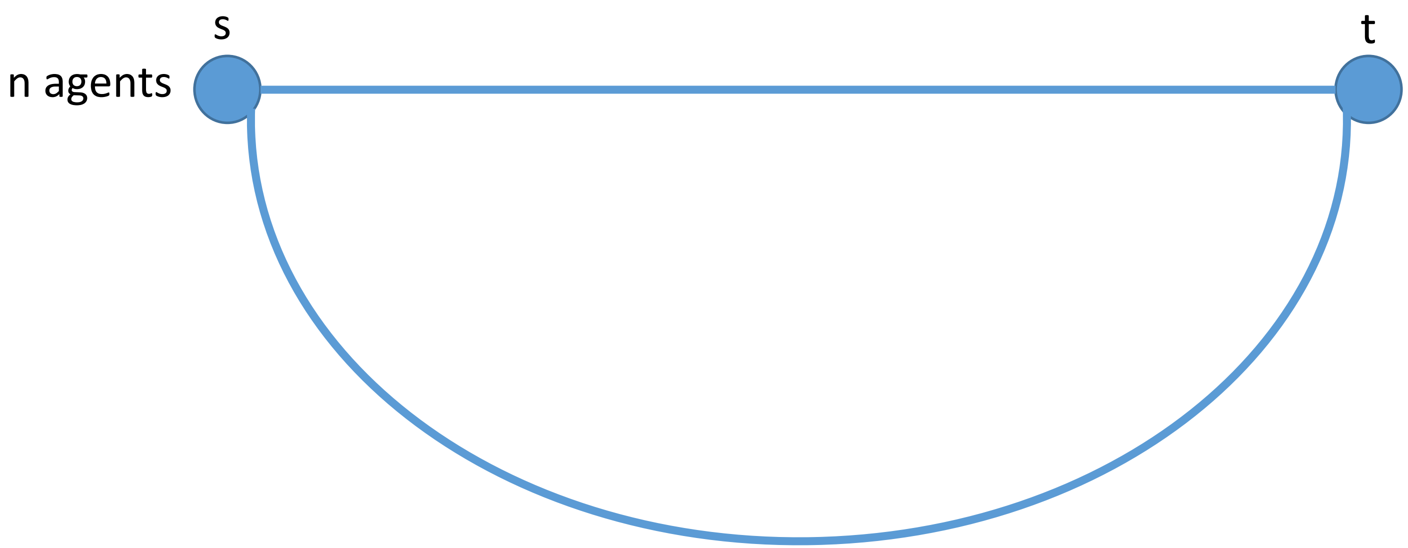

what role can the controller play?

how bad is the **best** equilibrium?
i.e., controller chooses routing paths
but they need to be **in equilibrium**

price of stability

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but they need to be **in equilibrium**

this is a **<u>potential game</u>**
(corollary: equilibrium always exists)

[Anshelevich, Dasgupta, Kleinberg, Tardos, Wexler, Roughgarden ’04] edge **e** used by $\mathbf{n_e}$ agents

potential of edge **e** is $\phi_e = c_e\ (1 + 1/2 + 1/3 + \ldots + 1/n_e)$

in the example, if agent moves from 1 to 2

$\Delta\ \phi = c_2/(n_2+1) - c_1/n_1$

= difference in shared cost

Initialize with optimal solution and run to equilibrium

[Anshelevich, Dasgupta, Kleinberg, Tardos, Wexler, Roughgarden ’04]

OPEN: Can this logarithmic ratio be improved?

[Li ’09: **O(log n / log log n)**]
[Best lower bounds are small constants]

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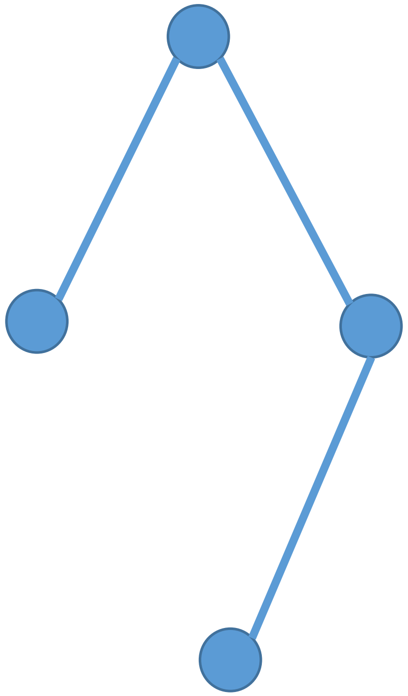
<u>special case: **broadcast games**</u>
each vertex has an agent
all agents route to a common gateway destination

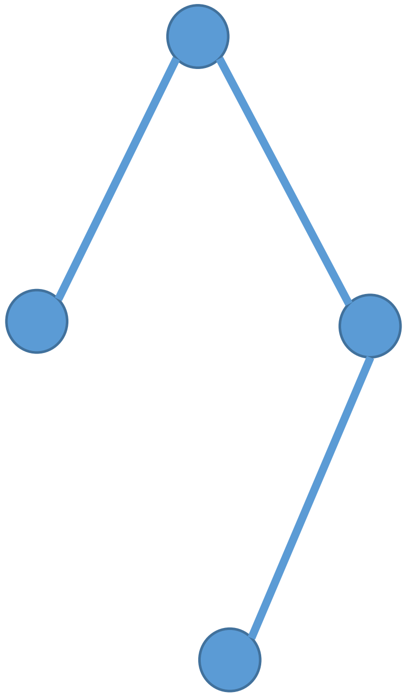
broadcast games



v is responsible for edge e_v

Fiat-Kaplan-Levy-Olonetsky-Shabo ’06: **O(log log n)**
Liggett-Lee ’13: **O(log log log n)**
Bilo-Flammini-Moscardelli ’13: **O(1)**

broadcast games



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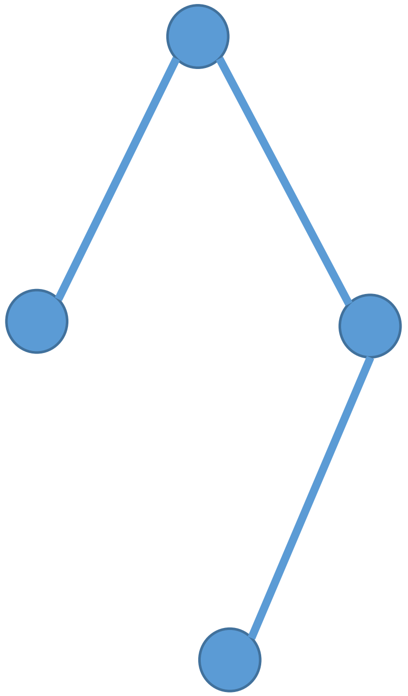
broadcast games

what about multicast games?

<u>Main challenge</u>

Mechanism for transferring responsibility



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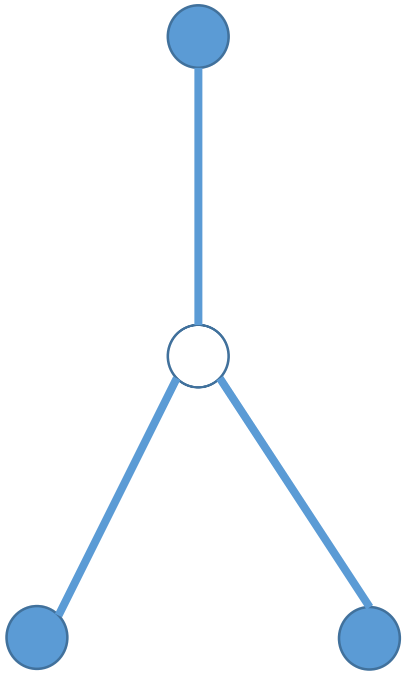

who is responsible for edge e?

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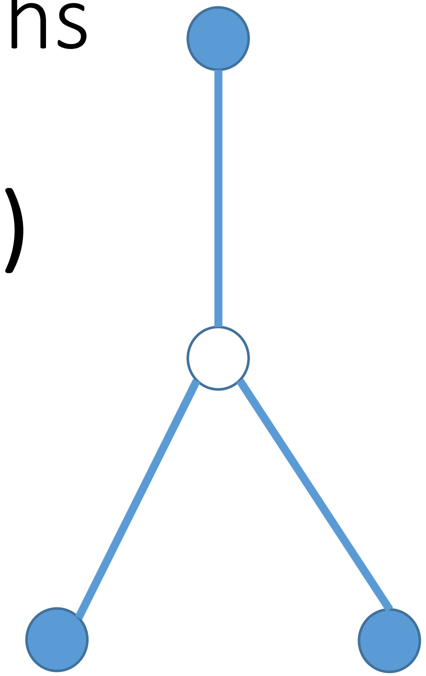
recent progress [Freeman, Haney, P.]

multicast games on quasi-bipartite graphs

price of stability is O(1)



agent-agent path is of length ≤ 2

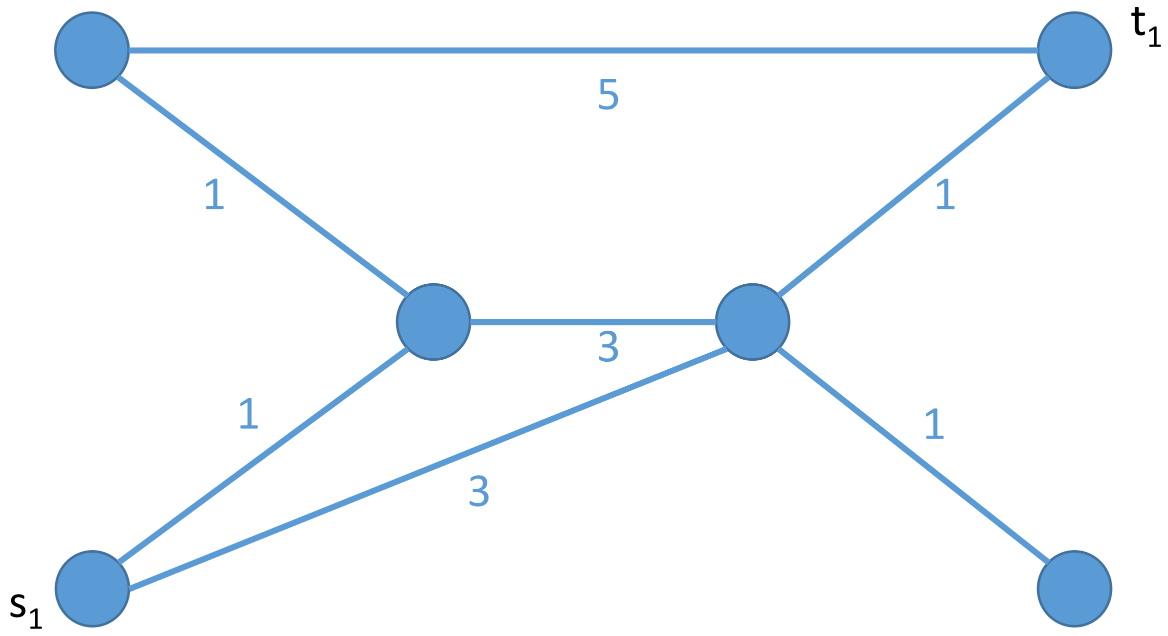
exponential gap between best and worst equilibria

which of these equilibria is achievable?

OPEN: Find <u>**any**</u> equilibrium in polynomial time.

changes in potential can be exponentially small

what if agents can join and leave the network?



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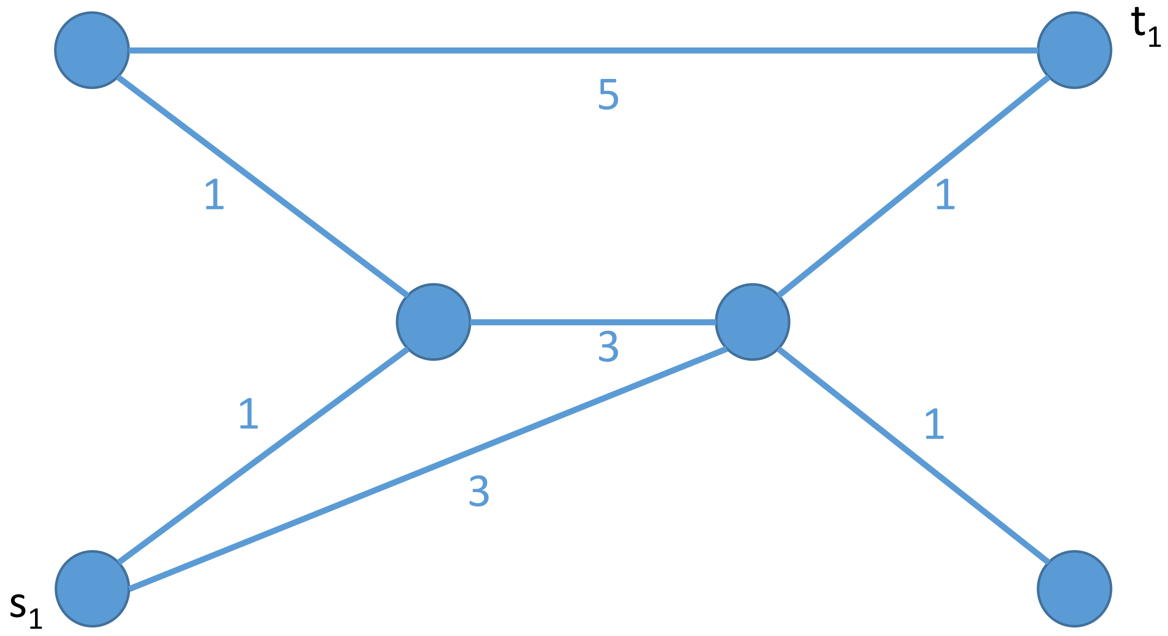

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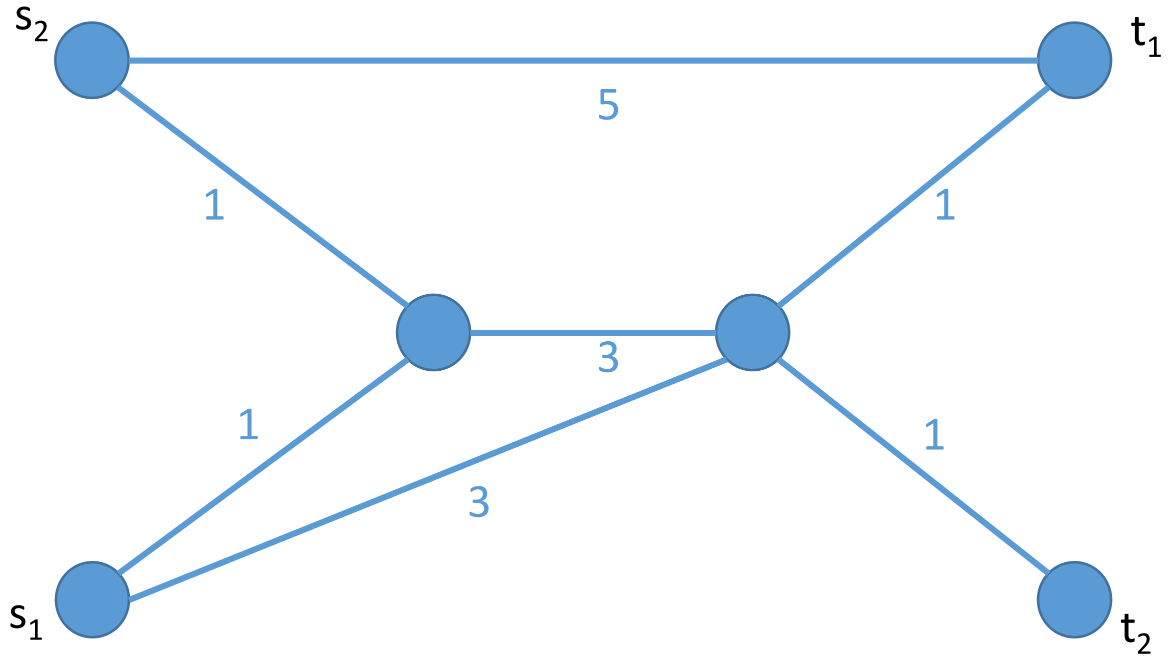

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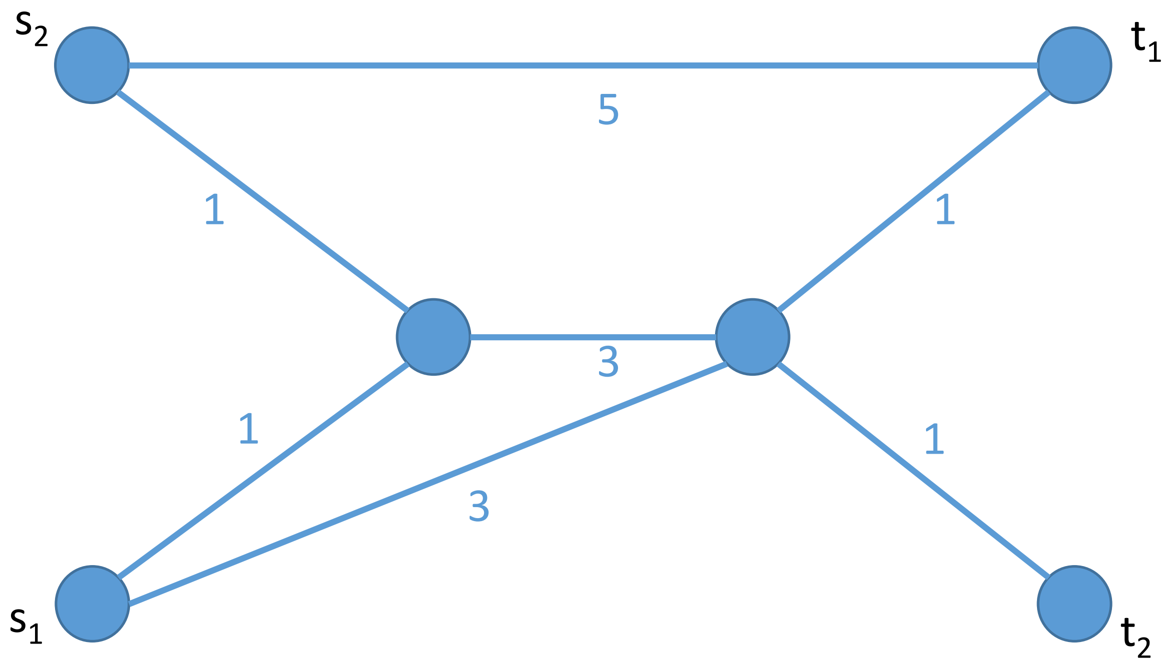

OPEN: What is the quality of the equilibrium reached if there are no departures?

if arrivals and moves are not interleaved, then $O(\log^3 n)$
[Charikar, Karloff, Matheiu, Naor, Saks '08]

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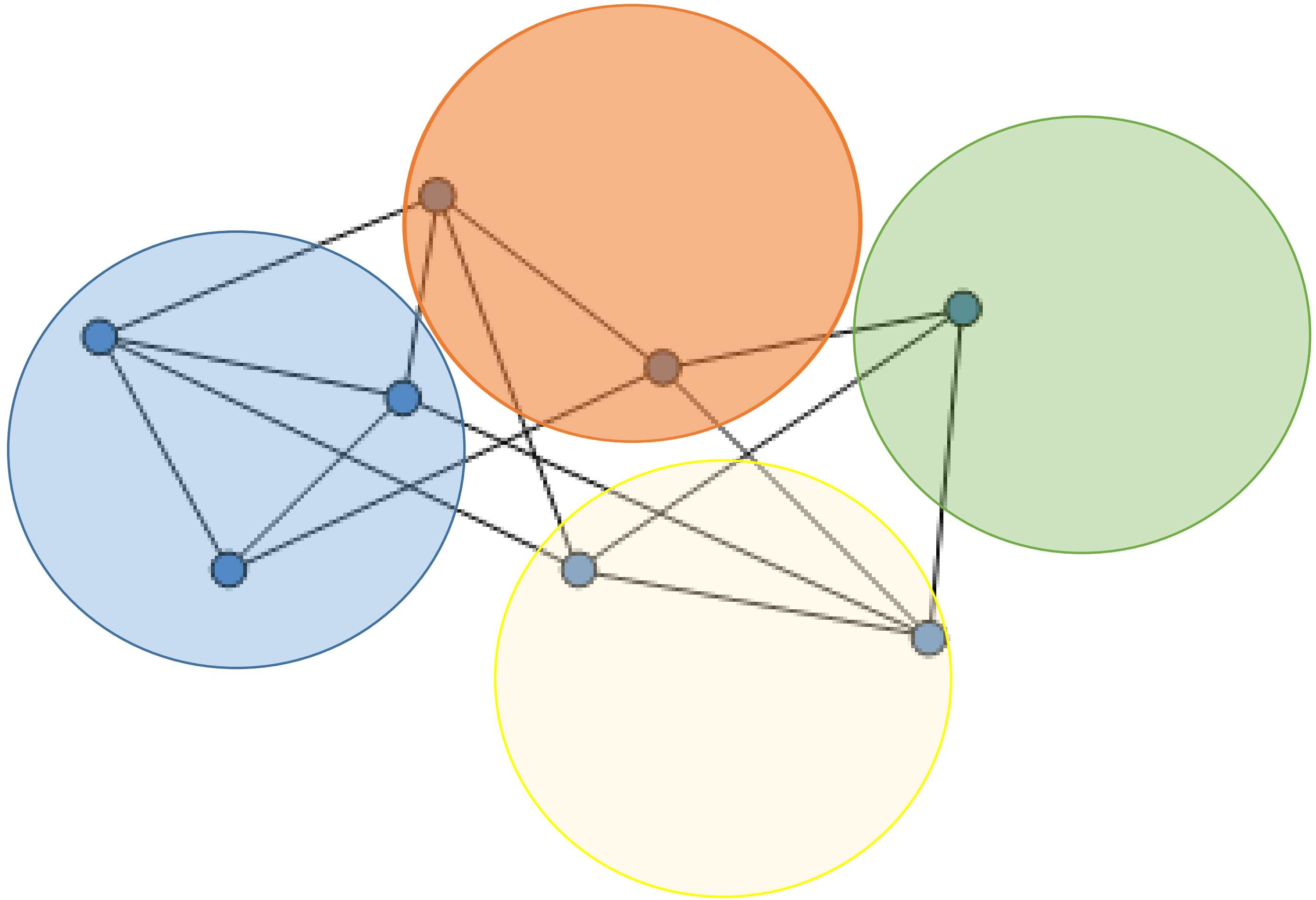
what can the controller do?

if the controller suggests (improving) moves to attain equilibrium between arrival/departure phases

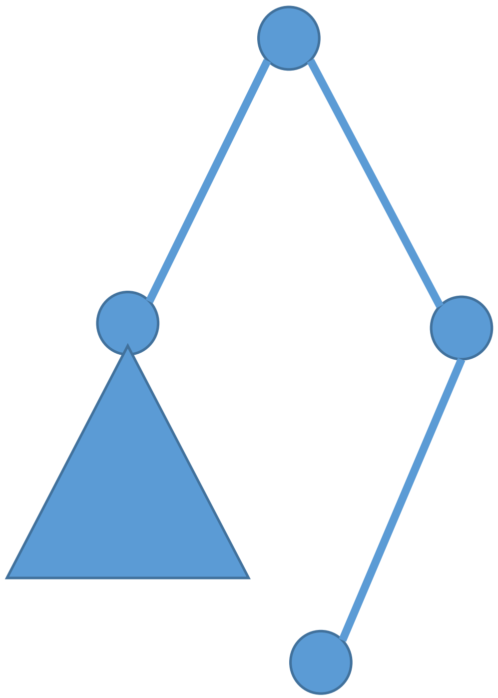
theorem: equilibrium within **log n** of optimal

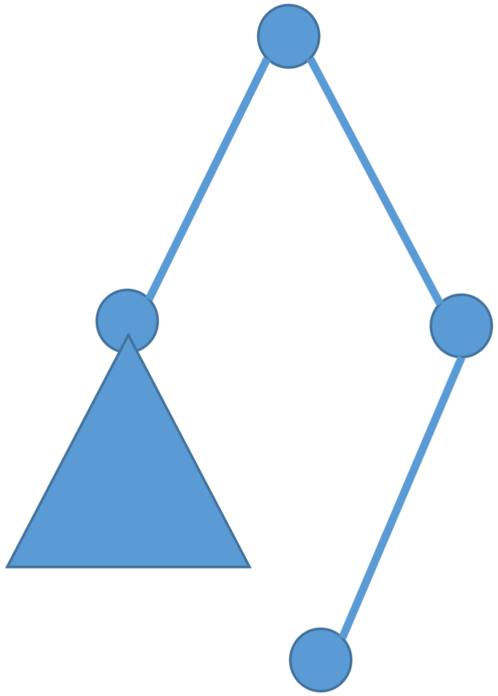
[Chawla, Naor, P., Singh, Umboh]

partition graph into subgraphs of diameter $\mathbf{2^k}$, for $\mathbf{1 \le k \le \log n}$
(embed into a distribution of **HST**s)

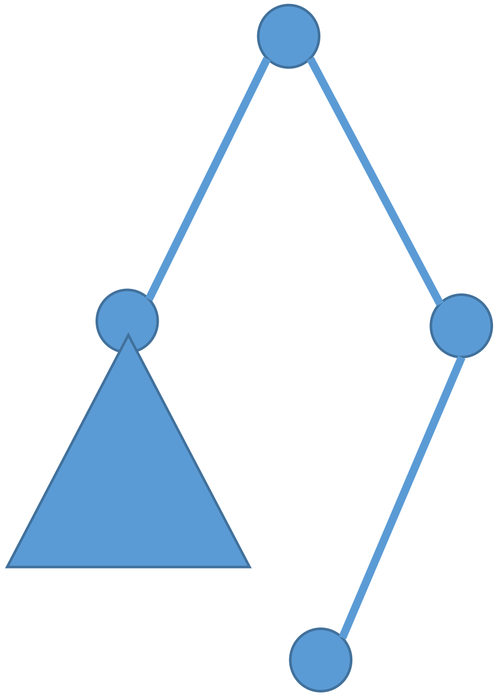


hope: vertices with edges of same length are **well-separated**





improving move removes an overcharge



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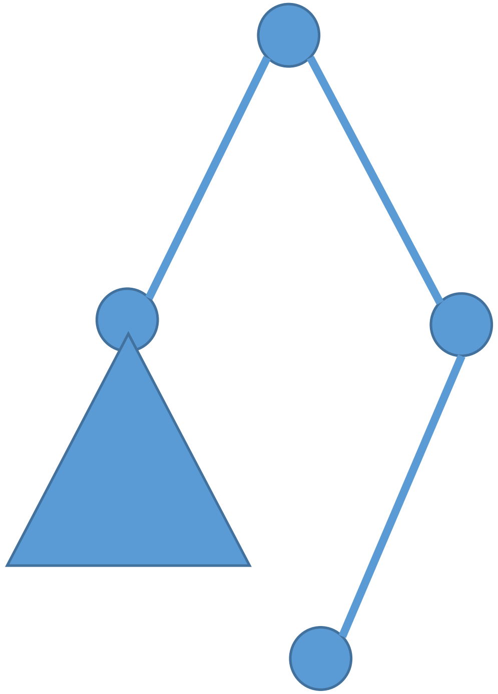

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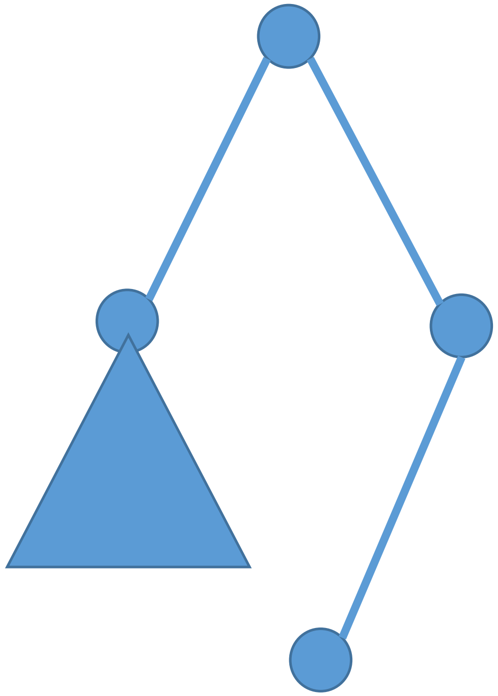

improving move removes an overcharge
but can create a different one

repeat

potential argument shows sequence is finite
eventually, there is no overcharging

how do we extend to multiple arrivals/departures?

now, overcharging on multiple subgraphs

(1) overcharging only done by leaves of the routing tree
except possibly one subgraph charged by 2 non-leaves

(2) if there is overcharging, then there is an improving move
that maintains invariant (1)

(3) potential decreases over time

(4) eventually, there is no overcharging

summary

equilibria in network games can have linear inefficiency
but the best equilibrium has **log** inefficiency
open: does it only have **constant** inefficiency?
yes, for broadcast and multicast on quasi-bipartite

open: can we find **any** equilibrium in polynomial time?

if agents join/leave/move **arbitrarily**, inefficiency can be **linear**
but controlling the moves yields **log** inefficiency

thank you

questions?