Shock Scaffold Segregation and Surface Recovery

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The shock scaffold is a hierarchical organization of the medial axis ($\mathcal{MA}$) in 3D consisting of special medial points, and curves connecting these points, thereby forming a *geometric directed graph* [7]. We will describe a new method for segregating the shock scaffold ($\mathcal{SC}$) for an unorganized cloud of points in 3D in two sub-groups, one of which is used to mesh the point generators into a surface interpolant.[1] In the practical scenario where point generators are sampling the surfaces of 3D objects, we expect intuitively that part of the symmetry structure in the resulting $\mathcal{SC}$ closely approximates the original *surface symmetries*, while the remaining part arises from symmetries pertaining to the *interaction of nearby sampled points*. (*e.g.*, compare Figure 1 (c) and (e-f)).

We segregate an initial $\mathcal{SC}$ in two sub-groups in three main steps: *(S1)* rank shock curves by a geometric measure of the triangle (*e.g.*, area) interpolating their associated triplet of generators; *(S2)* select a threshold to obtain a first segregation of the $\mathcal{SC}$ in the "initial surface and medial scaffolds;" *(S3)* examine the topology of each surface interpolant and clean-up the resulting surface (Fig.1).

The construction of the initial segregation of $\mathcal{SC}$ in *S2* is a "one shot," *i.e.*, non-iterative, process. We march through the sorted list of shock curves of the $\mathcal{SC}$ until an *end condition* is reached, *e.g.*, all generators are interpolated as vertex of at least one surface triangle. In general, whichever (global) *threshold* we pick, we will have certain triangles which are not part of the desired interpolant to the surface. Typically, these *extraneous* triangles occur near concave, saddle-like regions, necks of the shape, *i.e.*, where remote surface patches can be arbitrarily near each other. Our strategy is then to seek in *S3* triangles which if removed do not change the connectivity of adjacent triangles, *i.e.*, lead to no new holes in the mesh (Figure 2). Once this triangle removal step is completed, we obtain a final surface scaffold, $\mathcal{SC}_s$, and a final medial scaffold, $\mathcal{SC}_m$. Each shock curve of $\mathcal{SC}_s$ has an associated triangular surface interpolant; together, these triangles constitute the final surface interpolant, S. An optional fourth step *(S4)* uses $\mathcal{SC}_m$ to construct coarse-scale axial and and rib curves (Figure

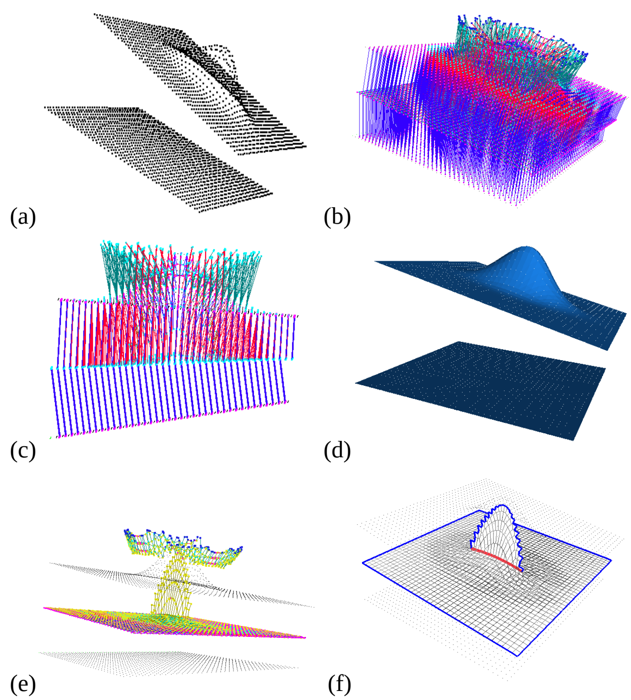

Figure 1: Example of a $\mathcal{SC}$ before and after segregation. (a) A set of 3200 generators are uniformly distributed on a pair of planes, one of which is deformed by an elongated Gaussian kernel. (b) The $\mathcal{SC}$ for (a) where those shock curves which shoot off to infinity are not shown, for greater visibility. (c) A side-view of the *surface scaffold*. (d) The automatically reconstructed surface arising from the shock curves in (c) and their associated triangular interpolants. (e) The *medial scaffold* structure which approximates the $\mathcal{MA}$ of the original continuous surface. NB: The "sum" of (c) and (e) gives back (b). (f) The largest components of the medial scaffold. The recovered "axial" shock curve sits at the intersection of the two large medial sheets, one vertical due to the pull of the Gaussian bump, which is bounded by a shock curve corresponding to the *ridge* of the bump.

[1] We join to this abstract a draft paper, which covers most computational aspects of our method (see [7] for more details).

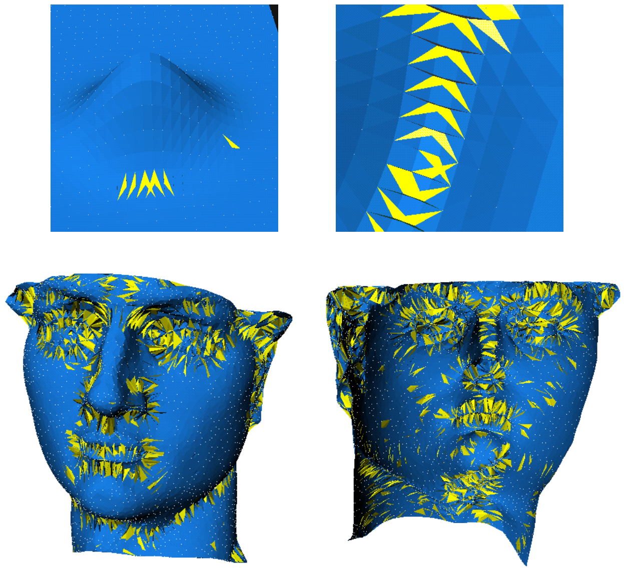

Figure 2: *Top row:* Outside and inside views of the top of the "two planes with Gaussian bump" dataset which was used in Figure 1. *Bottom row:* Front and back views of the face of Michelangelo's David (dataset shared by the Graphics group of Stanford University [6]). *Color code: extraneous triangles* are shown in **yellow**, remaining pre-surface interpolants in **blue**, and generators as **white** dots. Notice how extraneous triangles accumulate near ridges, like at the inside of the elongated Gaussian bump (top-left), and for the lips, eye lines, neck, *etc.*, of the David.

1.(f)) approximating those of the $\mathcal{MA}$ of S [7].

Our approach makes no special assumption about the input generators and requires no user interaction. In particular we do not require *a priori* knowledge of the local geometry of the surface at each generator locus as is done in approaches based on differential geometry [3, 5] or on level sets [9]. Our approach is most closely related to those based on the use of the Voronoi diagram [2, 1, 8, 4]. However, there are two main assumptions in these recent "combinatorial" approaches for solving the problem of surface reconstruction from point clouds: *(i)* assume some local knowledge of the geometry of the surface (such as normals), or *(ii)* assume some knowledge of proximity to an idealized $\mathcal{MA}$, which in effect imposes strict requirements on the sampling density.

Our presentation consists of three main parts. *(i)* We summarize the definition of the $\mathcal{SC}$, which is based on the notion of contact with maximal spheres and singularities of shock flows [7, Ch.3], and explain its relation to the $\mathcal{MA}$ and Voronoi diagram. *(ii)* We present the main aspects of our new method to segregate the shock scaffold of unorganized points clouds in 3D [7, Ch.6]. *(iii)* We illustrate the application of this technique on artificial and real datasets from the domain of cultural heritage and medical imaging (Figure3), and discuss some of its present limitations.

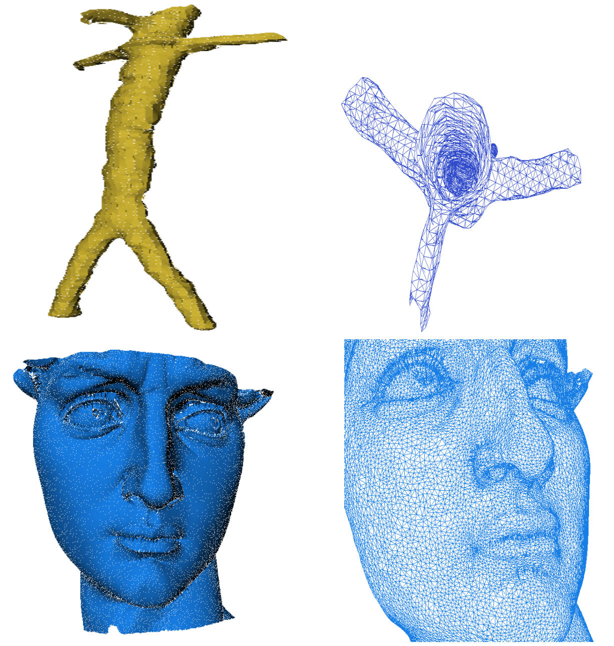

Figure 3: Examples of surface recovery (*Left*: shaded triangulation, *Right*: triangular wire-mesh) from unorganized data (white dots on left). *Top*: *(i)* human aorta (CT scan, 7691 points), *(ii)* Michelangelo's David (laser scan, 31043 points).

References

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