Extrapolating tumor invasion margins for physiologically determined radiotherapy regions

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1 Abstract

In radiotherapy, the constant margin taken around the visible tumor is a very coarse approximation of the invasion margin of cancerous cells. In this work, a new formulation to estimate the invasion margin of a tumor by extrapolating low tumor cell densities in magnetic resonance images (MRIs) is proposed. Current imaging techniques are able to show parts of the tumor where cancerous cells are dense enough. However, tissue parts containing smaller number of tumor cells are not enhanced in images. There have been several values proposed in literature for this threshold in CT images in terms number of tumor cells per unit volume or area, [1–3]. Although there is no threshold been proposed for MRIs, it is noted that the tumor seen CT images and the MRIs are very similar hence, it is a common practice to assume the same threshold.

We propose a way to estimate parts of the tumor not enhanced using the tumor mass visible in the image. Our formulation is based on the Fisher-Kolmogorov equation, which is been widely used to model the growth of brain tumors, [4, 5, 2, 3]:

$$\frac{\partial u}{\partial t} = \nabla \cdot (D(\mathbf{x})\nabla u) + \rho u(1-u) \tag{1}$$

where u is the tumor cell density, ρ is the average proliferation rate of tumor cells and D is the diffusion tensor, which characterizes the diffusion behaviour of cancerous cells. D is taken to be spatially varying with a higher determinant on the white matter yielding a higher diffusion rate, [6]. It is also taken to be an anisotropic tensor following the fibres in the brain using the diffusion tensor images (DTI), as proposed in [2]. Using this setting we derive the formulation describing the profile of the tumor front at low cell density parts at a single time instant:

$$\frac{\sqrt{\nabla u \cdot (D\nabla u)}}{\sqrt{\rho} u} = 1,\ u(\Gamma) = u_0 \tag{2}$$

where u_0 is the lowest cell density threshold for images and Γ is the last visible contour of the tumor in the image. Using this formulation, given a single image with a tumor and a set of estimated parameters (D and ρ), we were able to extrapolate the unseen parts of the tumor, see figure 1.

Advantages this formulation brings is that it extrapolates the low density profile of the tumor using a single time instants, providing a tool for medical

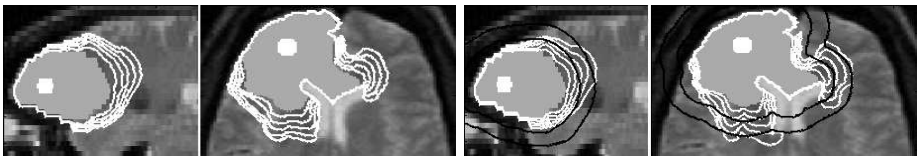

Fig. 1. Left: Sagital and axial views of the T2-weighted image for the synthetic tumor including its iso-density contours from 0.05 to 0.003. Right: Sagital and axial views of the same T2-weighted image with estimated iso-density contours based on our method. Black contours correspond to constant margin radiotherapy regions for 1cm and 2cm.

doctors to use in radiotherapy planning. Moreover, the formulation constructs a patient-specific tumor profile based on the underlying tissue structures and the current location of the tumor, which provides a realistic initial condition for tumor growth models in trying to predict the evolution of the tumor.

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