

**New Jersey**

**Mathematics**

**Curriculum Framework**

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**(with contributions from many other New Jersey educators)**

**December 1996**

**A Collaborative Effort of the**

**New Jersey Mathematics Coalition**

**and the**

**New Jersey Department of Education**



## PREFACE

The *New Jersey Mathematics Curriculum Framework* is based on the *Mathematics Standards* adopted by the New Jersey State Board of Education on May 1, 1996. The *Mathematics Standards* are part of the *Core Curriculum Content Standards*, developed by the New Jersey State Department of Education, including standards in seven content areas and cross-content workplace readiness standards. Taken together, the *Core Curriculum Content Standards* describe what every New Jersey student needs to understand and be able to do at the completion of the 4th, 8th, and 12th grade.

**PURPOSE:** **New Jersey’s *Mathematics Standards* describe a vision of mathematics teaching and learning which involves high expectations for *all* students, and insists that *all* students can achieve these expectations.**

**The *New Jersey Mathematics Curriculum Framework* provides information and guidance to teachers and districts on how to help make that vision a reality.**

Achieving this vision is an ambitious, long-term undertaking; there is no simple path to the goal. Achieving this vision will take time, effort, and a commitment to change. The recommendations of the *Mathematics Standards* cannot be implemented overnight, and results will not appear overnight. Changes will be needed in all areas — in curriculum, instruction, assessment — and will involve rethinking school practices and extensive professional development. The changes will require the commitment of teachers, administrators, school boards, parents, and policy makers, and the effort of the entire New Jersey community.

The *New Jersey Mathematics Curriculum Framework* is intended to be a resource, providing practical guidance to implement the *Mathematics Standards*. It includes information and resources for teachers at all grade levels and for school and district administrative personnel. Each chapter contains much information, and can serve as a basis for extended discussions involving teachers and administrators.

The *New Jersey Mathematics Curriculum Framework* is not intended to be read straight through. It is intended to be user-friendly; but to achieve that purpose, the user also has to be friendly, warming up to its contents a little at a time, and not shying away from it because of its bulk.

New Jersey’s *Mathematics Standards* and the *New Jersey Mathematics Curriculum Framework* are directed toward one crucial goal:

**GOAL:** **To enable all of New Jersey’s children to move into the twenty-first century with the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives.**

The *Mathematics Standards* are based on the twin premises that all students can learn mathematics and that all students need to learn mathematics. They set high achievable expectations for all students, and call for teachers and parents to help all students strive toward and achieve those standards.

New Jersey’s *Mathematics Standards* and the *New Jersey Mathematics Curriculum Framework* call for

major changes, both in terms of what mathematics will be taught, and in how it will be taught. The recommendations provided here are very specific. Yet, it is not intended that they be implemented dogmatically; different situations call for different responses and different strategies. In education, as in other areas, there is a tendency to swing from one extreme to another. We hope that educators will utilize their common sense, judgment, and experience in finding appropriate ways of using the recommendations in this *Framework* to inform their decision-making. We expect that this *Framework* will be a major resource to teachers seeking to implement the *Mathematics Standards* in the classroom; we also expect it to be valuable to districts which are seeking to introduce mathematics curricula based on the *Mathematics Standards* and to provide professional development to their teachers based on the *Mathematics Standards*.

The publication of this document is the culmination of the New Jersey Mathematics Curriculum Framework Project, a collaborative effort of the New Jersey Mathematics Coalition and the New Jersey Department of Education, which was funded by an Eisenhower grant from the United States Department of Education. This effort is also a component of New Jersey's Statewide Systemic Initiative to Improve Mathematics, Science, and Technology Education. The *Framework* and the *Mathematics Standards* build on the Standards published by the National Council of Teachers of Mathematics in 1989 and 1991.

A *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* was published in January 1995. That *Preliminary Version* reflected the efforts of hundreds of New Jersey mathematics educators who worked together during 1993 and 1994 to develop materials that would be appropriate for a world-class mathematics curriculum framework. During the last two years, the *Preliminary Version* has been reviewed and used by many teachers, schools, and districts throughout the state. This new version of the *New Jersey Mathematics Curriculum Framework* reflects their comments and suggestions, and follows the organization of the *Mathematics Standards* adopted by the New Jersey State Board of Education.

Though published, the *New Jersey Mathematics Curriculum Framework* is not completed. We anticipate that it will continue as a living document on the Web site of the New Jersey Mathematics Coalition, where it is available at [http://dimacs.rutgers.edu/nj\\_math\\_coalition/framework.html/](http://dimacs.rutgers.edu/nj_math_coalition/framework.html/). We hope to post additional resources relating to the *Mathematics Standards*, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

The efforts of all those who have contributed to the development of the *New Jersey Mathematics Curriculum Framework* are acknowledged below. This has been truly a state-wide effort of which we can all be proud. Let us all continue to work together to make the vision of New Jersey's *Mathematics Standards* a reality in the coming years!

For further information, please call the New Jersey Mathematics Coalition at 908/445-2894 or contact the New Jersey State Department of Education, Office of Standards and Assessment, CN 500, Trenton, NJ 08625-0500. We welcome your comments on the *Framework* and your suggestions about its future; please send them to [joer@dimacs.rutgers.edu](mailto:joer@dimacs.rutgers.edu) or the New Jersey Mathematics Coalition, P.O. Box 10867, New Brunswick, NJ 08906.

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## ACKNOWLEDGEMENTS

The development of the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* was a broad-based effort. As evidence, I submit the following section (pages v-xi) which appeared in the *Preliminary Version* acknowledging all those who had roles in the development of that version of the *Framework*. They should be pleased to see that all their efforts have now come to fruition. Many thanks to all of those who played a role in developing the *Framework*!

By contrast, this version was the result of an intense effort by a small number of people. Warren D. Crown, Professor of Mathematics Education at Rutgers, Janet H. Caldwell, Professor of Mathematics at Rowan College of New Jersey, and Joseph G. Rosenstein, Professor of Mathematics at Rutgers University wrote this entire document, based on the materials in the *Preliminary Version* and responding to the comments and suggestions offered by all those who used and reviewed that document.

Assisting in the writing process was Karin Rupp who collected and organized all of the comments and suggestions, and developed additional information that grew out of those comments and suggestions. Also assisting in the writing process were those who carefully read various chapters of the *Preliminary Version*, and recommended many changes. This includes Robert Davis, Frank Gardella, Evan Maletsky, and Maureen Quirk. Especially important was the contribution of those who reviewed the chapter dealing with Discrete Mathematics, a topic which has never before been subjected to a K-12 grade-level analysis; this includes Valerie DeBellis, Emily Dann, Bobbie Goldman, Janice Kowalczyk, Evan Maletsky, Claire Passantino, and Michael Saks, as well as the many teachers in the Leadership Program in Discrete Mathematics who shared their classroom experiences with these topics.

Before this version of the *New Jersey Mathematics Curriculum Framework* could be written, the *Mathematics Standards* had to be adopted by the New Jersey State Board of Education. So acknowledgements are appropriate here to those who served on the Governor's Review Panel for the Mathematics Curriculum Standards — Janet Amenhauser, Joyce Baynes, Janet Caldwell, Warren Crown, Barbara Graham, Patricia Klag, Paul Lawrence, Evan Maletsky, Paula Norwood (Panel co-Chair), Jean Paige, Robert Riehs (Department of Education), Joseph G. Rosenstein, William Smith (Panel co-Chair), Dorothy Varygiannes (Department of Education), and Allen Wesley.

The editing of the *New Jersey Mathematics Curriculum Framework* was the work of Joseph G. Rosenstein, with the dedicated assistance of Karin Rupp. Both have read each word of this document ... over and over. Meeting the deadline imposed by the Eisenhower grant period — this document had to be printed by December 31, 1996 — involved, simply put, many long days and nights.

The document was prepared by the staff at the Center for Mathematics, Science, and Technology Education at Rutgers University, including the staff of the New Jersey Mathematics Coalition. Most of the document was put into its final form by Stephanie Micale and Debby Toti who have cheerfully put up with the *Framework*, and its Editor, full-time for the last three months. Chris Magarelli did all the computer graphics, and others provided important assistance when it was needed — Janet DeBellis, Valerie DeBellis, Lisa Estler, Bonnie Katz, Stephanie Lichtman, and Peter Sobel. There were times when six people were working on the document simultaneously! Thank you for all your help.

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Education who has helped ensure that the contents of this document reflected the initiatives of the Department. Thanks also to Stephen Bouikidis for providing expert advice on the format of this document and to him and Keith Kershner for ensuring that its cover would do justice to its contents. In addition to their assistance, the Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education at Research for Better Schools also provided a graphic artist for the cover and contributed to the distribution of the *Framework*. Thanks also to Rutgers University and the Center for Mathematics, Science, and Computer Education for serving as a home for this project, and to its Director, Gerald A. Goldin, for his consistent support.

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December 9, 1996

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(from *Preliminary Version* — 1995)

The development of the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* was made possible by a grant from the United States Department of Education to the New Jersey Department of Education and the New Jersey Mathematics Coalition. Joseph G. Rosenstein has served as co-Director of the New Jersey Mathematics Curriculum Framework Project for the New Jersey Mathematics Coalition; serving as co-Directors for the New Jersey Department of Education have been Charles Mitchel, Karen Sanderson, and Dolores Keezer.

In addition to a collaboration between the New Jersey Mathematics Coalition and the New Jersey Department of Education, this document represents a collaboration involving many of the most knowledgeable mathematics educators in New Jersey and many other members of the community. Some served on the New Jersey Curriculum Standards Panel that developed the draft version of the *New Jersey Mathematics Standards*. Others were active members of the Curriculum Framework Project Advisory Committee of the New Jersey Mathematics Coalition. Still others served as members of Task Forces which developed recommendations and drafted materials for the framework. Lists of members of these groups are provided on the following pages<sup>1</sup>; please bring corrections or omissions to our attention so that modifications can be made in subsequent versions.

We also acknowledge the 294 individuals who submitted comments on the draft version of the *New Jersey Mathematics Standards* and the 29 District Leadership Teams (DLTs) who reviewed an earlier version of the framework as part of their participation in the pilot implementation program of the New Jersey Mathematics Curriculum Framework Project. Serving as Project Coordinator of the pilot implementation program have been Irwin Ozer and Karin Rupp.

Special mention must be made of the following individuals who have served the Leadership Team in various capacities, attending long and arduous planning meetings, chairing Task Forces, and writing, reviewing, and editing endless drafts of sections of this document.

Janet Caldwell, Professor of Mathematics, Rowan College of New Jersey  
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<sup>1</sup> Members of the New Jersey Mathematics Curriculum Standards Panel are listed on page vii, of the Curriculum Framework Project Advisory Committee on page v, and of the various Task Forces on pages viii-ix. A list of the 60 districts comprising the District Leadership Teams (DLTs) and the DLT Coordinators appears on pages x-xi.

Joseph G. Rosenstein, Professor of Mathematics, Rutgers; Director, NJ Mathematics Coalition  
Maureen Quirk, Center for Communications Research  
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**Karen Sanderson**, currently Assistant Principal, Hamilton (Atlantic County) Public Schools, who served for nearly two years as Project co-Director for the New Jersey Department of Education;

**William Smith**, Mathematics Supervisor, Haddonfield Public Schools, who served as Facilitator of the *New Jersey Mathematics Standards* Panel;

**Janet Caldwell**, Professor of Mathematics, Rowan College of New Jersey, and  
**Warren Crown**, Associate Professor of Mathematics Education, Rutgers University, and  
Associate Director, New Jersey Mathematics Coalition

who were principal authors of the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* with Joseph G. Rosenstein, writing, rewriting, and weaving together the contributions of many others; and

**Joseph G. Rosenstein**, Professor of Mathematics, Rutgers University, and  
Director, New Jersey Mathematics Coalition

who served as Project co-Director, and headed the team and managed the effort to create this document.

Congratulations and thanks to all who have participated in developing the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework*.



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(continued)

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### **New Jersey Mathematics Curriculum Framework Project**

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East Orange DLT	Howard Walker, James Leutz
Dover DLT (including Mine Hill)	Gary Meyer
Jefferson Township DLT	Mary Ann Tierney
Sparta Township DLT	Pat Spagnoletti, Karl Mundi

#### **Regional Leadership Team VI**

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Franklin Township DLT	Jane Taglietta
Bernardsville DLT	Louis Rodriguez
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