Number of distinct sites visited on a percolation cluster: Characterization of fluctuations

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In continuation of previous work [L. K. Gallos, P. Argyrakis, and K. W. Kehr, Phys. Rev. E **52**, 1520 (1995)], we present two additional analyses in the search for possible multifractality of the distribution of $\langle S_n \rangle$, the mean number of distinct sites visited by a random walk in n steps on percolation clusters at their critical point. The first analysis utilizes the number of the different origins of the random walk as the relevant size parameter. This analysis shows that the distribution of $\langle S_n \rangle$ over different disorder environments is *not* a multifractal but rather exhibits constant gap scaling. In the second analysis, the moments of the distribution $W(\langle S_n \rangle)$ are studied with respect to their n dependence. Here also constant gap scaling is observed. [S1063-651X(96)02110-1]

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In a recent study [1], the transport properties of percolation clusters at their critical point were investigated. The statistics of S_n, the number of distinct sites visited in n steps by a random walk, was investigated, starting from an arbitrary site r on an infinite percolation cluster. In the first part of [1], the quantity $\langle S_n(r) \rangle$ was considered, where $\langle\,\rangle$ denotes averaging over an ensemble of random walks starting from r, and the nature of its fluctuations from one disorder environment to the other was studied. In the second part the distribution of S_n was investigated. In this paper, we reconsider the first part of [1], propose two additional analyses of the fluctuations, and demonstrate unambiguously that these fluctuations do not obey multifractal scaling.

Let $s = \langle S_n(r) \rangle$, for a given initial site r on the percolation cluster. We consider N_r different initial sites (or equivalently disorder environments) and calculate $\{s_i ; i=1,\dots,N_r\}$. For convenience, we consider the scaled variables

$$\tilde{s}_i = \frac{s_i - s_{\min}}{s_{\max} - s_{\min}}, \tag{1}$$

where $s_{\max}$ and $s_{\min}$ are the maximum and the minimum values of s in the set $\{s_i ; i=1,\dots,N_r\}$. Thus each $\tilde{s}_i$ can be represented as a dot on the unit line segment $(0,1)$. Our aim is to study the nature of the density distribution of these dots. In [1], a finite but large value of N_r was considered and the distribution of the dots was analyzed as follows. The unit line was divided into nonoverlapping intervals, each of size, say, l. Let $p_i(l)$ denote the fraction of the total number of dots that are contained in the ith interval. The partition function is given by

$$Z(q,l) = \sum_i p_i^q(l), \tag{2}$$

where the sum is taken over nonempty intervals only. A scaling ansatz was made

$$Z(q,l) \underset{l\to 0}{\sim} l^{\tau(q)}, \tag{3}$$

where $-\infty < q < \infty$, and the limit $l \to 0$ was taken by setting $l = 2^{\nu}$ and taking ν to be $1,2,\dots$. This amounts to dividing the unit line segment into $2,4,8,16,\dots$ intervals and calculating the scaling behavior of the partition function with respect to the index ν. The scaling exponents are given by $\tau(q)$. Such an analysis reported in [1] showed that the $\tau(q)$ curve was not a straight line, nor did it vary with a monotonically decreasing slope, characteristic of a multifractal measure. No definitive conclusions could be drawn about the nature of the distribution. The reason can perhaps be traced to finite-size effects and to the fact that the partitions used were not compatible with the structure of the set.

A natural way to partition the unit line segment, for describing the set of N_r dots, is to consider equal intervals, each of width $1/N_r$. The reason for such a choice is simple. If the dots were distributed uniformly, then each interval would contain one dot. On the other hand, if there is a tendency for the dots to cluster, then we would find several intervals with more than one dot and several empty intervals. To study the nature of clustering of dots, if any, we calculate the partition function [see Eq. (2)] by setting $l = 1/N_r$ and investigate its scaling behavior by letting $N_r \to \infty$. The idea is that if we want to probe a structure (of points) with a given resolution, we should also generate the structure at the same, if not finer, resolution. Thus, achieving the $l \to 0$ limit, by setting $l = 1/N_r$ and letting $N_r \to \infty$, meets this requirement in a simple and straightforward manner; see, e.g., [2]. Such an analysis would help make unambiguous statements about the nature of the distribution of the dots. To this end we turn our attention below.

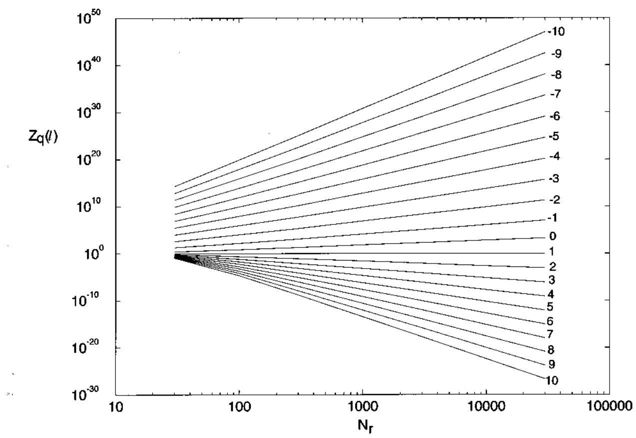

FIG. 1. log-log plot of $Z(q,l)$ vs N_r, for different q values. The q values are shown at the right of every line.

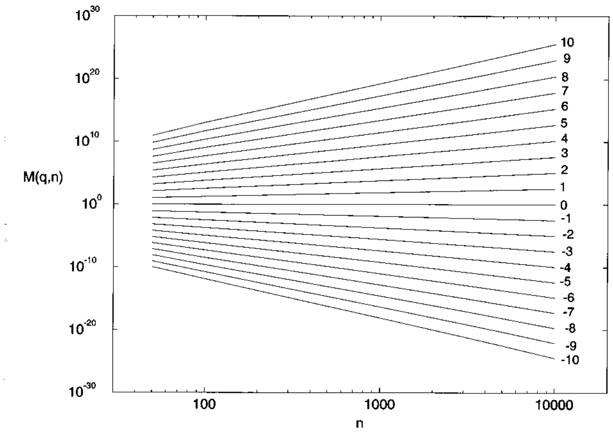

FIG. 3. log-log plot of the moments $M(q,n)$ as a function of n, for different q values.

We consider the same data used in [1] and check the validity of the scaling ansatz, [see Eq. (3)] by plotting $Z(q,l=1/N_r)$ versus N_r on a log-log graph. The plots for different values of q are shown in Fig. 1. In the present calculations, we have kept the number of steps at a constant value of $n=1000$. The partition function depicted in Fig. 1 has been obtained by averaging over several realizations. The first observation we make is that the data for each value of q fall on a straight line verifying the scaling ansatz. The slope of the straight line for a given q gives the scaling exponent $\tau(q)$. Figure 2 depicts $\tau(q)$ versus q for values of q ranging from -10 to $+10$. We find that $\tau(q)$ varies linearly with q and the variation is described by the equation $\tau(q)=1-q$. This implies that the N_r values of the mean number of distinct sites visited are distributed uniformly over its range.

The second analysis we propose here (see for example, [3]), considers the moments of the random variable $s=\langle S_n\rangle$ and their asymptotic $(n\to\infty)$ scaling behavior. In this analysis, the number of steps n takes the role of the system size parameter. The moments of s are calculated by

$$M(q,n)=\frac{1}{N_r}\sum_{i=1}^{N_r} s_i^q, \tag{4}$$

where the sum runs over a large number of different realizations of the disorder. We have taken $N_r=10\,000$. We first make a scaling ansatz

$$M(q,n) \underset{n\to\infty}{\sim} n^{\xi(q)}. \tag{5}$$

Figure 3 depicts the $M(q,n)$ versus n on a log-log graph, for various values of q. The data for each q fall on a straight line and thus the scaling ansatz is verified. The slope gives the scaling exponent $\xi(q)$. Figure 4 depicts $\xi(q)$ versus q. It is linear and is described by $\xi(q)=0.628q$. This implies that the distribution of $\langle S_n\rangle$ over disorder obeys a constant gap scaling with respect to the number of steps n.

Summarizing, we employed two methods of analysis toward a multifractal characterization of the fluctuations of the

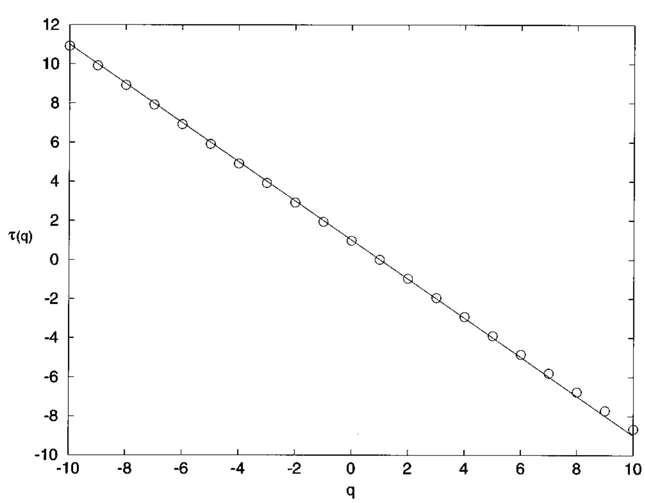

FIG. 2. Plot of the mass exponents $\tau(q)$ as a function of the moment order q. The symbols represent the slopes of the lines in Fig. 1, while the continuous line represents the homogeneous behavior $\tau(q)=1-q$. It is obvious that the two curves are identical.

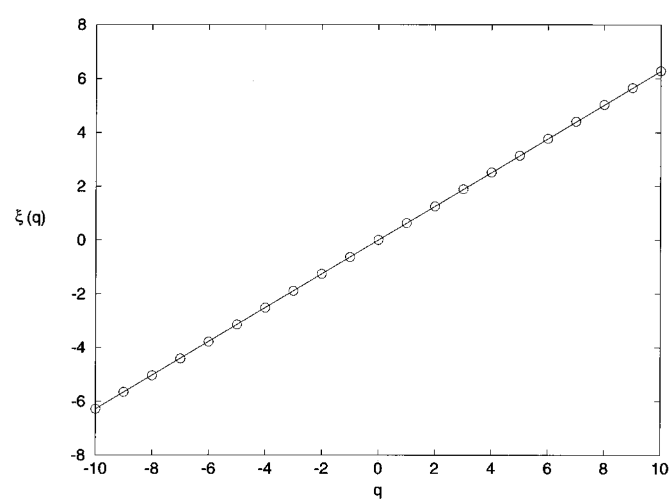

FIG. 4. Plot of the scaling exponent $\xi(q)$ as a function of the moment order q.

mean number of distinct sites visited (by a particle diffusing on a percolation cluster) from one disorder environment to the other. We first studied the scaling of the partition function with the number of disorder configurations for a fixed but large number of steps n. In the second method we investigated the asymptotic ($n \to \infty$) scaling behavior of the moments of $\langle S_n \rangle$. In both we found a constant gap scaling of the pertinent quantities. We conclude that the number of distinct sites visited by a particle diffusing on an infinite percolation cluster belongs truly to the constant gap scaling class.

[1] L. K. Gallos, P. Argyrakis, and K. W. Kehr, Phys. Rev. E **52**, 1520 (1995).

[2] K. P. N. Murthy, S. Rajasekar, and K. W. Kehr, J. Phys. A **27**, L107 (1994).

[3] K. P. N. Murthy, K. W. Kehr, and A. Giacometti, Phys. Rev. E **53**, 444 (1996).