Looking for 14-Cycles in the Cube

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Given graphs P and Q the generalized Turan number $\mathrm{ex}(P, Q)$ denotes the maximum number of edges of a P-free subgraph of Q. We consider the case when P is C_k, the cycle of length k and Q_n is the hypercube, (i.e., Q_n is n-regular and it has 2^n vertices).

Erdős conjectured that

$$\mathrm{ex}(C_4, Q_n) = (\frac{1}{2} + o(1))e(Q_n) \quad (?)$$

Fan Chung showed an upper bound 0.623 and that $\mathrm{ex}(C_6, Q_n) \geq (1/4)e(Q_n)$, moreover that $\mathrm{ex}(C_{4k}, Q_n) = o(e(Q_n))$. There are futher results concerning C_{10} by Alon et al., by Axenovich et al., by A. Thomason et al., and more. Here we deal with the next unsolved case, and show that

$$\mathrm{ex}(C_{14}, Q_n)/e(Q_n) \to 0.$$

This is a joint work with Lale Özkahya.