

## The Erdős-Rényi Phase Transition

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In their great 1960 paper *On the Evolution of Random Graphs* Paul Erdős and Alfred Rényi expresses a special interest in the behavior of the random graph  $G(n, p)$  when  $p$  was near  $n^{-1}$ . Today we view it through the prism of Percolation Theory. If  $p = cn^{-1}$  and  $c < 1$  the process is subcritical and all components are small and simple. But for  $c > 1$  the process is supercritical and a complex giant component has emerged. We now understand the fine structure: the critical window is parametrized  $p = n^{-1} + \lambda n^{-4/3}$ , with  $\lambda \rightarrow -\infty$  and  $\lambda \rightarrow +\infty$  represently the barely subcritical and barely supercritical phases. We discuss the behaviors and the arguments, making particular use of the similarities to the Galton-Watson Birth Process when the expected number of children is near one.