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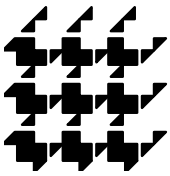

DIMACS EDUCATIONAL MODULE SERIES

MODULE 06-1
Some problems are NP-harder than others

Date Prepared: May 26, 2006

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Module Description Information

- **Title:**
  Some problems are NP-harder than others

- **Authors:**
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- **Abstract:**
  This module discusses two problems from graph theory that have important applications - the Vertex Cover problem and the Dominating Set problem. While the problems appear to be similar, they differ in the amount of time needed to find a solution. Although both problems seem to require exponential time relative to the size of the graph, the Vertex Cover problem has special properties that allow it to be solved faster than the Dominating Set problem. We introduce an integer programming representation for each problem and discuss some methods to solve integer programs. To illustrate the difference between the Vertex Cover and Dominating Set problems, we present rules for preprocessing the Vertex Cover problem that effectively reduce its complexity.

- **Informal Description:**
  This module introduces students to the Vertex Cover problem and the Dominating Set problem to illustrate the use of approximation algorithms and heuristics on problems that are computationally difficult. To provide flexibility, the module includes introductory material on graph theory, computational complexity, linear programming and integer programming. The background of the students using the module will help determine which portions of the module will be emphasized.

  Sections 1-6 comprise the core of the module and would normally be covered by all students. Section 1 introduces basic definitions from graph theory and complexity of algorithms that are needed throughout the module. Section 2 presents the Vertex Cover and Dominating Set problem, and Section 3 discusses simple solution techniques. Section 4 introduces the use of linear programming and integer programming techniques to solve the two problems. Students who have taken a course in linear programming may cover this section quickly or skip it altogether. This section includes some small problems that may be solved by hand. Students with access to optimization software can solve larger instances of these problems, but this is not required. Section 5 introduces several preprocessing rules that may be used to reduce the time required to solve the Vertex Cover problem. While these rules are most useful for large problem instances, the exercises included are on smaller problems so that the concepts can be illustrated clearly and so that students without background in computing can solve them. Section 6 discusses the use of the preprocessing heuristics to solve the Vertex Cover problem, and presents a more theoretical discussion of computational complexity, including the concept of fixed-parameter tractability. Section 7 is an optional section containing some programming exercises. Section 8 is also optional. While the introduction includes some simple application problems, this section describes a more substantial application from bioinformatics on phylogenetic trees. Finally, Section 9 includes a few additional exercises that may be assigned at the discretion of the instructor.

- **Target Audience:**
  This module is intended for upper division undergraduate students in mathematics or computer science. It may be used as a supplement for courses such as graph theory, linear programming, or analysis of algorithms, or as the basis for an independent study course.

- **Prerequisite:**
  The prerequisite for this module is an introductory course in discrete mathematics. Section 7, which is optional, requires knowledge of elementary computer programming.

- **Mathematical Field:**
  Graph Theory, Combinatorial Optimization, Computational Complexity

- **Applications Areas:**
  An optional section discusses an application to phylogenetic trees.

- **Mathematics Subject Classification:**
  Primary: 05C69, 90C57, 90C60
  Secondary: 68Q15

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- **Other DIMACS modules related to this module:**
  03-5: Communications Network Design