Shortest Paths and Probabilities on Time-Dependent Graphs - Applications to Transport Networks

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Workshop DIMACS. – Paris

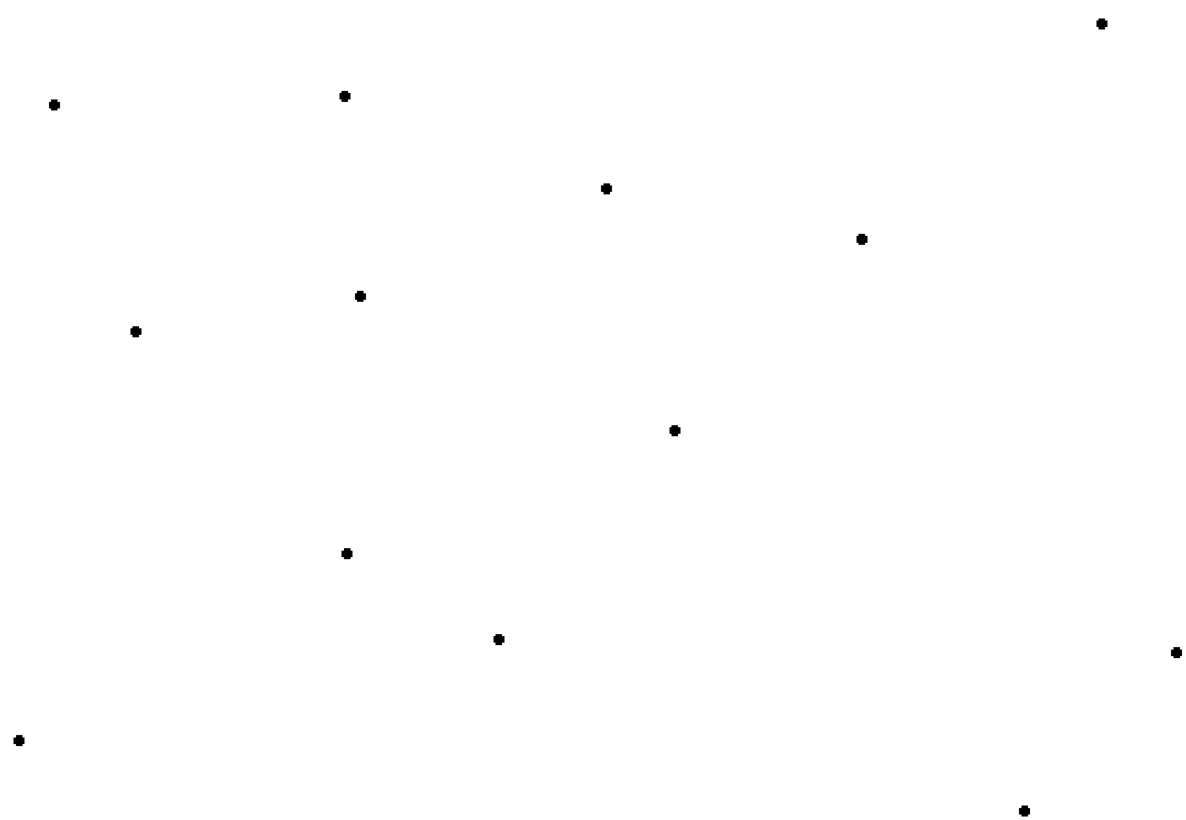
Agenda

- Our framework
- How do we model a Network?
- Shortest Path algorithms
- Why do we use time dependence?
- Our algorithm
- Conclusion and further work

Our framework

- Model transport networks
- « How long will last my journey with a probability of at least 99 % ? »
- « What is the path from v_i to v_j which length is the lowest guaranteed with a probability of at least 99 % ? »

How do we model a Network?



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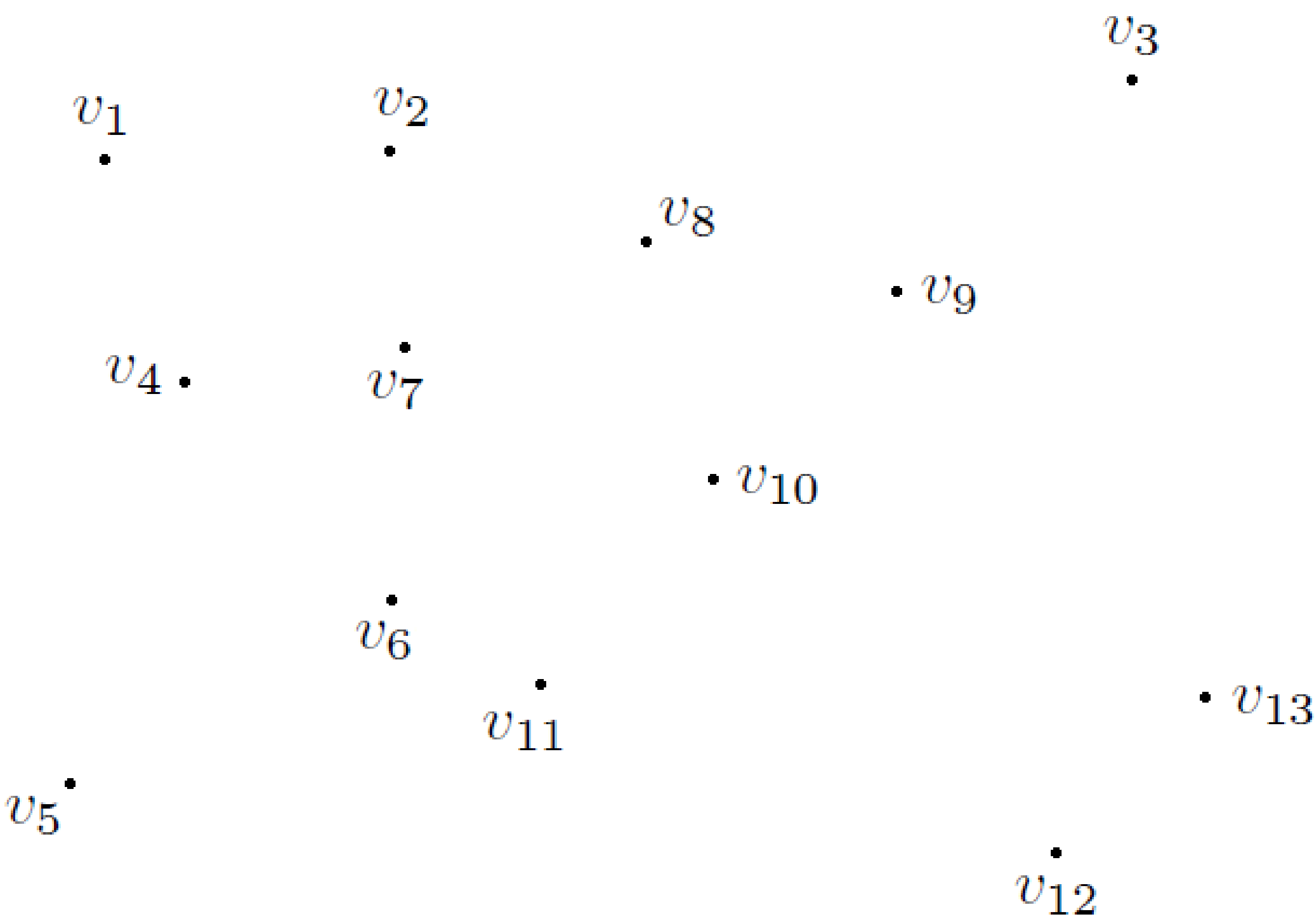

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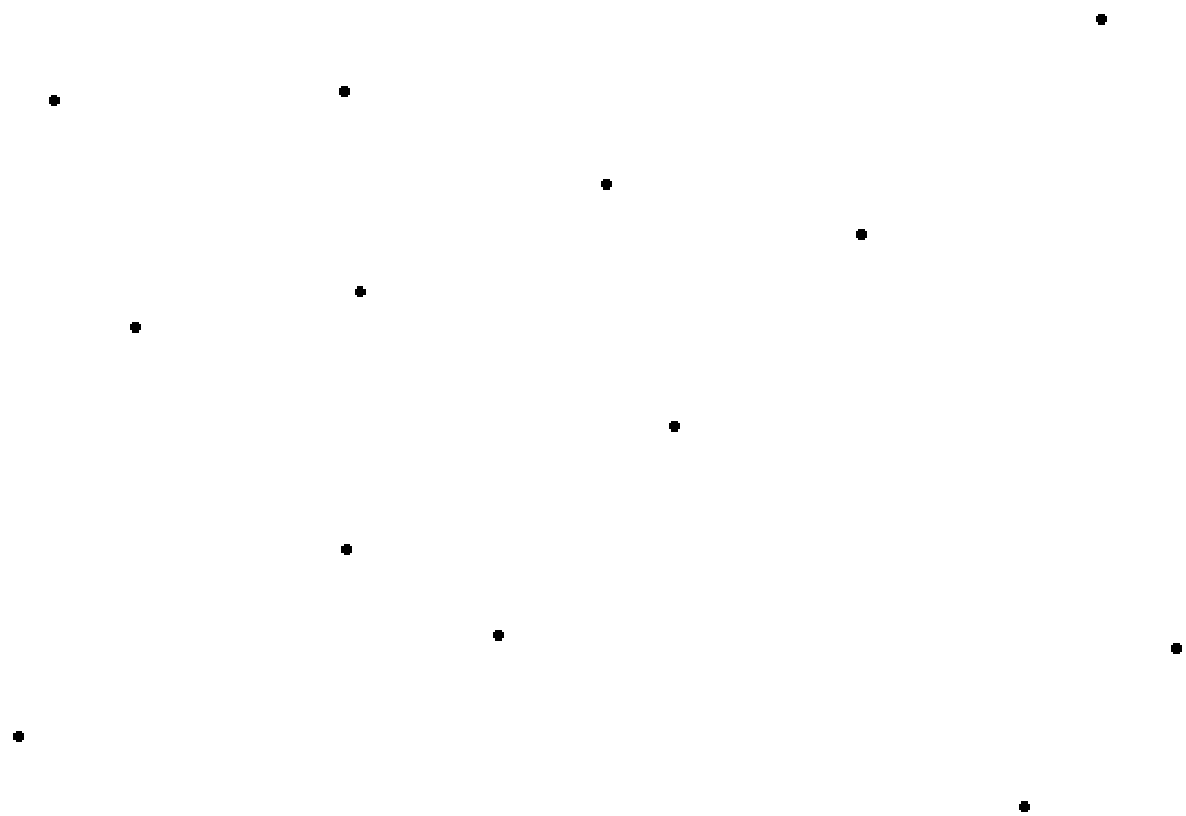

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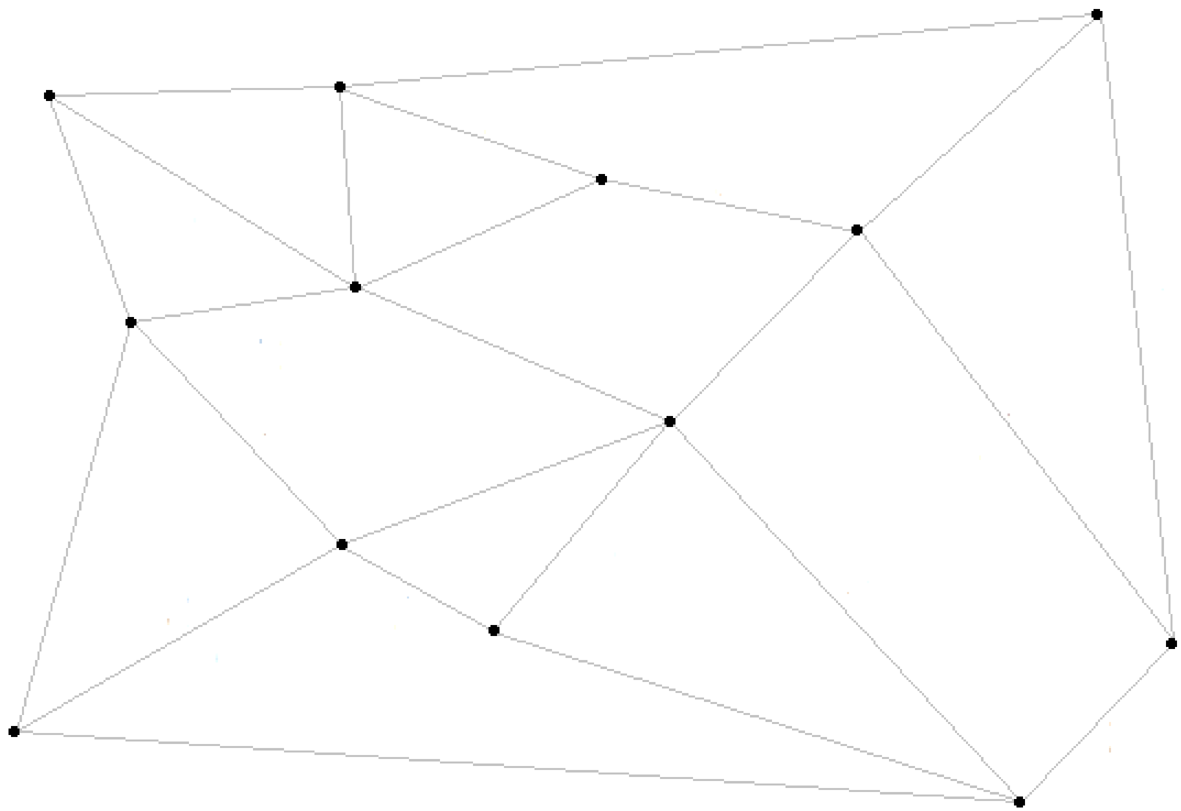

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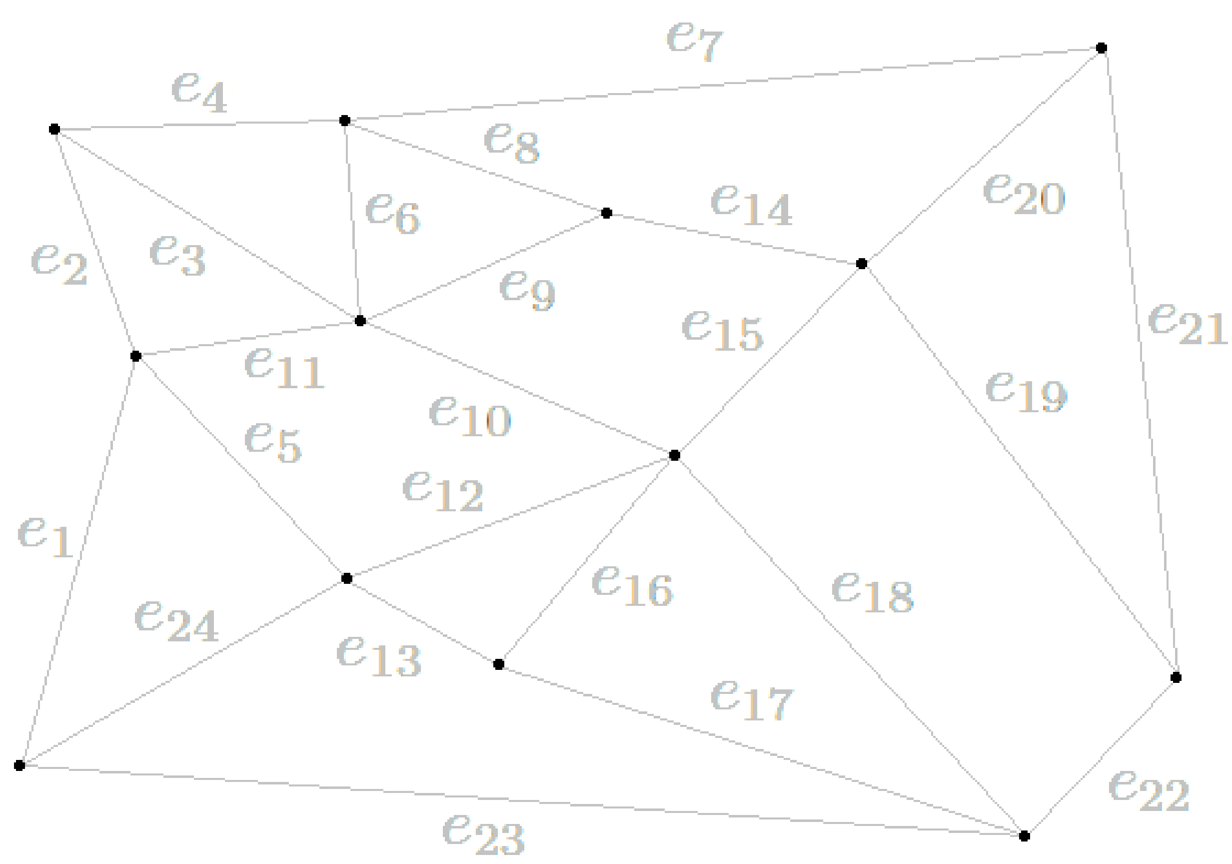

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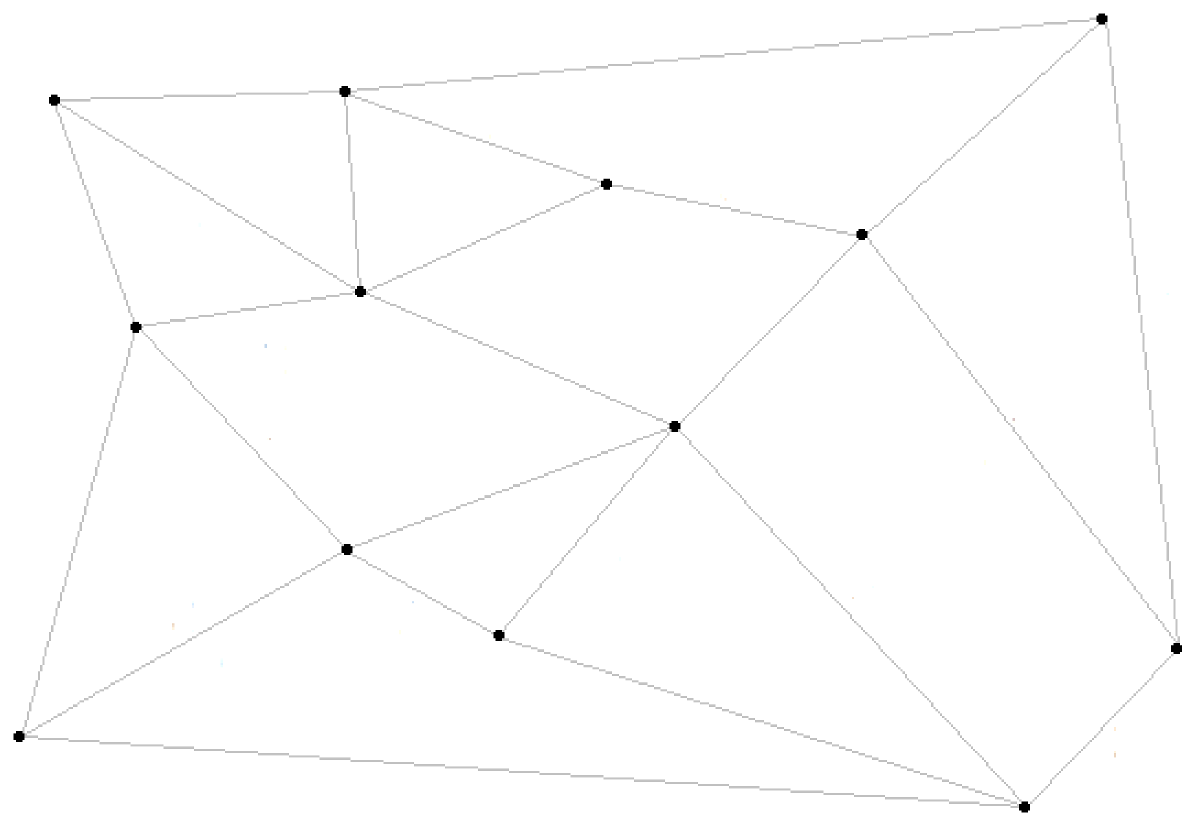

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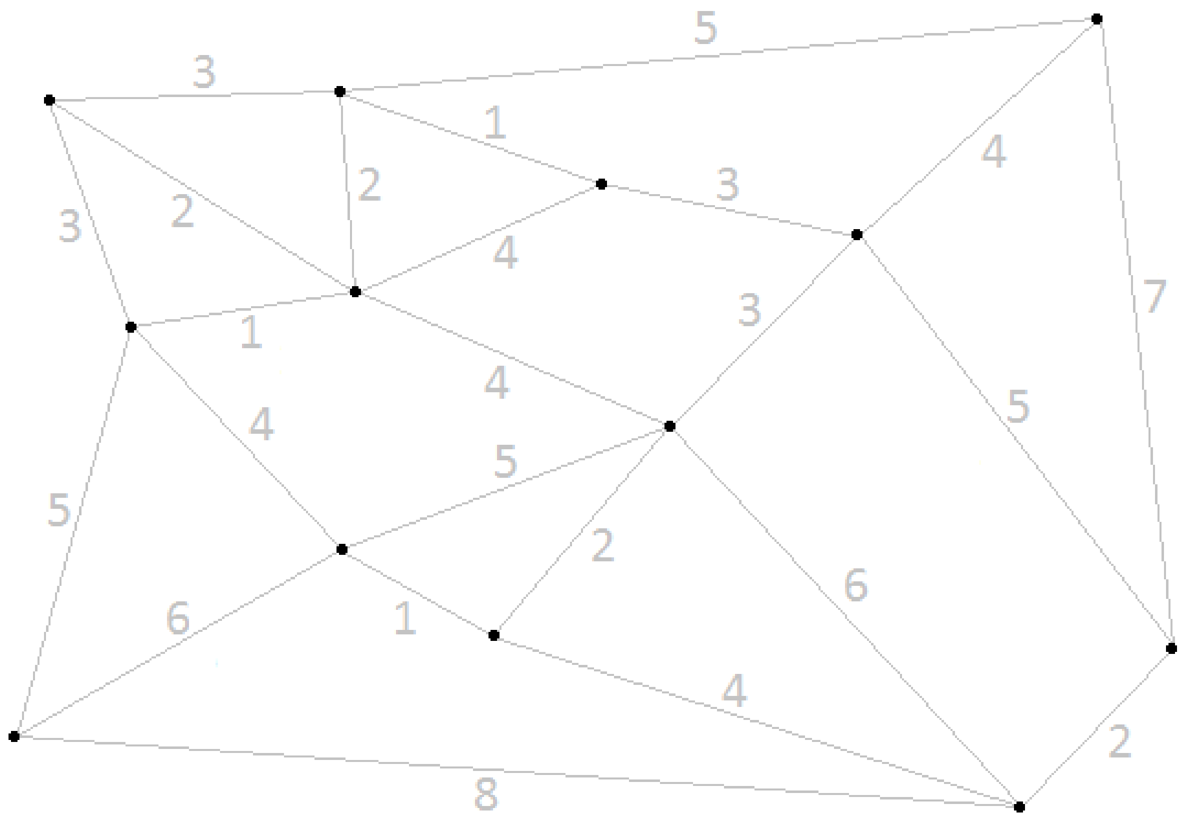

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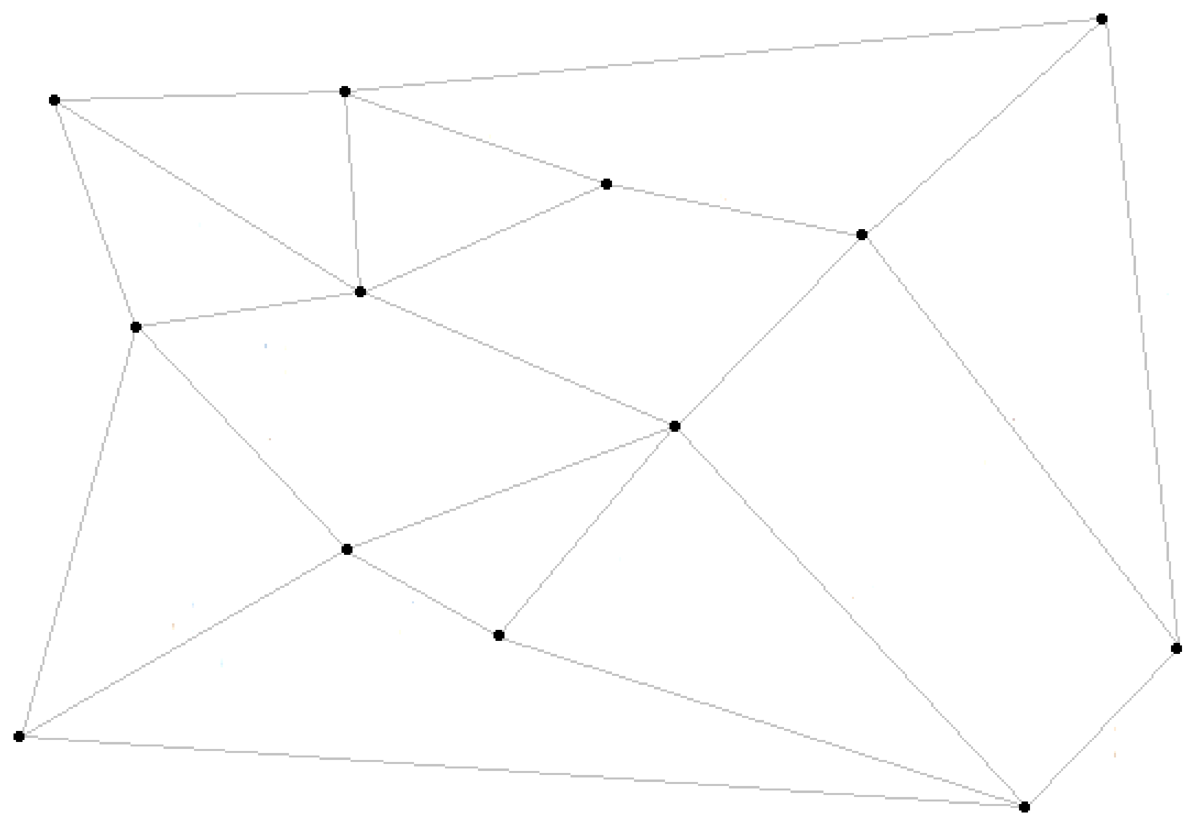

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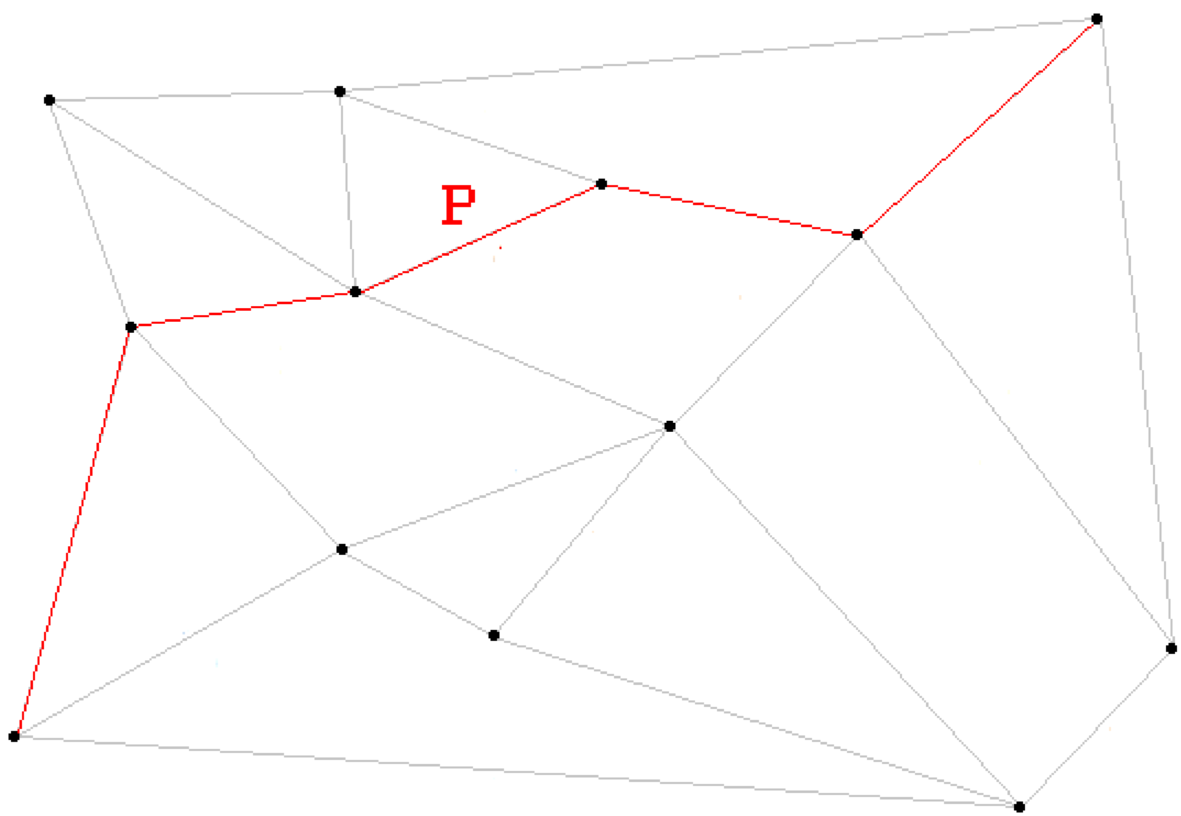

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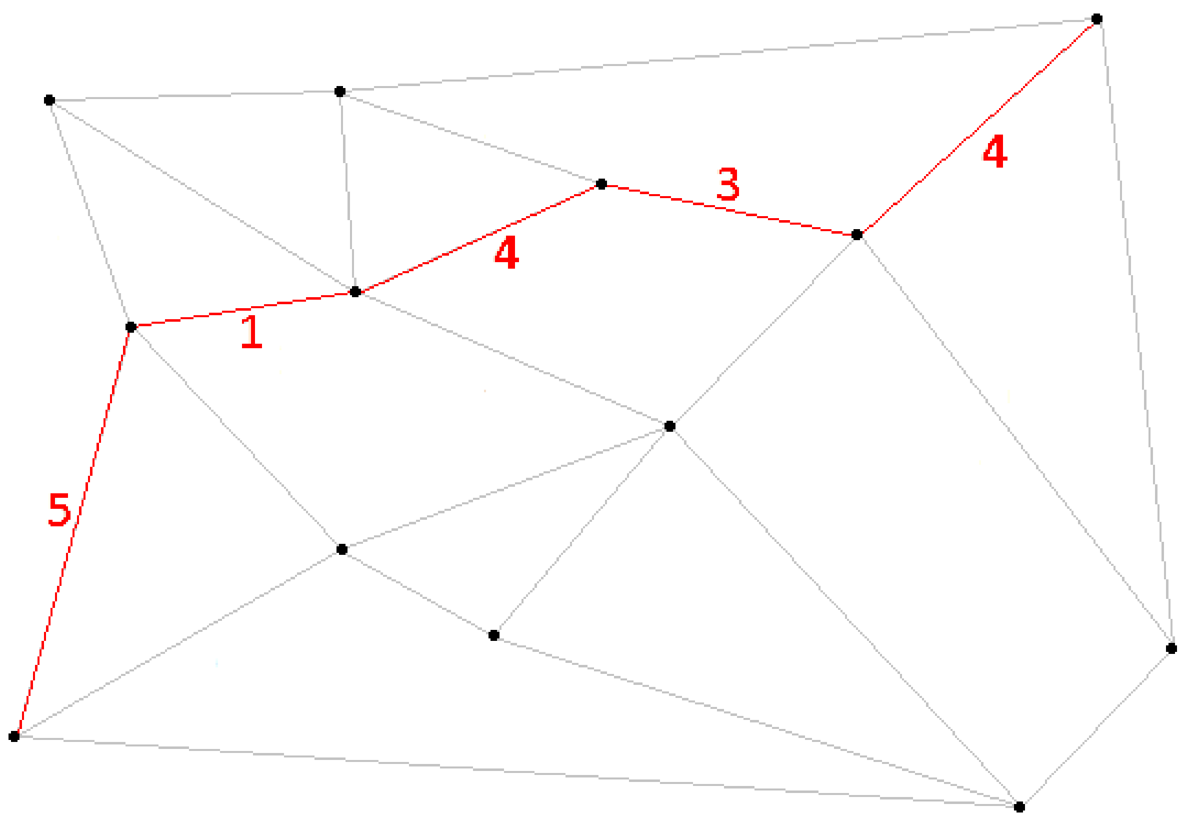
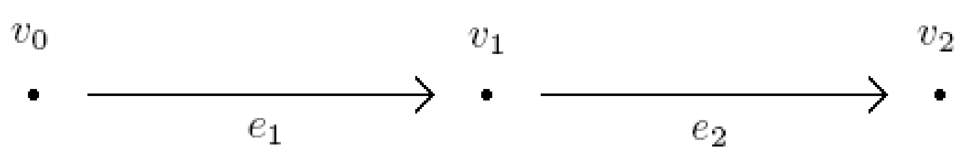
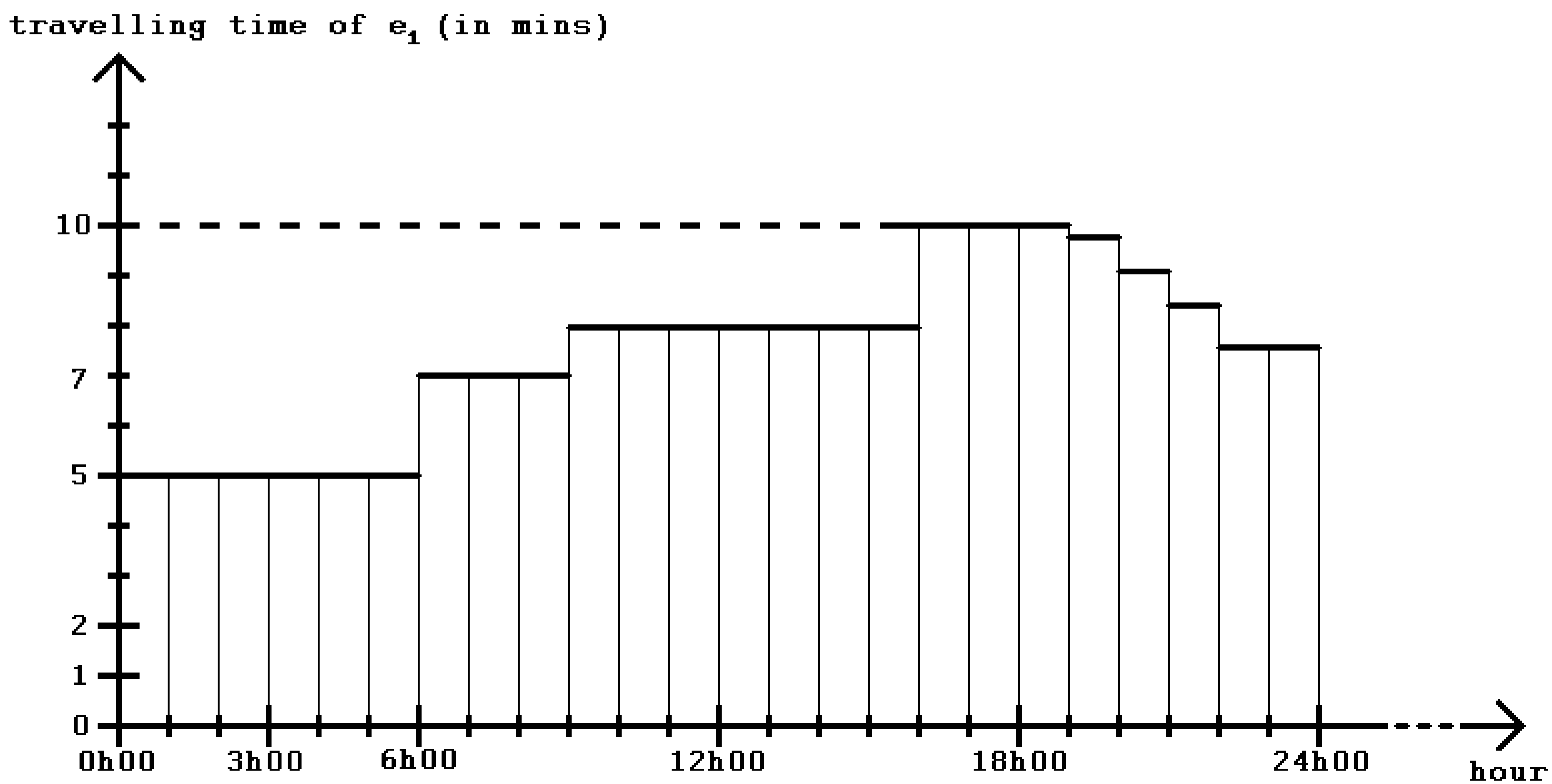


How do we model a Network? Time-dependent Graphs

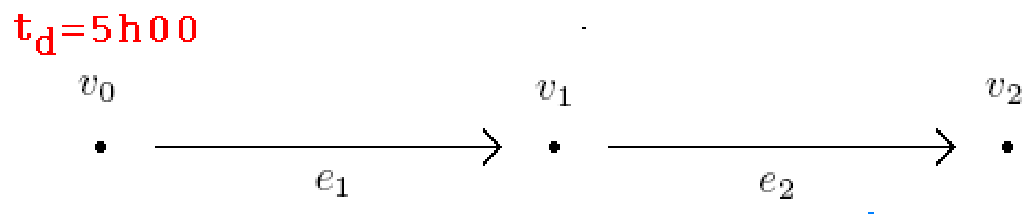
$$v_0 \xrightarrow[e_1]{} v_1 \xrightarrow[e_2]{} v_2$$

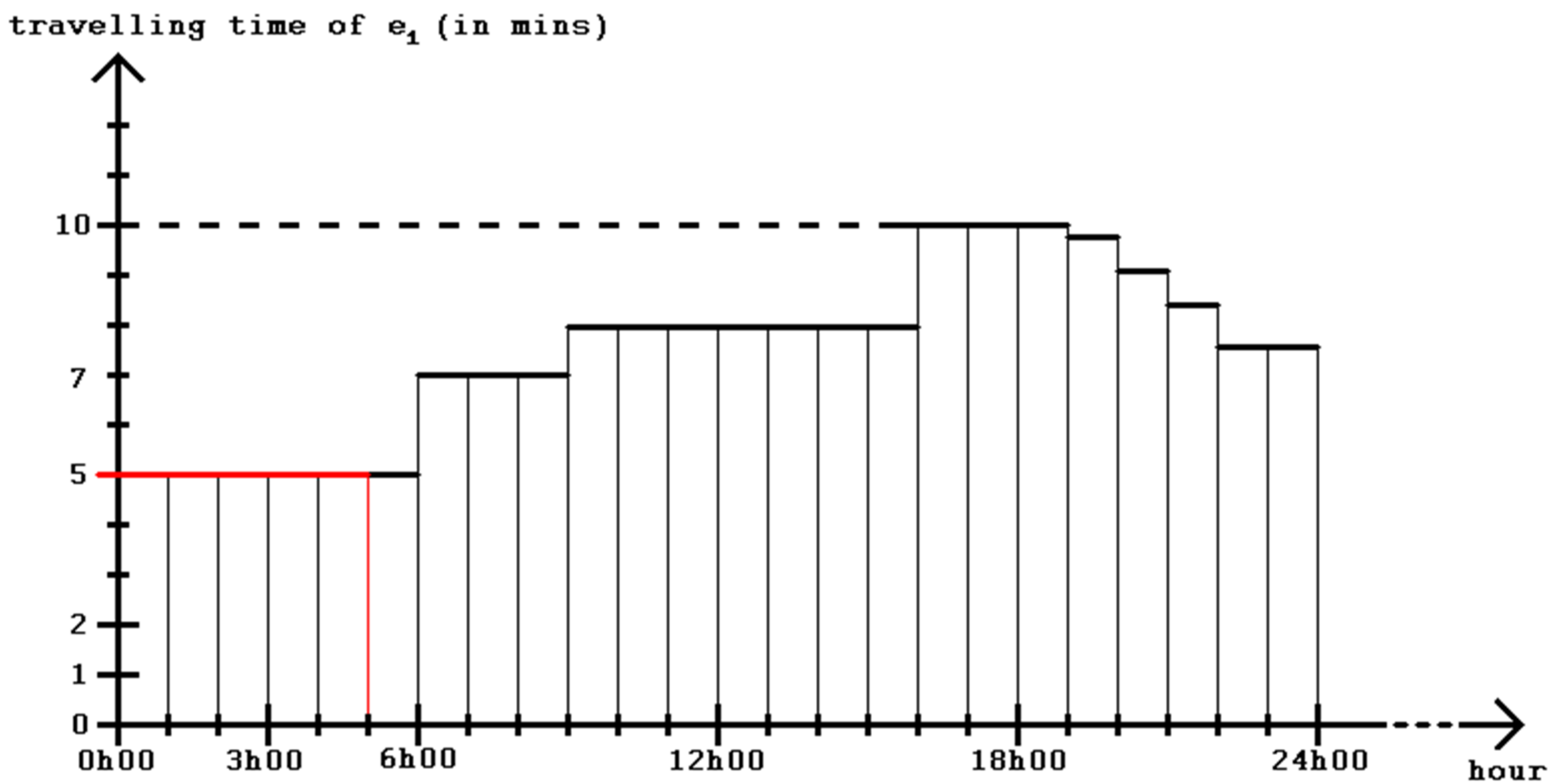
How do we model a Network? Time-dependent Graphs





How do we model a Network?
Time-dependent Graphs





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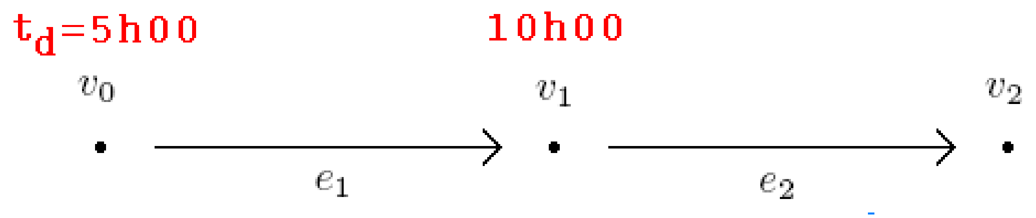

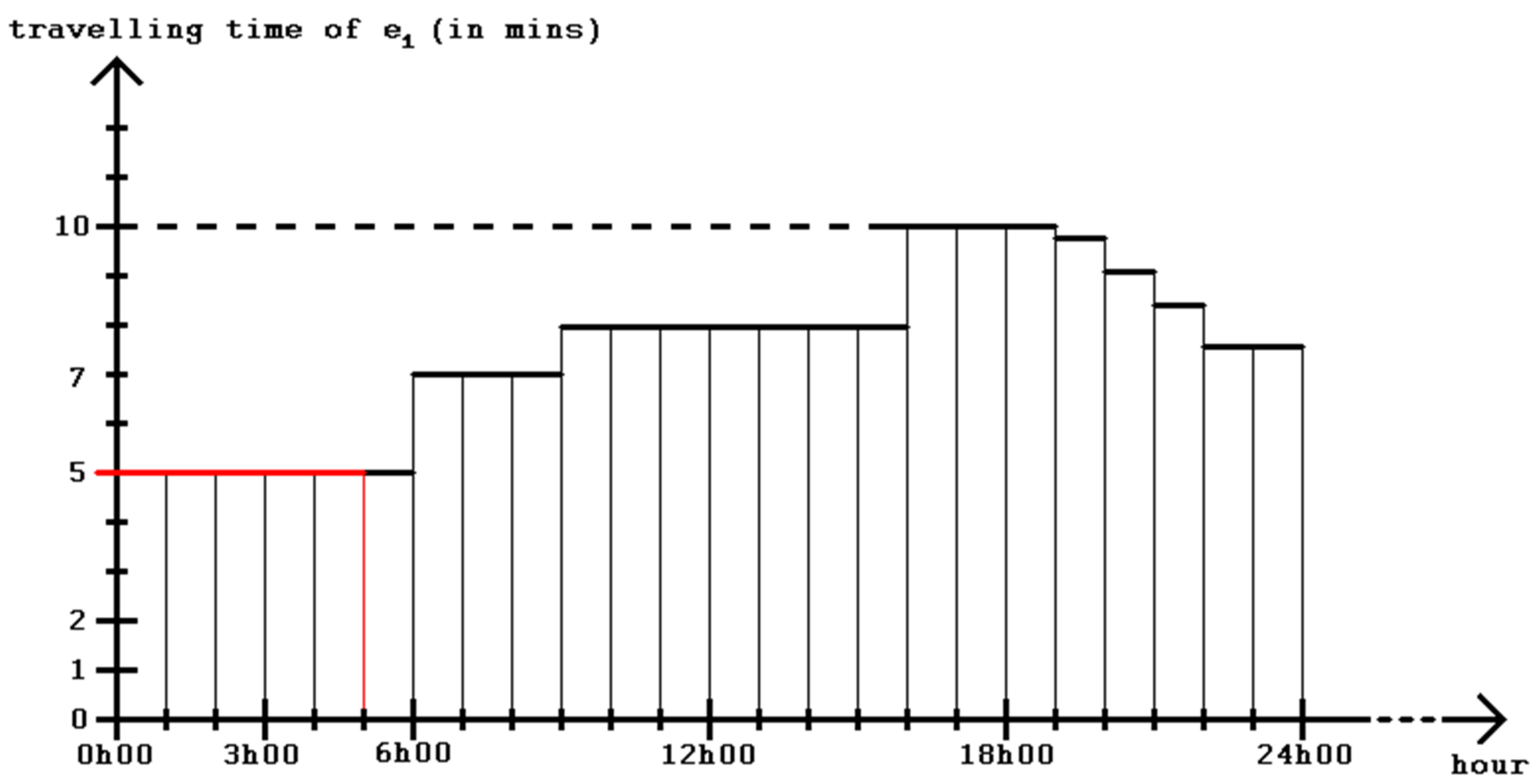

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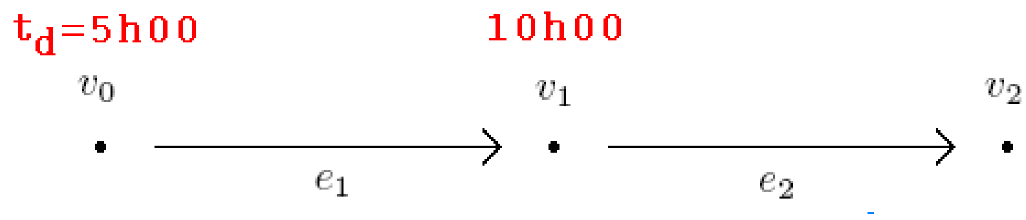

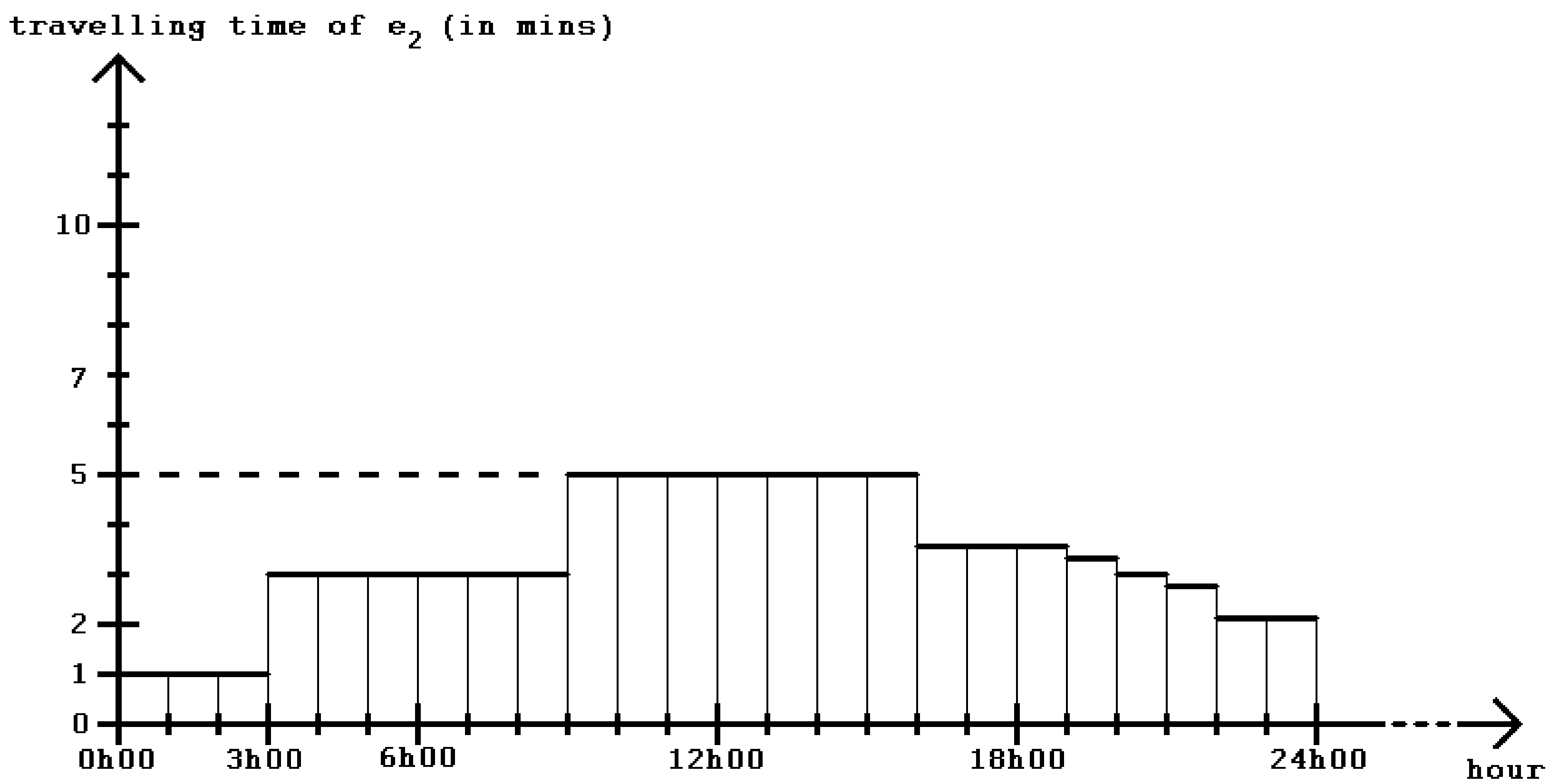

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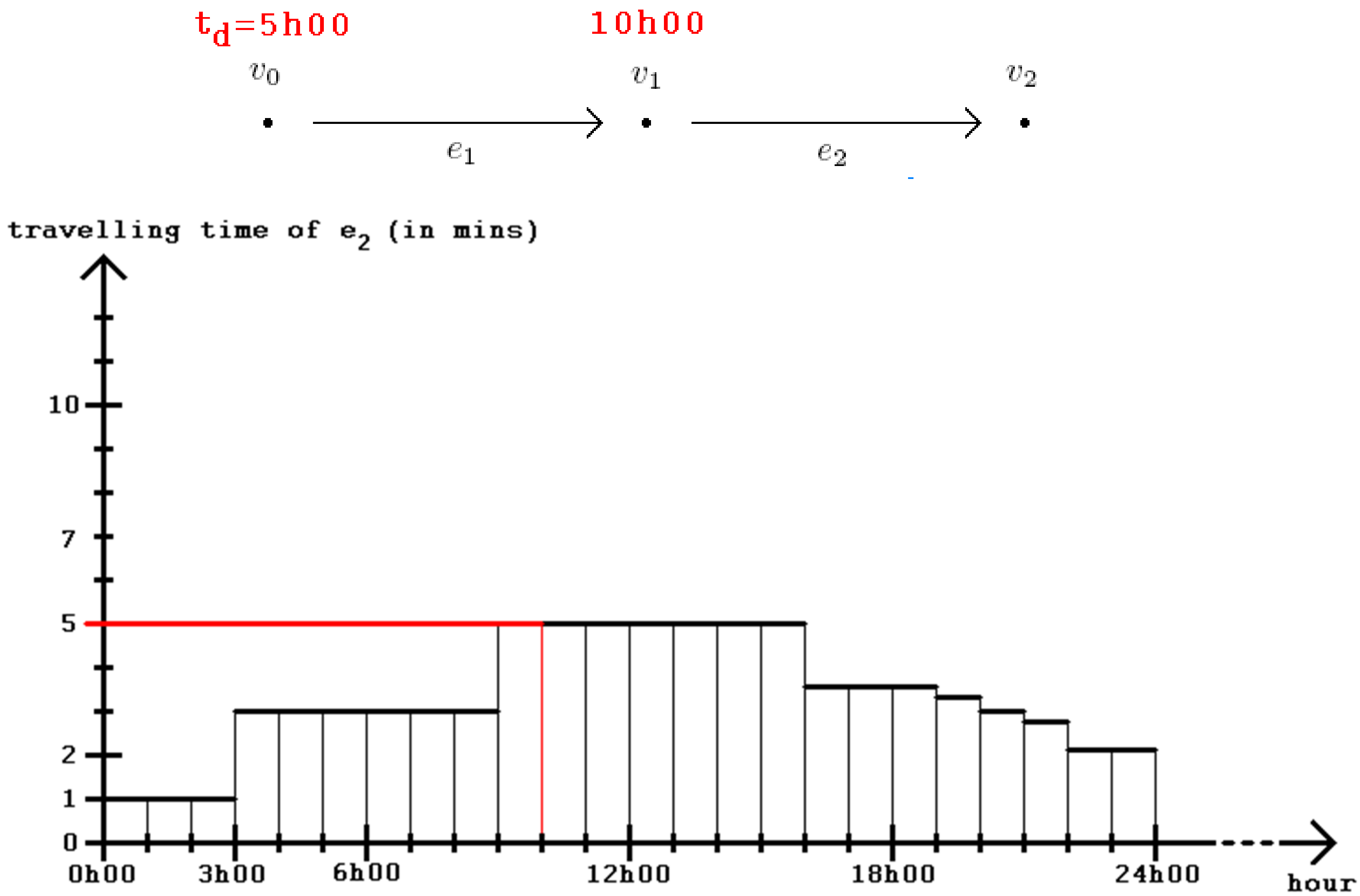

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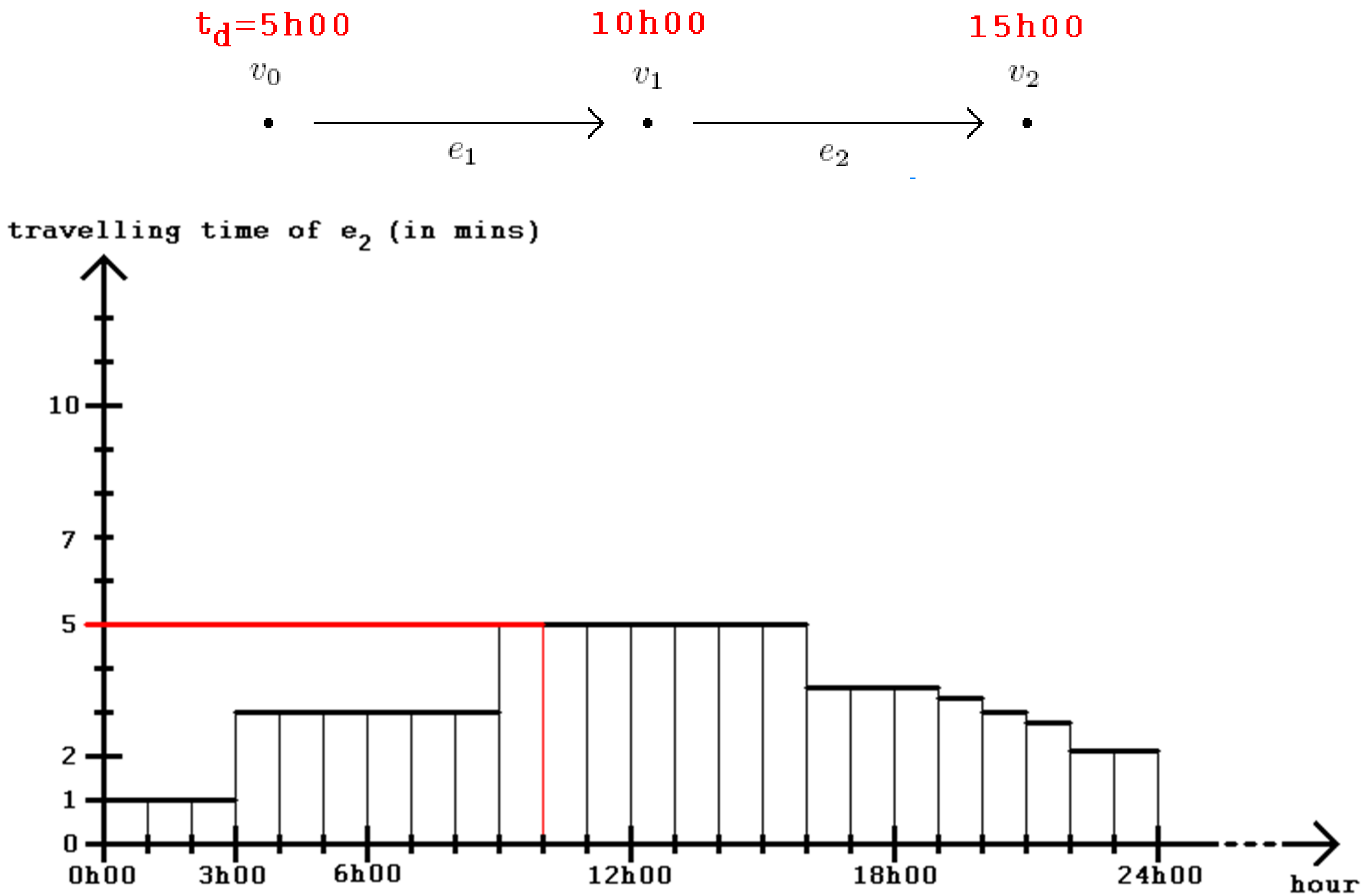


Shortest path algorithms

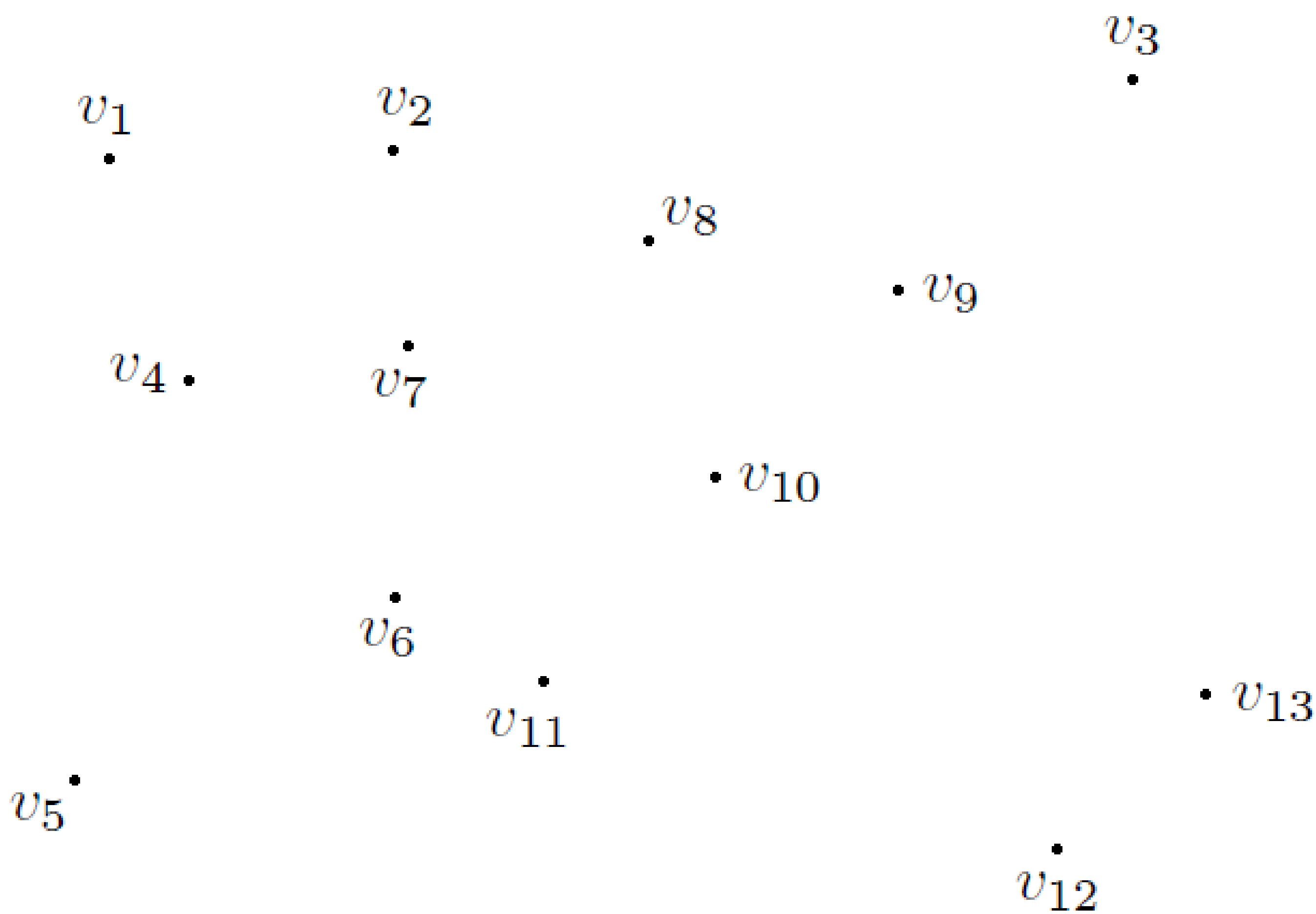
- Principle of Optimality :

  Every subpath of a shortest path is a shortest path

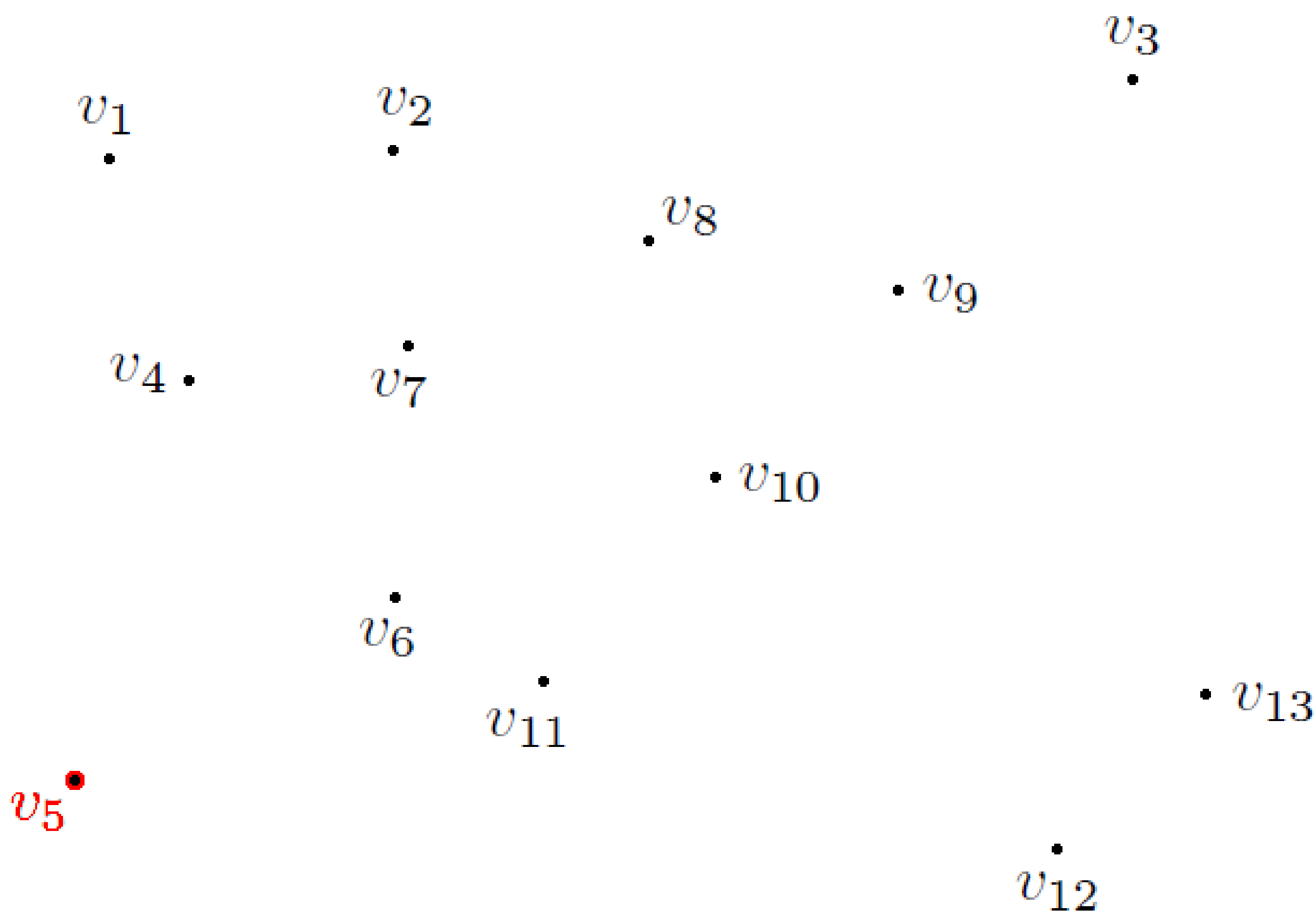
Shortest path algorithms

- Dijstra : complexity of $O(m+n.\ln(n))$
- Bellman-Ford : complexity of $O(n^3)$

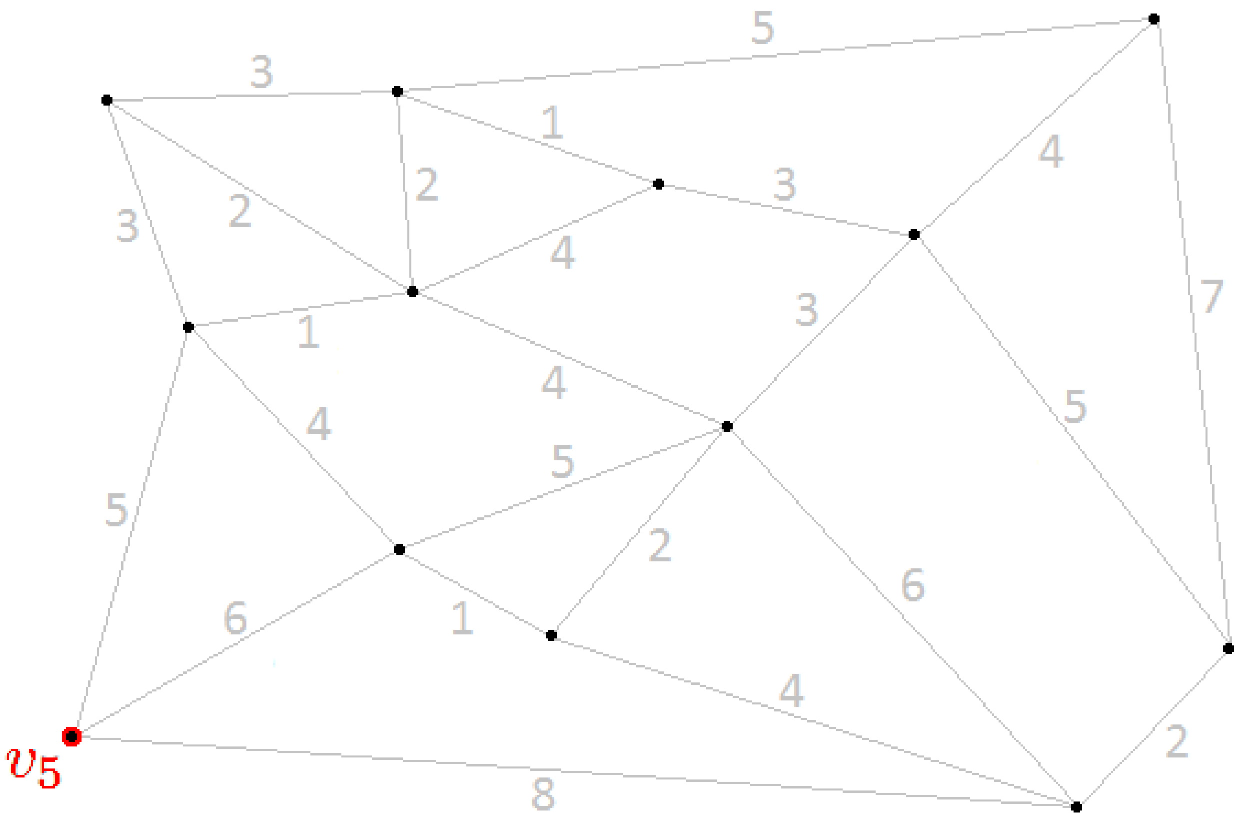
Shortest path algorithms
Dijkstra algorithm



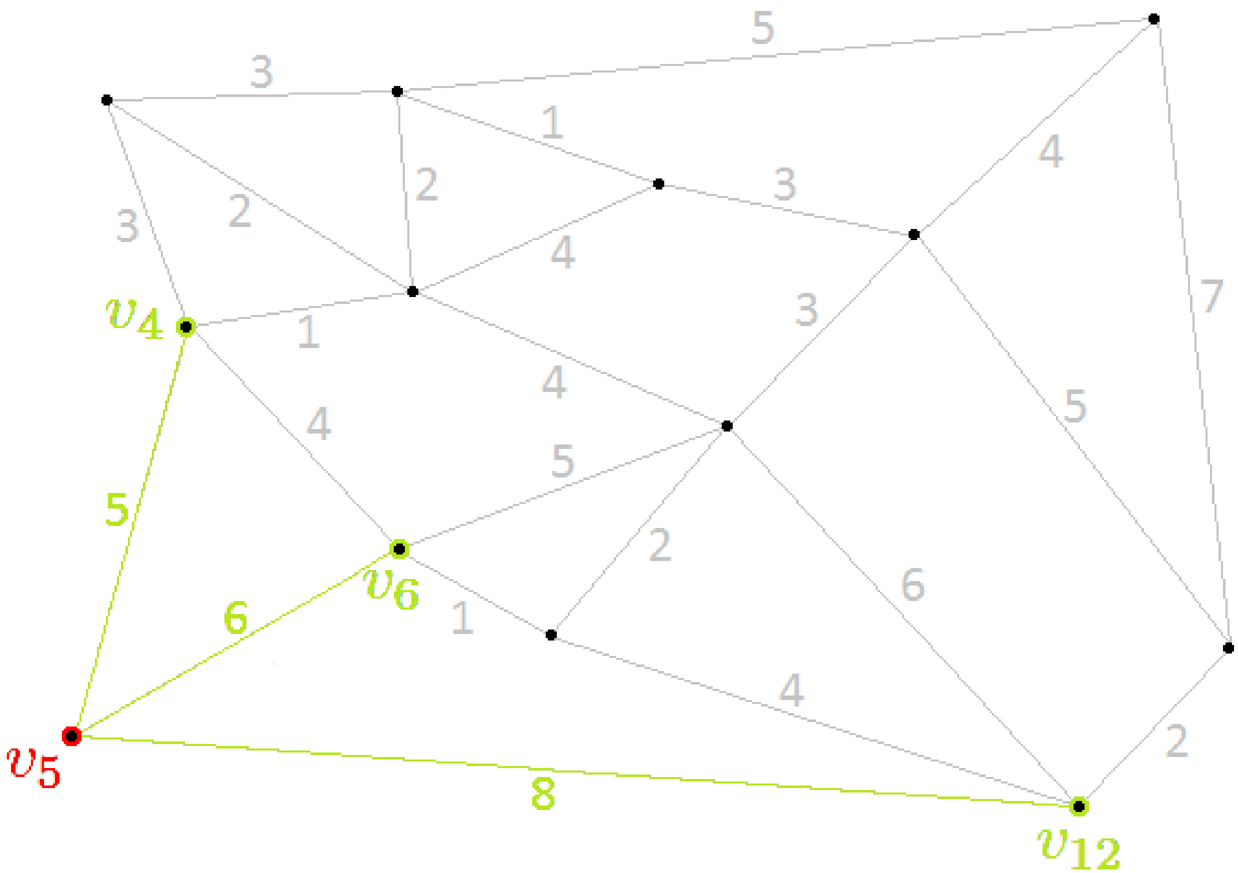
Shortest path algorithms
Dijkstra algorithm



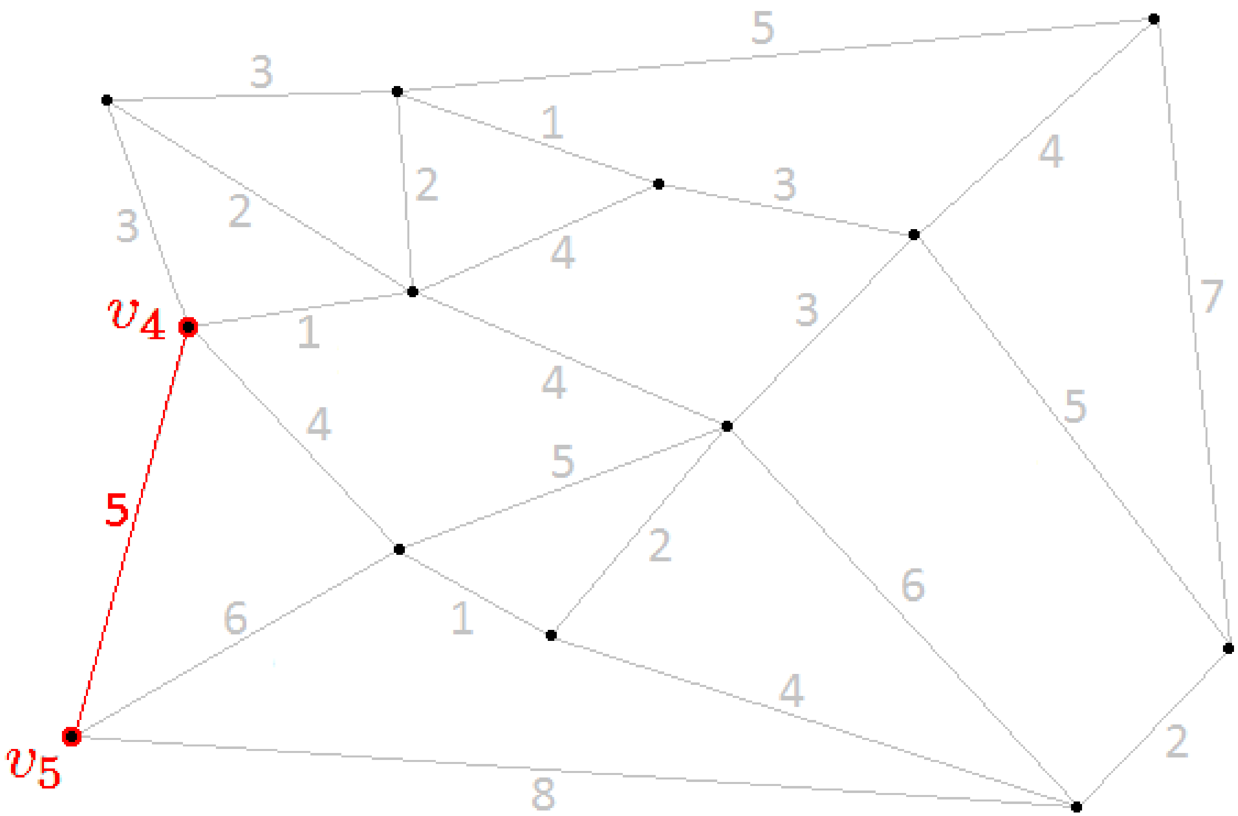
Shortest path algorithms
Dijkstra algorithm



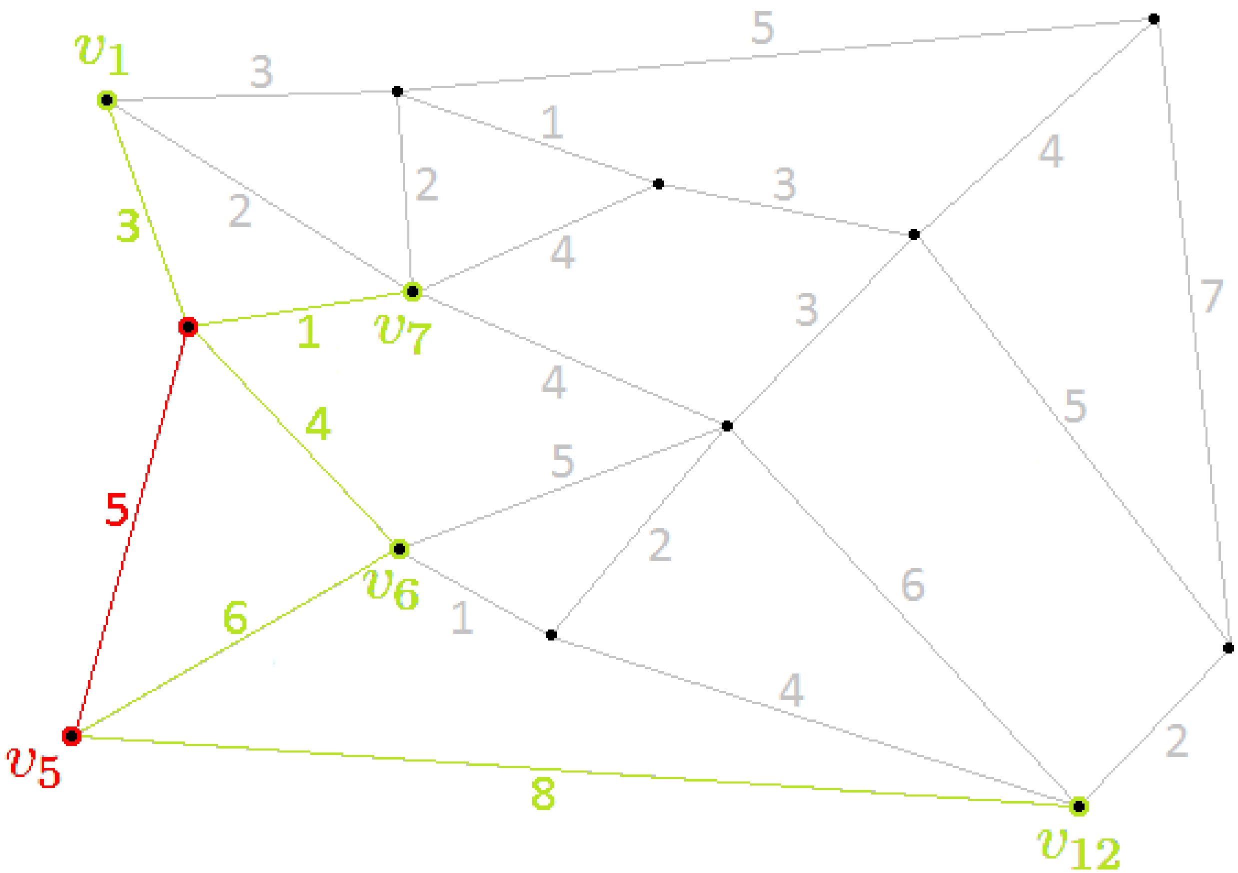
Shortest path algorithms
Dijkstra algorithm



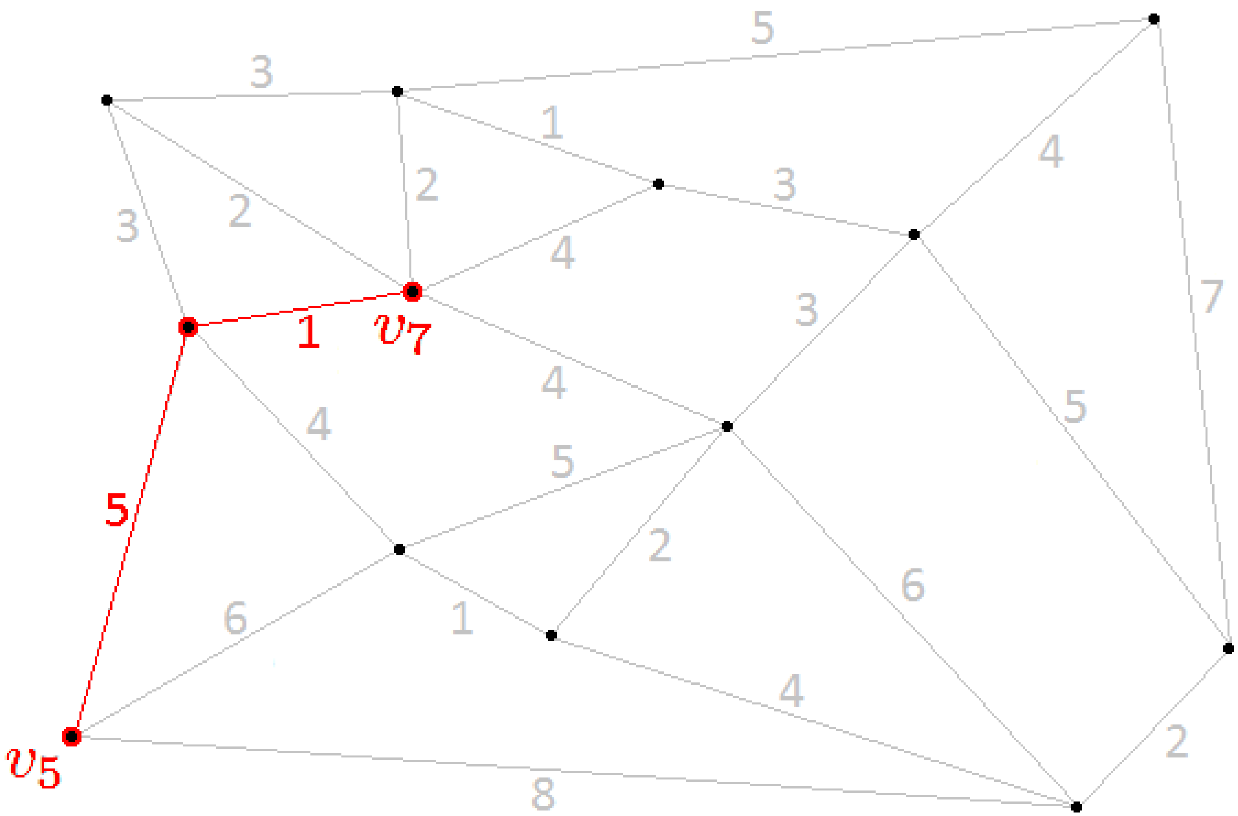
Shortest path algorithms
Dijkstra algorithm



Shortest path algorithms
Dijkstra algorithm



Shortest path algorithms
Dijkstra algorithm



Shortest path algorithms
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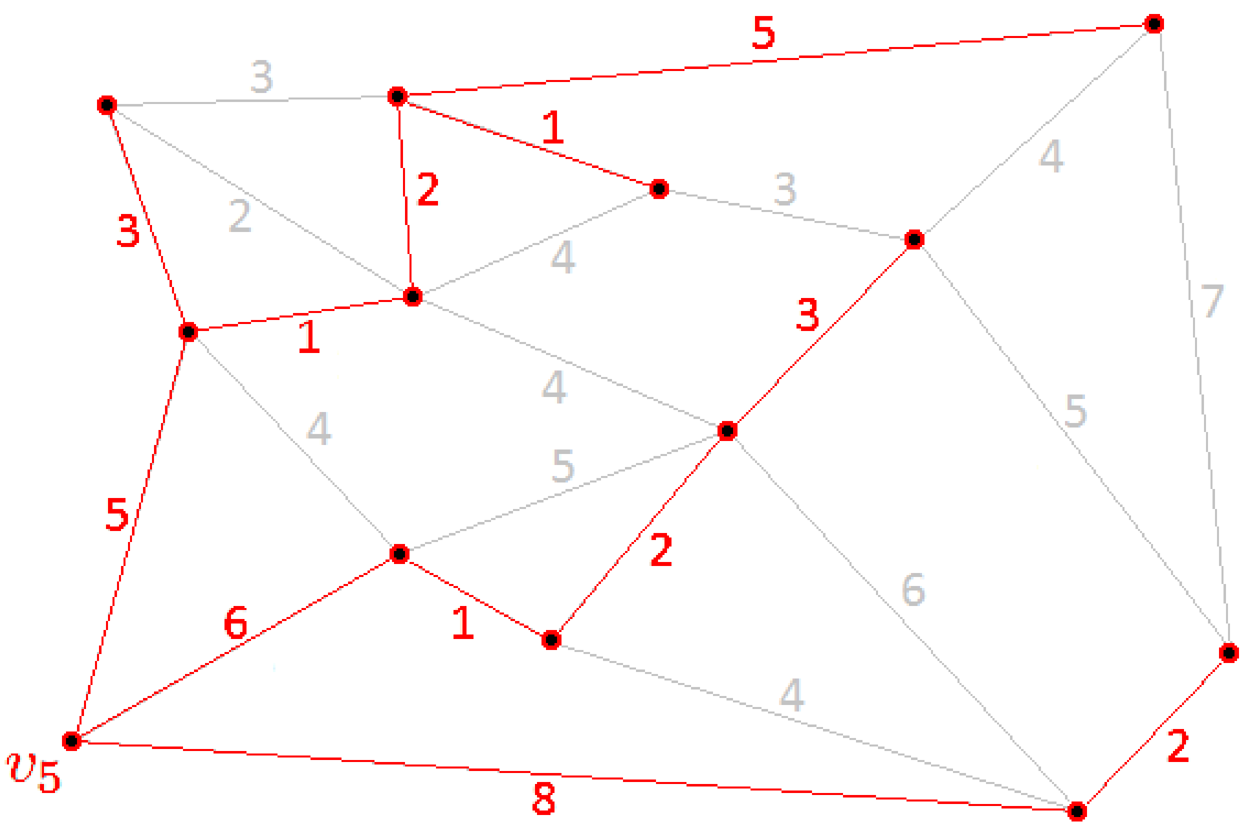


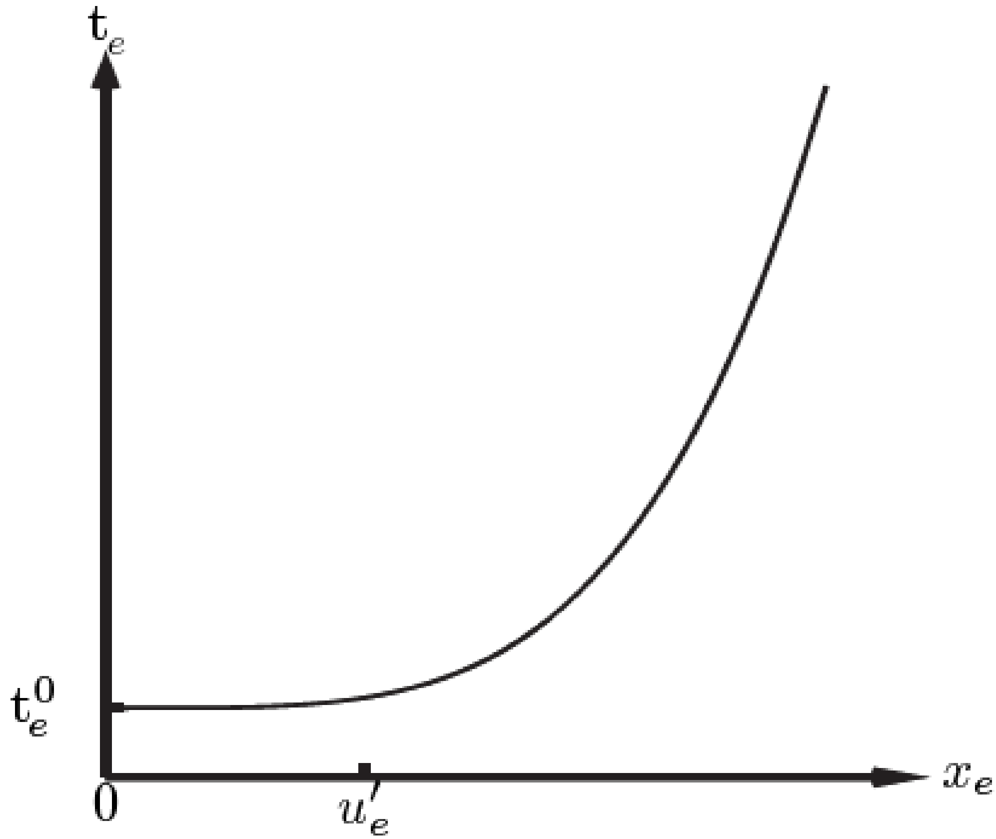
Why do we use time dependence?

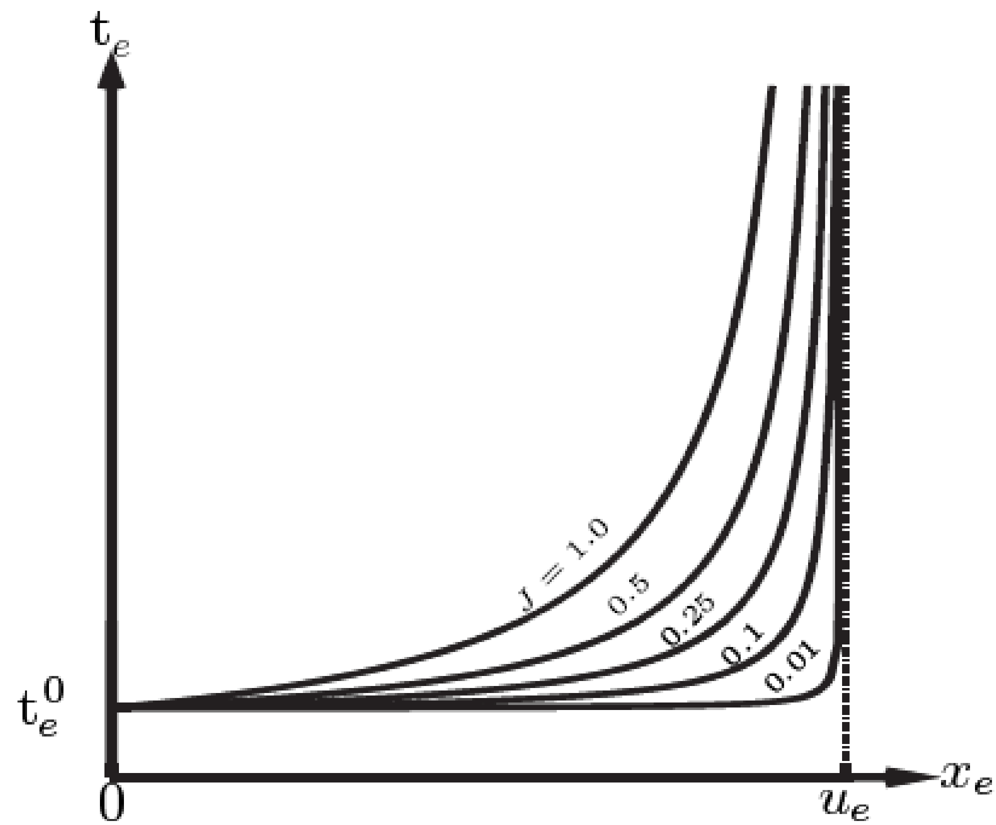
Why do we use time dependence?

- Number of sensors is increasing :
  - GPS on mobile phones
  - CCTV cameras
  - Magnetic loops
  - Embedded accelerometers and compasses

- load and travelling times are linked

load and travelling times are linked





$$t_e = t_e^0(1+0.15(x_e/u'_e)^4)$$

$$t_e = t_e^0(1 + Jx_e/(u_e - x_e))$$

Link between the load x_e and the travelling time t_e of a road e

(pictures from « Flows over time with load-dependent transit times », E. Kôhler and M. Skutella)

Our algorithm

- FIFO property
- Principle of optimality

Probability density

$$\delta_e(t_d, \cdot) = \text{probability density function of the random variable that gives the time needed to traverse } e \text{ starting at } t_d.$$

$$p_e(t_d, t_p) = \int_{-\infty}^{t_p} \delta_e(t_d, \epsilon)\, d\epsilon$$

Probability density

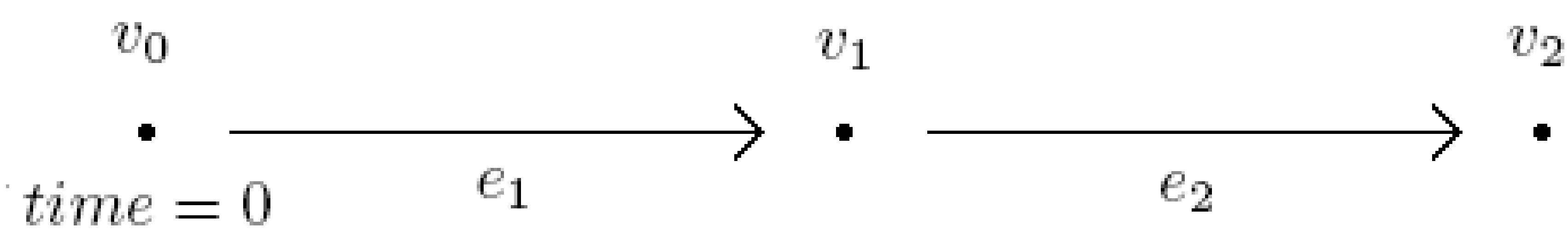
$$\delta_{e+e'}(t_d, t_p) = \int_{z=-\infty}^{+\infty} \delta_e(t_d, z)\delta_{e'}(t_d + z, t_p - z)dz$$

$$p_{e+e'}(t_d, t_p) \geq p_e(t_d, t_1) * p_{e'}(t_d + t_1, t_p - t_1)$$

Principle of our algorithm

$$v_0 \xrightarrow{\;e_1\;} v_1 \xrightarrow{\;e_2\;} v_2$$

Principle of our algorithm



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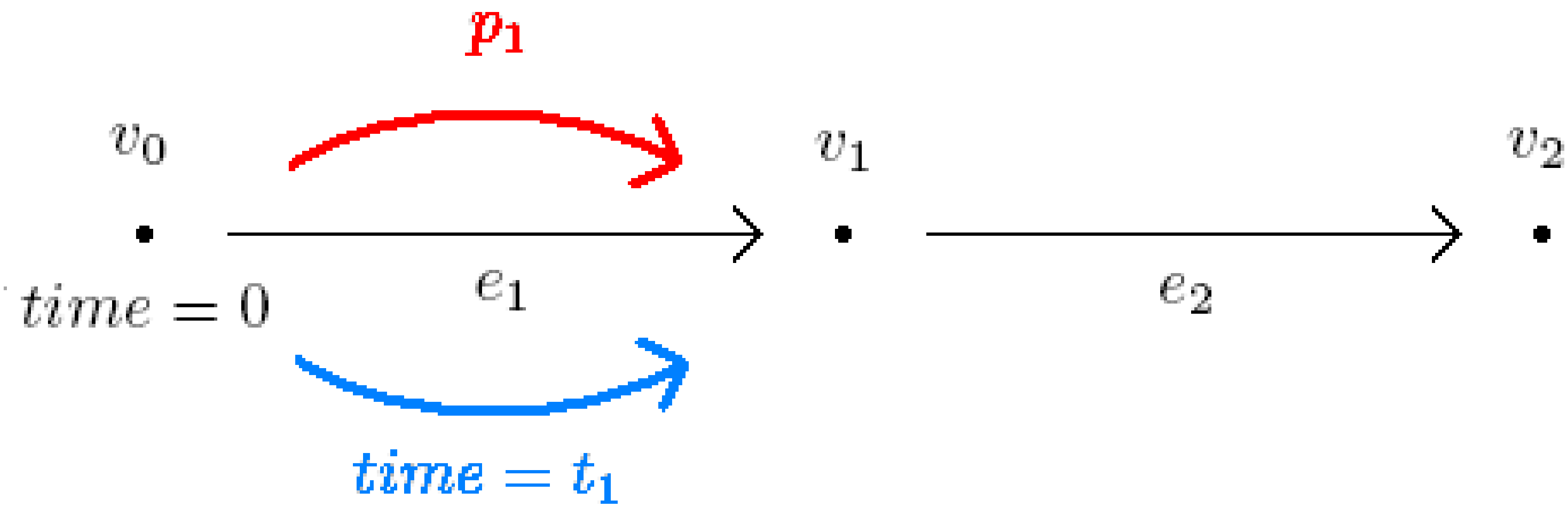

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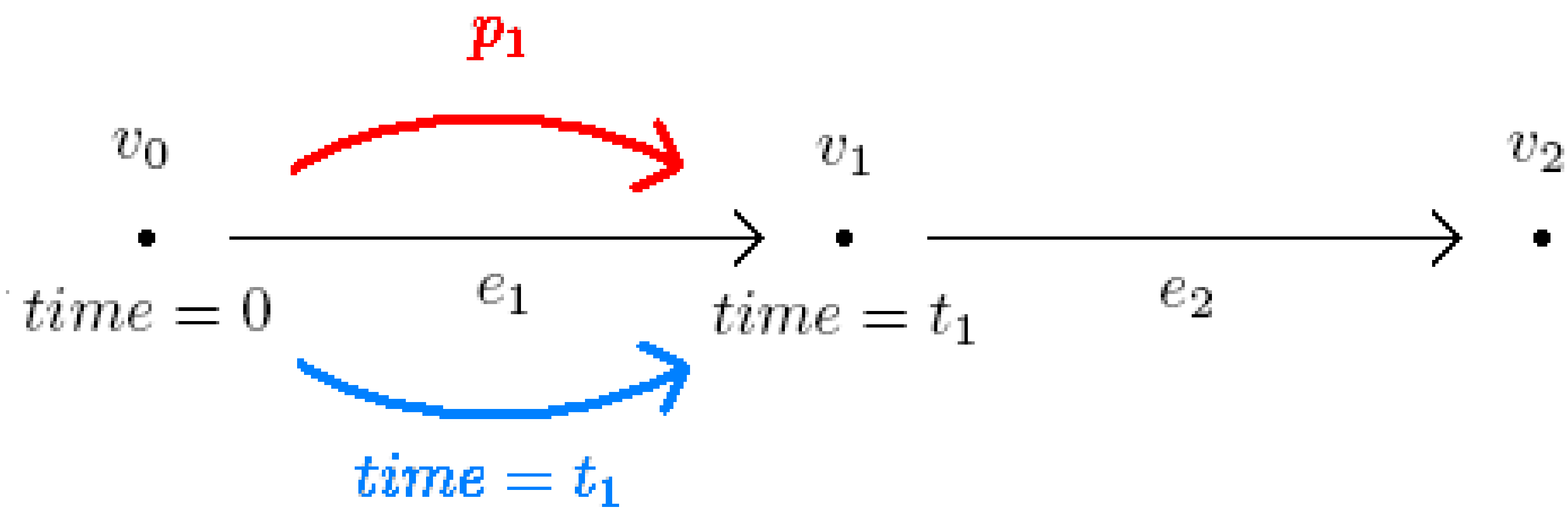

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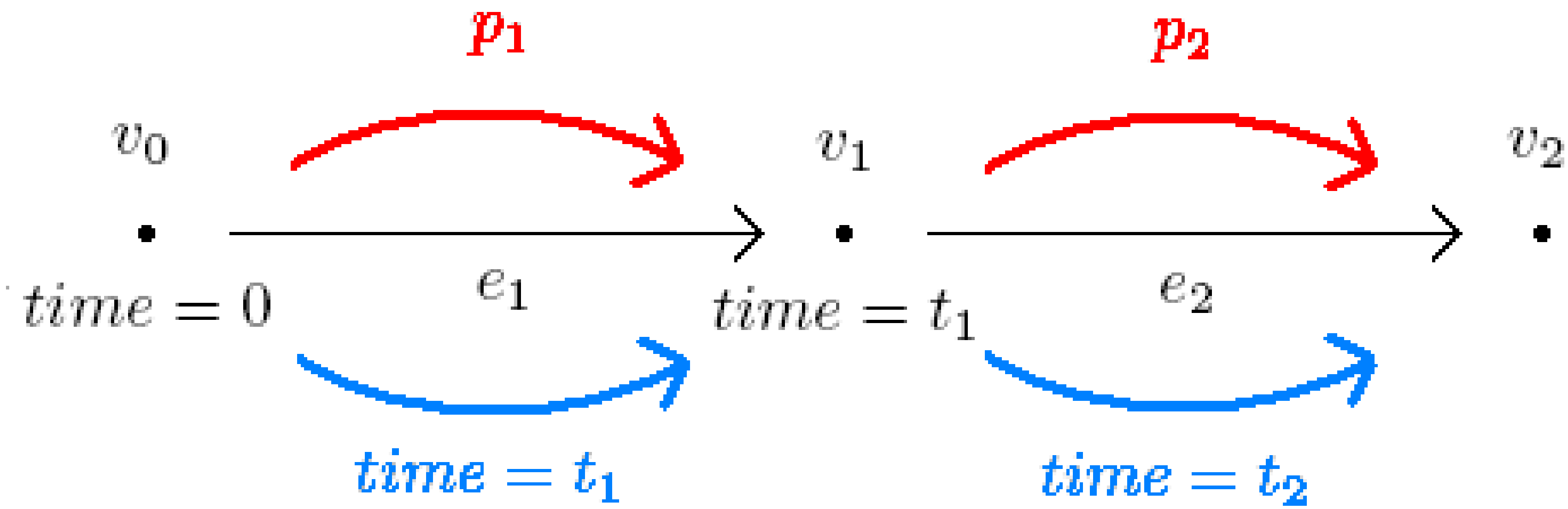

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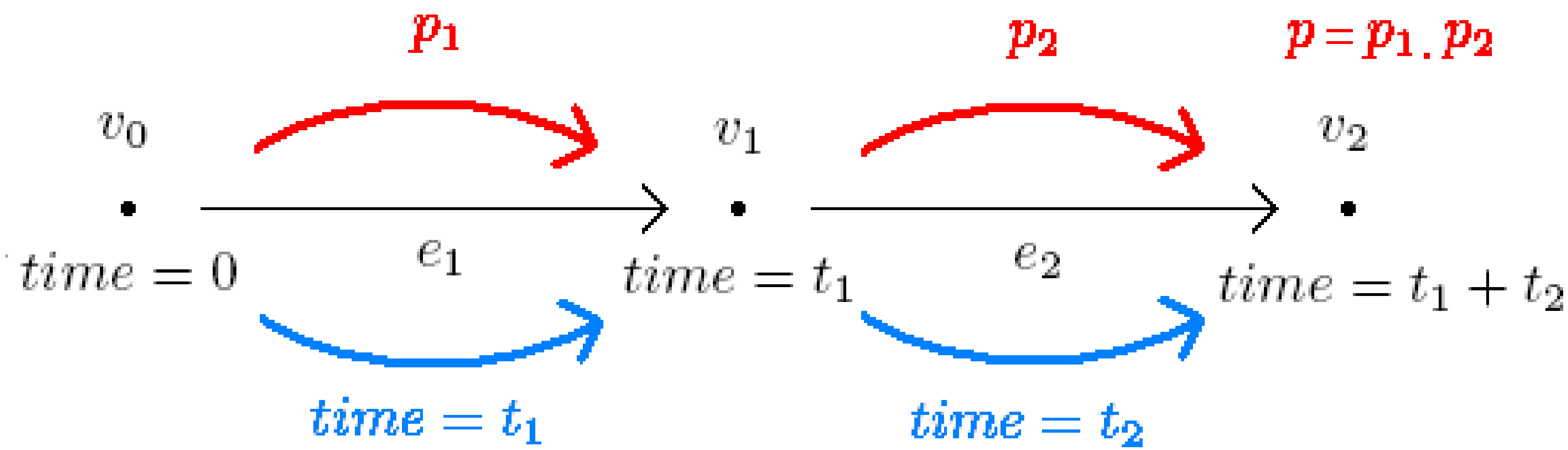
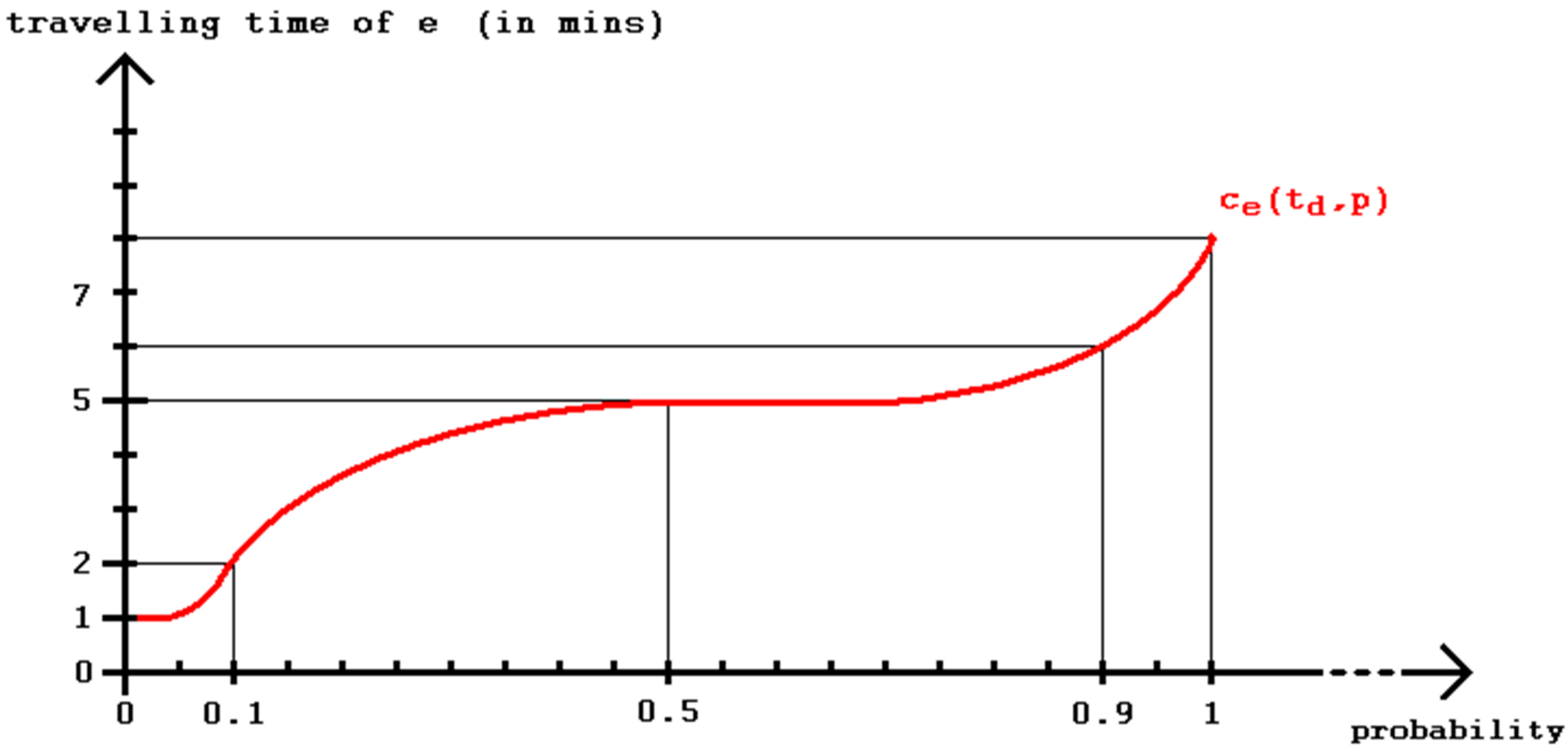


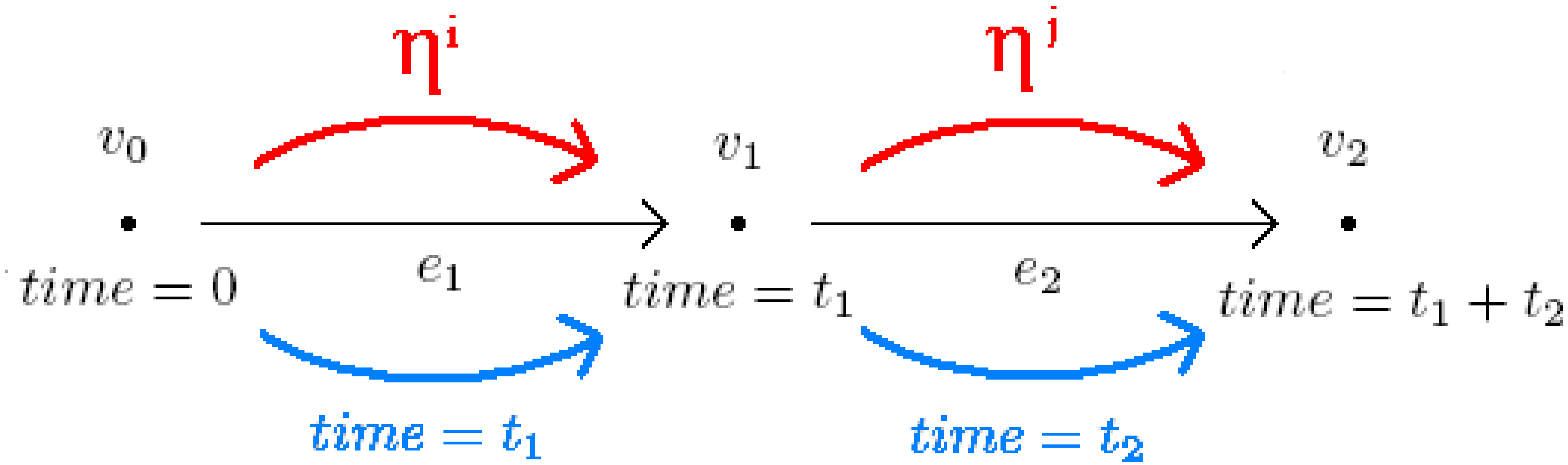
Our input data



Our algorithm - Example

- Let $\eta=1-10^{-N}$, with $N > 0$
- Let $N = 2$:
  - $\eta^0 = 1$
  - $\eta^1 = 0.99$
  - $\eta^2 = 0.9801 \approx 0.98$
  - $\eta^3 = 0.970299 \approx 0.97$

Principle of our algorithm



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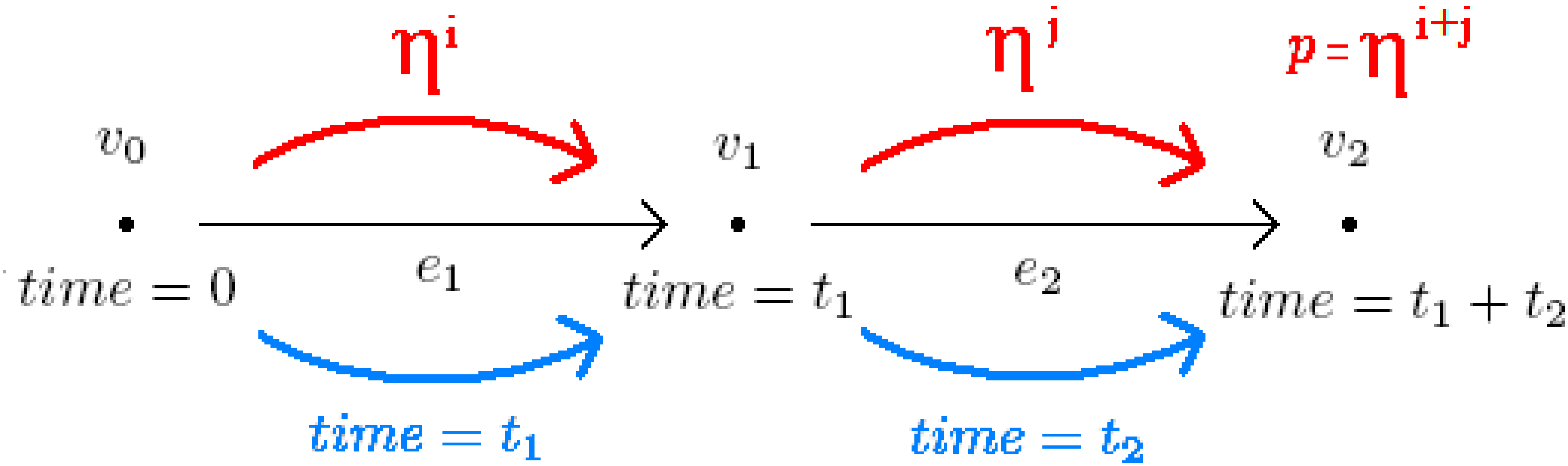


Our algorithm - Example

$$v_0 \xrightarrow[e_1]{} v_1 \xrightarrow[e_2]{} v_2 \xrightarrow[e_3]{} v_3$$

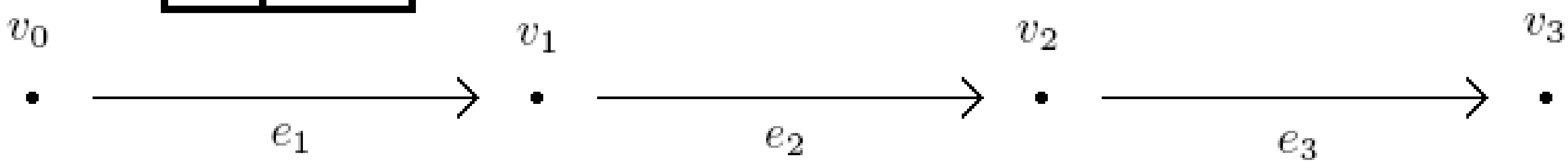
Our algorithm - Example

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5



p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	
η^1	
η^2	

Our algorithm - Example

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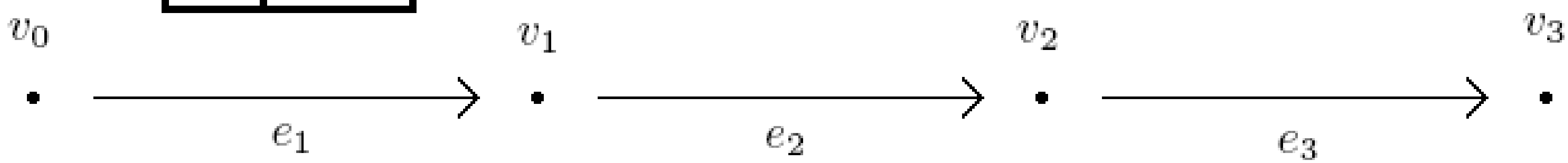

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow[e_1]{} v_1 \xrightarrow[e_2]{} v_2 \xrightarrow[e_3]{} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

Our algorithm - Example

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow[e_1]{} v_1 \xrightarrow[e_2]{} v_2 \xrightarrow[e_3]{} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	
η^2	

Our algorithm - Example

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	20

Our algorithm - Example

p	$c_{e_1}(p)$
η^0	8
η^1	7
η^2	5

p	$c_{e_2}(p)$
η^0	6
η^1	4
η^2	2

p	$c_{e_3}(p)$
η^0	10
η^1	9
η^2	7

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \xrightarrow{e_3} v_3$$

p	$l_{v_0}(p)$
η^0	0
η^1	0
η^2	0

p	$l_{v_1}(p)$
η^0	8
η^1	7
η^2	5

p	$l_{v_2}(p)$
η^0	14
η^1	12
η^2	10

p	$l_{v_3}(p)$
η^0	24
η^1	22
η^2	20

Conclusion and further work

- Shortest path algorithms on time-dependent graphs
- Result given with a certain accuracy
- Computing $c_e(t_d,p)$ functions

- Analyse data from sensors
- Get closer to the continuous model

Questions?