DIMACS

Center for Discrete Mathematics &
Theoretical Computer Science

DIMACS EDUCATIONAL MODULE SERIES

MODULE 07-2
Rehabilitating after a Forest Fire
Voronoi Diagrams and Scheduling Algorithms

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Module Description Information

- **Title:**
  Rehabilitating after a Forest Fire
  Voronoi Diagrams and Scheduling Algorithms

- **Authors:**
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- **Abstract:**
  Students are introduced to a problem involving scheduling crews to rehabilitate a forest after a
  fire. Initial discussion of the problem gives way to a guided exploration of Voronoi diagrams
  and some potential use of Geometer's Sketchpad. Scheduling algorithms involving priority
  lists are considered as a means of solving the problem.

- **Informal Description:**
  This module uses the pedagogical technique of "guided design" to direct students through a
  solution to a multi-step problem involving Voronoi diagrams and scheduling.   Problem solving
  techniques and stamina should be developed. The module provides a venue to improve group
  process and communication skills.   Individuals printing this module will need a color printer.

- **Target Audience:**
  The module is designed for sophomore and junior students in courses such as Discrete
  Mathematics, Geometry or Mathematical Modeling.

- **Prerequisites:**
  The only required background is high school geometry. We use theorems related to
  perpendicular bisectors and circumcircles of triangles along with basic terminology from graph
  theory (vertex, edge). It would be helpful, but not mandatory, if instructors have had some
  experience with scheduling algorithms involving prerequisites and priority lists. A reference
  for scheduling algorithms is listed.

- **Mathematical Fields:**
  Planar Geometry, Discrete Mathematics.

- **Applications Areas:**
  This module addresses Voronoi diagrams and a schedule algorithm.

- **Mathematics Subject Classification**
  68U05, 90B50

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- **Other DIMACS modules related to this module:**
  None

Notes to the Instructor

Individuals printing this material will need a color printer.

This module is written in a "guided design" format in which a series of instructions and feedback sheets are given to groups of students. Guided design was developed to assist engineering students in developing problem solving, group process and communication skills.   The process is described below and further information on the process of guided design is available on the website listed in the references.

This module will take three to four one-hour class periods to complete. It is helpful if instructors and students have access to Geometer's Sketchpad. If, however, this software is not available, diagrams can be estimated by hand and the feedbacks used to give exact answers. This option will also save a day of work on Geometer's Sketchpad.

Have the students form groups of 2 to 4 students. The students have 13 instructions to complete. The specific instruction is in italics on each instruction sheet. Upon completion of an instruction, have the group discuss their response with you. When satisfied with their response, give them the feedback sheet and the next instruction sheet.   Students should follow along with the printed feedback sheet even if it differs from their response.

Many of these questions do not have just one good answer, so students must be encouraged to think intelligently, critically, and creatively. Instructors should be open to interesting and useful responses that are not mentioned in this material. Only one possible solution is given on a feedback sheet. Students tend to try to rush through the instructions at first so instructors need to be encouraging and flexible, guiding students to develop more complete answers to the instructions. This often involves a couple of short interactions with the group before they create an answer that the instructor finds satisfactory.   Student solutions need to be informed and well-developed but they will seldom match the feedbacks exactly.

A few details can assist in managing the process.
1. Use several colors of paper. Copy each instruction/feedback pair on the same color paper.
2. One experienced teacher can usually handle 5 to 6 groups. For large classes, it is helpful to have a second instructor available in the classroom. At the college level, upper level students can do a good job with this.
3. Don't answer individual questions, just group questions. You can refer an individual question back to the group by asking, "What does your group think about that?" or "Have you asked your group that question? See what they have to say."
4. Have students verbally justify their answers.
5. Ask different students to give you the verbal feedback. This helps prevent one student from dominating and reinforces the importance of making sure that everyone in the group understands the group response.

The Story: Rehabilitation after a Forest Fire

Introduction and Variables: The summer of 2002 yielded one of the most ferocious wildfire seasons ever seen in the western United States. In June, the Rodeo and Chediski fires in eastern Arizona and the Hayman Fire in Colorado burned 330,000 and 137,000 acres, respectively. In August, the Biscuit Fire in southwestern Oregon burned approximately 495,000 acres. In October 2003, fires raged near San Diego, California. The Cedar, Paradise, and Otay fires burned a total of 376,237 acres of land. Maps of burn areas are updated regularly, indicating the severity of the burn in each region of the map. In Arizona, the brief rainy season followed closely on the heels of these wildfires. Denuded of vegetation, the topsoil is easily washed away in heavy rains. The early work to rehabilitate fire-damaged forests is dedicated to safety concerns and prevention of topsoil degradation. Workers first clear roadways of potential hazards, taking down burned and damaged trees that could fall across a road. Severe heat during fires causes soil to harden, preventing moisture from soaking in and increasing the run-off and the removal of even more soil. If necessary, hardened soil is raked to open access for moisture to penetrate. Logs and riprap are placed along steep slopes to prevent soil from washing away.   Fires can provide natural opportunities for forest reseeding as some types of trees require heat to open up fallen seeds. When needed, workers will plant seedling trees. Forest rehabilitation can take up to a year as crews move between burn regions, dealing with high priority concerns in order.

As experienced Forest Service workers, your team has been given the following memo from the U.S. Forest Service District Manager.

Memo

To: Aaron Zook, Brenda Yale, Cale Watson
From: S. Bronson, U.S. Forest Service District Manager
Re: Forest Rehabilitation Assignment

The outbreak of wildfires across the western United States is creating a need to be able to efficiently schedule Forest Service crews to work on rehabilitating areas of the forest.

For the past few years we have been using an algorithm to schedule crews to rehabilitate the forest. As you know, our crews stay in established campgrounds in the burn area while working on rehabilitation. Each crew is assigned to a region that is subdivided for purposes of scheduling the rehabilitation work.   Currently these regions are given priority based on their distance from a campground. You can see this on the attached map. Because the crews do more work near the campground, those higher priority areas take longer to rehabilitate.. This method of subdividing the forest and assigning priorities is dependent upon the size of the grid used and does not consider many of the practical reasons for prioritizing a region for rehabilitation.   Our crew leaders are convinced that there must be a more effective way to subdivide the forest and a more useful way to prioritize the regions for rehabilitation.   This is what I would like the three of you to work on.

I have attached an example of a forest burn area. Once you have developed your method for subdividing the forest and prioritizing the regions, could you demonstrate your technique on this particular map and apply the scheduling algorithm so we can see where each crew should work on each day until the initial rehabilitation concerns have been dealt with?   We would like to compare this to our previous method for subdividing the forest.

I know the three of you work well together and trust that your years of experience working on forest rehabilitation crews will prove useful.

S. Bronson

The Scheduling Algorithm

Terminology:

Crew region –region associated with a campground site for a crew. Each campground houses exactly one crew.

Rehabilitation subregion – a subdivision of a crew region that is rehabilitated by the same crew in the same block of time.

Open region – A region on which rehabilitation has not been started.

High priority – priority 8, 7 or 6

Medium priority – priority 5 or 4

Low priority – Priority 3, 2 or 1

Rehabilitation rates:

High Priority regions – 80 acres per day

Medium Priority regions – 120 acres per day

Low Priority regions – 160 acres per day

Basic Idea:

At first emphasis will be placed on completing high priority rehabilitation subregions. Later, the emphasis will shift to having crews work in their crew regions. For simplicity, we will not allow two crews to work on the same rehabilitation subregion at the same time.

Algorithm:

Primary Rules:

A crew will work first on high priority rehabilitation subregions in their crew region. They will always work on the subregions in order 8, 7 and 6.   After their high priority subregions are completed the crew will work on high priority subregions in other crew regions.

A crew will work next on medium priority subregions in their crew region. Crews will work on the rehabilitation subregions in order 5 and then 4. After their medium priority subregions are completed the crew will work on medium priority subregions in other crew regions <u>unless</u> the crew associated with that rehabilitation subregion can begin working on that region within 1 day.

Crews will end by working on low priority subregions within their own crew regions in order 3, 2 and 1. Crews will <u>never</u> move to work on low priority rehabilitation subregions in other crew regions. Those crews will be assigned to other forests at that point.

Secondary Rules:

If the above rules yield one or more choices of rehabilitation subregions for a crew to work on, the crew will select the region that takes the least time to complete. Ties can be broken arbitrarily.

When there is a choice to start more than one crew to work on various open rehabilitation subregions within a single crew region, assign outside crews to the higher priority open regions. Assign outside crews to the region that takes the most time among equal priority open regions.

The map. The gridlines represent the crew region (dark lines) and rehabilitation subregions (lighter lines). Each rehabilitation subregion is labeled with a letter in green. The red numbers indicate the priorities. Applying the algorithm to this has crew 1 work for 43.8 days, crew 2 for 41.1 days and crew 3 for 41.7 days.


Plain Map

▲ Campground

Plain Map

yards

▲ Campground
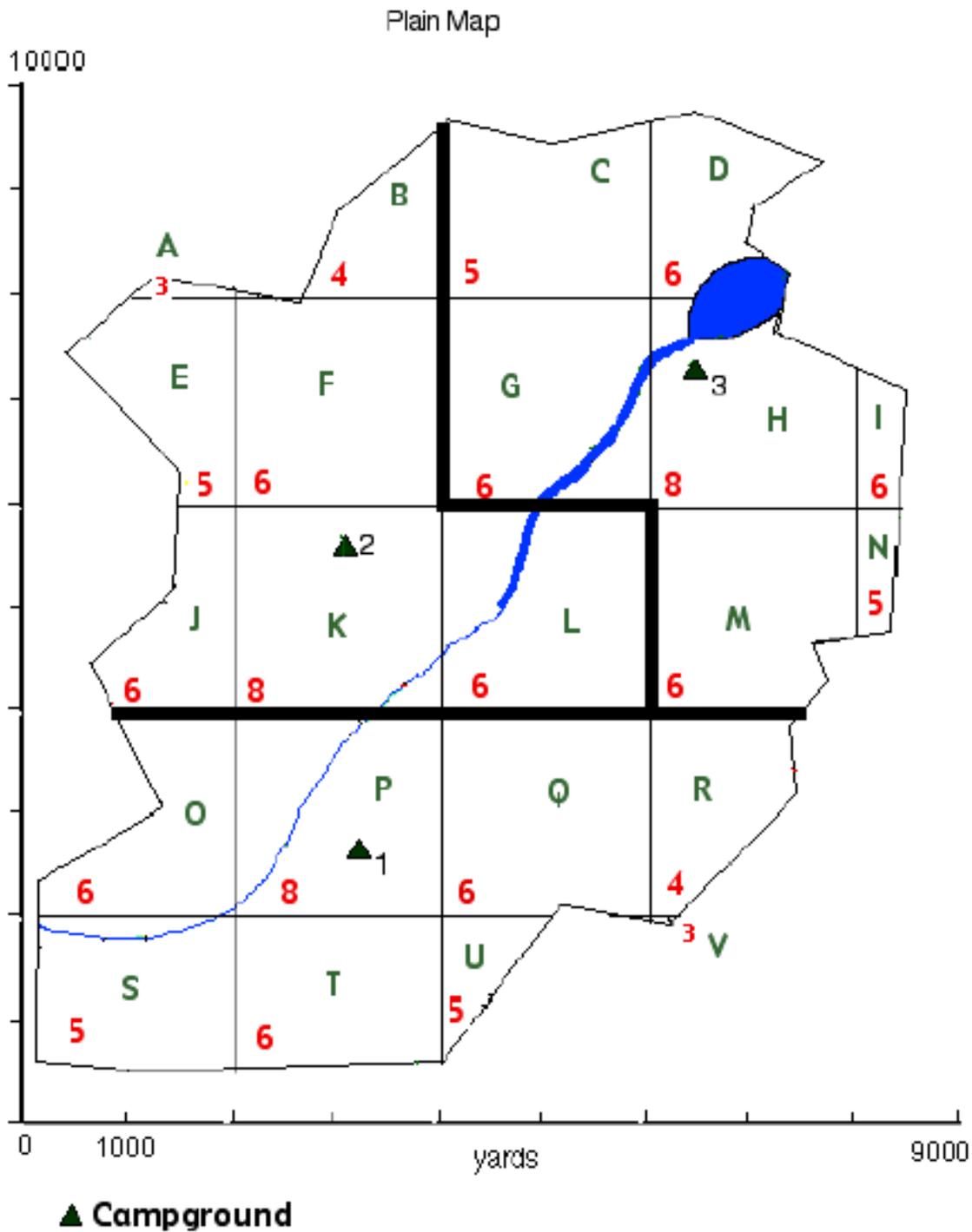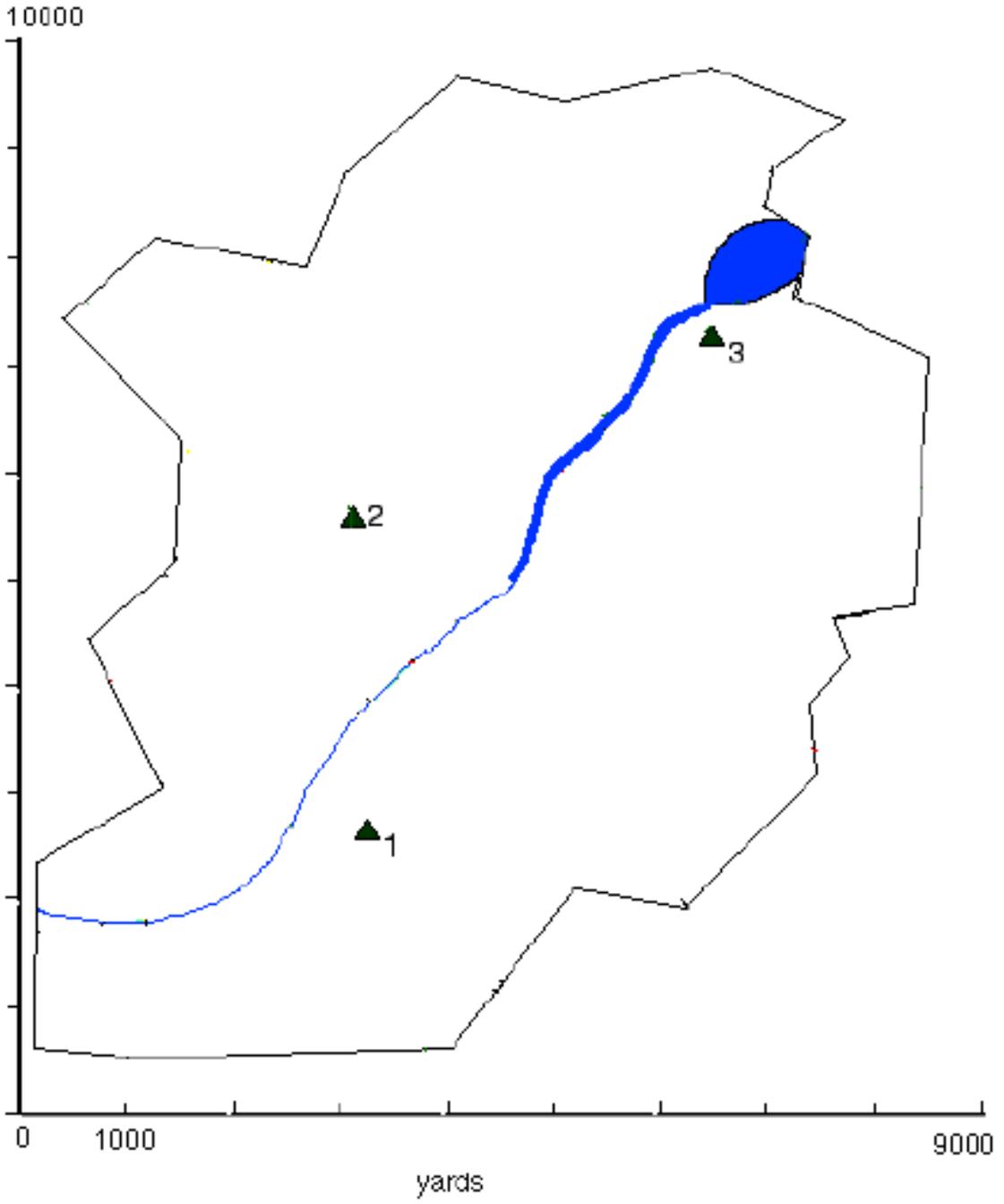
Instruction #1 – Crew Region Exploration I

Aaron: Okay, we have three tasks. Right now the forest is divided into a grid. We need to figure out a more effective way to subdivide the forest. Currently the regions are prioritized based on their distance from the campsite. We need to find a better way to prioritize the regions. Finally, we need to apply the scheduling algorithm to this sample problem to see how it compares to the current approach.

Cale: Let's start by thinking about how to effectively subdivide the forest. We can see from the example that crews are often moving between small regions. Fewer regions might be helpful.

Aaron: I think we should start off with something simple.

Brenda: So let's say we just have two Forest Service campgrounds where the crews are located. It seems reasonable to *have the <u>nearest</u> crew rehabilitate each position on the map. Let's divide the map into two areas, one for each crew, indicating which region each crew should rehabilitate.*
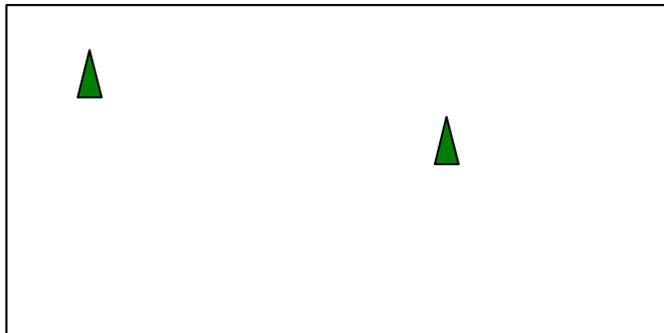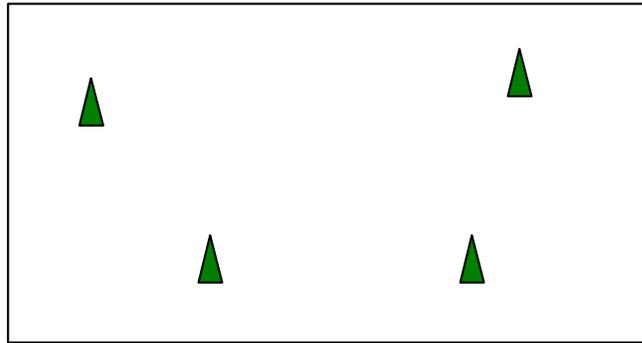

That looks good to me. Let's *try this again, with three campsites*.
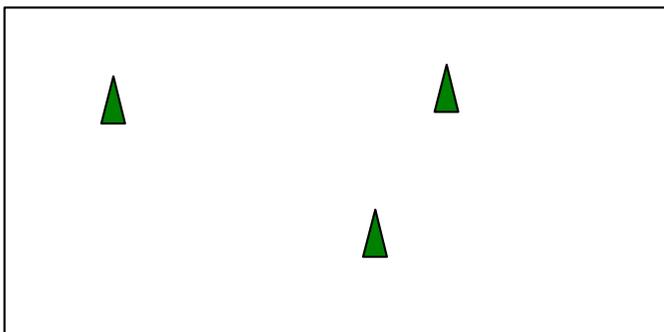
Instruction #2 – Crew Region Exploration II
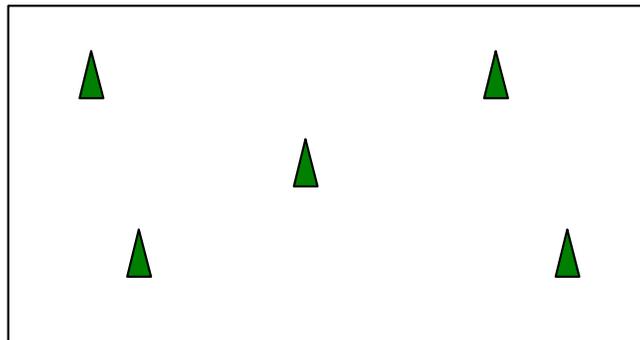
Aaron: I think this could get difficult with many campsites. What's it look like if we have four or five sites? Here's a map with four campsites. *How should we divide this into four regions, with each region associated with its nearest campsite?*

Now what happens if we have five sites?

Are there any characteristics of our diagrams that carry from one size to the next?

Instruction #3 – Mathematical Exploration

Brenda: Hey! I've seen this type of design before! In one of my geology classes they showed us some rocks that had this type of design on the surface of the rock! This type of design is called a Voronoi diagram. Let's see if I can remember some of the terminology, I remember there are Voronoi sites, regions, edges and vertices. I'll bet we can find these words defined on the internet.

(The students google "Voronoi" and on the website http://www.ics.uci.edu/~eppstein/gina/scot.drysdale.html#dna they find this sentence:

> "A Voronoi diagram of a set of "sites" (points) is a collection of regions that divide up the plane. Each region corresponds to one of the sites, and all the points in one region are closer to the corresponding site than to any other site.")

Brenda: *So what are Voronoi sites?*

Aaron: *And what is a Voronoi region?*

Instruction #4 – Check for Understanding

Cale: Let's make sure we agree on these definitions. Here's a picture. Brenda, would you *draw the Voronoi Diagram for the figure?*
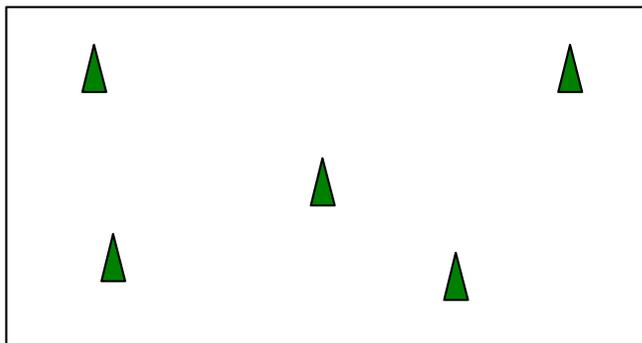
Cale: Seems okay to me. Aaron, will you *circle the Voronoi sites, color the Voronoi regions, make bold the Voronoi edges, and put a star at the Voronoi vertices?*

In classes in which Geometer's Sketchpad will be used, time should be taken now to learn how to make Voronoi diagrams. A GSP document is linked to Appendix B. This document includes instructions on using GSP to make Voronoi documents. Documents helpful later in the module are also included. If Sketchpad will not be used, the Voronoi diagrams can be estimated by hand.

Instruction #5 – Prioritization Factors

Brenda: It seems like this idea will work well for assigning a region to each crew. But now we need to think about whether there is a nice way to subdivide the crew regions. Still, at this point, we are thinking about using a grid. But maybe there is a better way to do this.

Aaron: Our second task was to find a better way to prioritize the smaller regions for rehabilitation. Right now the priority is just based on the distance from the campsite because we might want to work out from the campsite. But, in reality, *what factors would cause us to prioritize a region for rapid rehabilitation?*

Instruction #6 – Rehabilitation Subregions

Aaron: Okay, we've decided to schedule our crews based on
1. The severity of the burn.
2. The steepness of the terrain.
We're going to need three maps: a plain map, a burn severity map and a steepness map.

(Note: These maps are in Appendix A.)

Brenda: We want a schedule that has crews rehabilitate regions on the map that are nearest to their campsite. Let's use the plain map and Geometer's Sketchpad to *make a three-region Voronoi diagram using the campgrounds as sites.* We can call these regions <u>crew regions</u>.

Cale: So, if I understand correctly, crew regions indicate where each of the three crews will focus their forest rehabilitation work.

Aaron: Yes, that's right. Now we need to think about the burn severity and the steepness.

Brenda: This gets complicated! Instead of looking at a whole crew region or at each individual point on the map, let's just select ten sample sites (pink circles) and subdivide our crew regions. *Within each crew region, we can use the sample sites to create a second set of Voronoi diagrams.* These can indicate ten rehabilitation subregions. We could assume that the burn severity and steepness at these sites is representative of their whole region.

Cale: Cool! So our picture has "nested Voronoi diagrams"!

Aaron: And that gives us our subregions – our rehabilitation regions!

Instruction #7 – Rehabilitation Priority Data

Brenda: We're on a roll now! *For each of the ten sample sites, we can record the burn severity and steepness colors.*

Region	A	B	C	D	E	F	G	H	I	J
Severity										
Steepness										

Instruction #8 - Rehabilitation Priority Scheme

Cale: Um, what's next? We have this information about how bad the burn is and how steep the terrain is, but we need to use both of these to figure out which region needs to be rehabilitated first.

Aaron: You're right Cale. So we need to *develop a rule for determining the rehabilitation priority of each of the ten rehabilitation subregions.* The algorithm requires us to have high, medium and low priority regions, with numerical values ranging from the highest priority 8 to the lowest priority 1.

Instruction #9 – Rehabilitation Priorities

Brenda: Okay, the manager wants us to try this on a specific problem, so let's look at our maps and *apply our rule to assign a priority to each of the ten rehabilitation subregions.*

Region	A	B	C	D	E	F	G	H	I	J
Priority										

Instruction #10 - Rehabilitation Times

Cale: This problem just gets more complicated. We've got the regions figured out using Brenda's Voronoi diagrams and now we have priorities assigned using the steepness and severity maps. But if we're going to make a schedule, we need to know how much time each region takes to rehabilitate.

Aaron: Last year we had some crews working on a small prairie fire in Wyoming and we figured they were doing the initial rehabilitation of 130 acres per day. That would take care of removing any dangers on roads, raking the soil and laying logs and riprap where needed.

Brenda: That number is close to the numbers they used in the algorithm. They say that a crew can rehabilitate 80 acres per day in a high priority area. What priority do you think the land in Wyoming would have had?

Aaron: It was a prairie fire so the land was not very steep but the burn had a high severity, so it would have been a priority 5 or medium priority.

Brenda: So the algorithm is saying 120 acres a day for a medium priority fire and your crews were working a little faster than that. Let's just stick with what the algorithm tells us.

Cale: Sounds good to me. This geometry software will let us *determine the acreage of each of the rehabilitation subregions and then we can compute how many days each subregion will take to rehabilitate. I think we should round up to the nearest half day to account for travel time of the crew.* That will give a good buffer for our schedule.

Region	A	B	C	D	E	F	G	H	I	J
GSP cm^2	21.93									
Square yards	7641115									
Acreage	1579									
Time	**20.0**									

Note: If not using Geometer's Sketchpad, skip this instruction and go directly to the feedback.

Instruction #11 – Practice Schedule

Cale: I think we've dealt with the first two issues. We've figured out a way to create crew regions and rehabilitation subregions and have developed a way to prioritize the rehabilitation subregions based on the steepness of the terrain and the severity of the burn.

Brenda: Now we need to apply that long algorithm to schedule the crews to rehabilitate the forest. Can we try this on a smaller problem just to get some practice first?

Brenda quickly draws the diagram below showing 2 campgrounds, one for each of two crews, along with 7 sample sites (A to G) and the corresponding rehabilitation subregions. The chart indicates the completion times and the Priority List is given, with equal priority regions in parentheses. *The team uses the Scheduling Algorithm to schedule the crews to work on the rehabilitation subregions.*

Region	A	B	C	D	E	F	G
Priority	6	7	8	3	7	4	4
Time (days)	3	1	3	2	6	3	3

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Crew 1														
Crew 2														

Instruction #12 – Final Schedule

Cale: We have our map, our priorities and our completion times and we've practiced using the scheduling algorithm. I think we can finish this up now and *use the Scheduling Algorithm to schedule the crews to rehabilitate the original forest area.*

**Instruction #13 – Final Report**

Aaron: Alright! I think we've got this tackled.

Brenda: Um, Aaron, we need to write a report to the district manager.

Write a one to two page typed professional report to the U.S. Forest Service district manager. The report should include a brief explanation of the problem, the method for subdividing the forest, the method for prioritizing the regions, along with a recommended schedule for rehabilitating the forest area under consideration. In addition, briefly compare this method of subdividing and prioritizing to the grid method that was previously used.
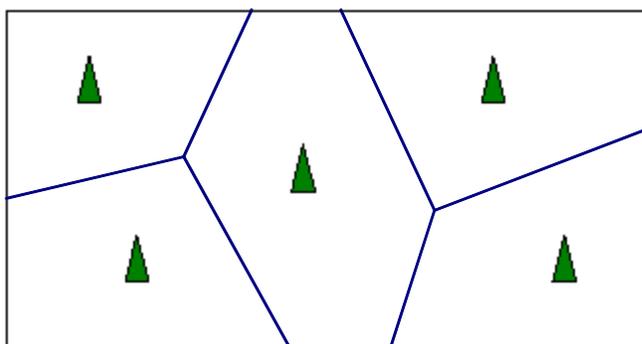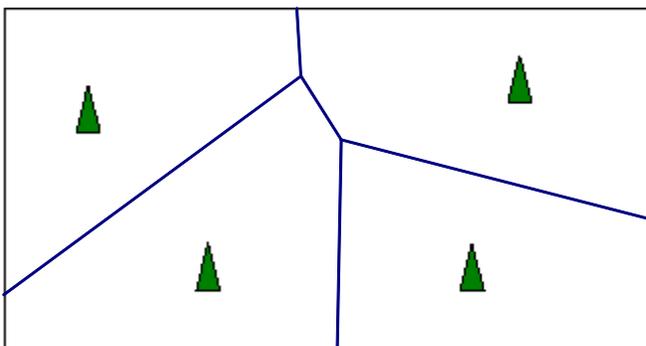
Feedback #1 Crew Region Exploration I

The maps are divided as shown. The crews will rehabilitate the areas that they are within.
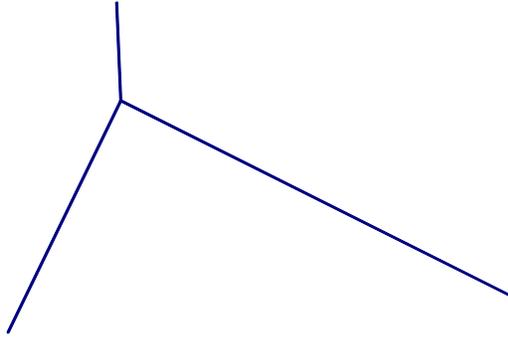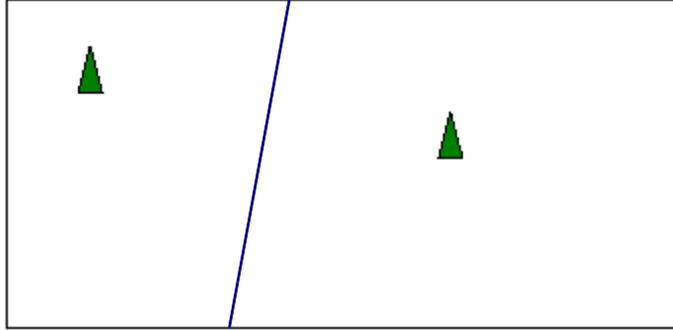
Feedback #2 Crew Region Exploration II

The maps are divided as shown. The crews will rehabilitate the areas that they are within.

The line seems to go right in between the two campsites. In fact, the line segment is the perpendicular bisector of the segment joining the two campsites. When there are more than two campsites, there is an intersection of three lines. The regions are convex polygons.

If the line segment between any two points on a polygon lies entirely inside the polygon, then the polygon is convex.
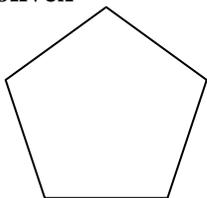
Convex   **Not Convex**
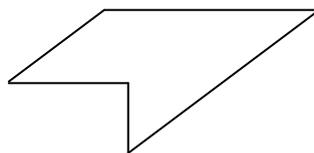
Students wanting more experience looking at Voronoi diagrams are encouraged to look at the applet at http://www.cs.cornell.edu/Info/People/chew/Delaunay.html

Feedback #3 Mathematical Exploration

The Petrosky Stone can be seen at http://www.statefossils.com/mi/mistone.html

Voronoi Sites- a set of points from which a Voronoi diagram is made. In this case, the campsites are the sites.

Voronoi Region- the area that a crew will rehabilitate. All the points in the region are closer to the site in that region than any other site.

Additional definitions:

Voronoi edge- two regions intersect at an edge. The points on the edge are equidistant from the sites of the two regions.

Voronoi vertex- Voronoi edges intersect at a vertex. The vertex is equidistant from at least three sites. A vertex has a degree of three or greater.

Feedback #4 Check for Understanding

The campsites are the Voronoi sites. The area assigned to a crew is the Voronoi region.
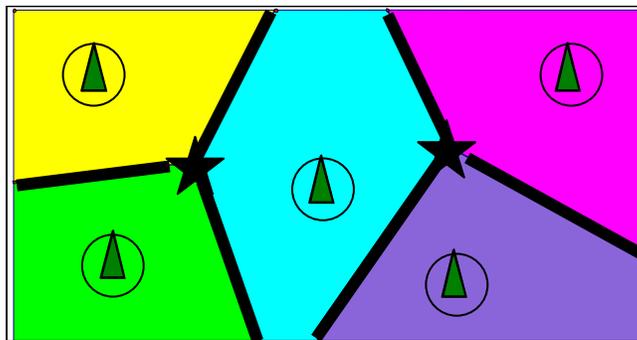
In classes in which Geometer's Sketchpad will be used, time should be taken now to learn how to make Voronoi diagrams.   A GSP document is linked to Appendix B. This document includes instructions on using GSP to make Voronoi documents. Documents helpful later in the module are also included.

If Sketchpad will not be used, the Voronoi diagrams can be estimated by hand.

Feedback #5 Prioritization Factors

The following are some of the factors affecting the prioritization for forest rehabilitation:

Steepness of the terrain (soil washes away more quickly on steep slopes)
Type of soil (some soils hold water better and are better for seedlings)
Topsoil degredation
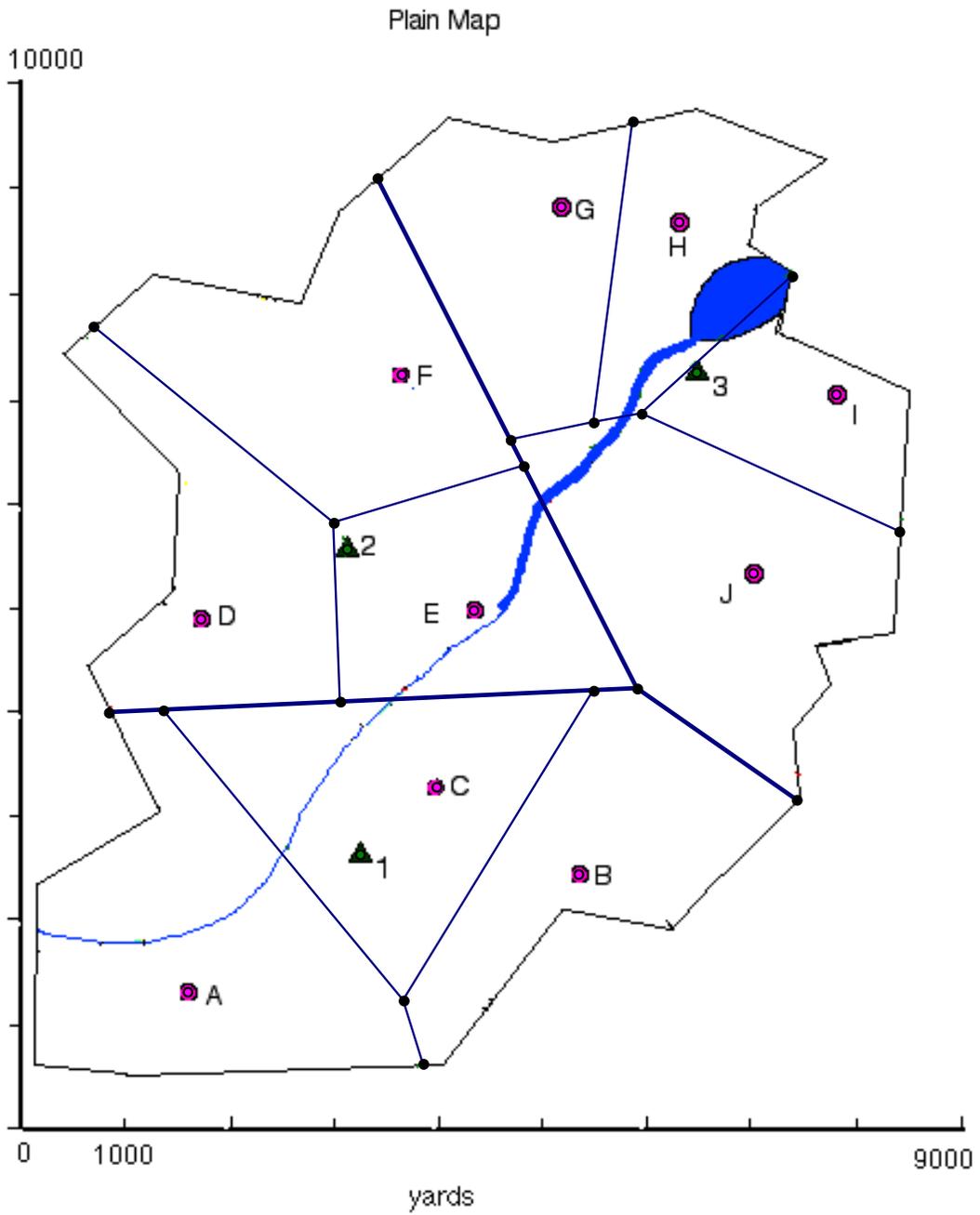Severity of the burn (severe burns harden the soil)
Roadway hazards
Distance from the crew's campsite

The team decides to prioritize each rehabilitation subregion based on the following information:
1. The severity of the burn.
2. The steepness of the terrain

Feedback #6 Rehabilitation Subregions

Plain Map

Feedback #7 Rehabilitation Priority Data

Region	A	B	C	D	E	F	G	H	I	J
Severity	Y	LG	R	LG	LG	R	R	Y	LG	Y
Steepness	R	G	Y	Y	G	R	Y	G	G	R

Feedback #8 Rehabilitation Priority Scheme

Assign each color a numerical value as follows:
Severity Points: Red = 4, Yellow = 3, Light Green = 1, Green = 0
Steepness Points: Red = 4, Yellow = 3, Green = 1
Priority number = Severity points + Steepness points
Special Case: If a location on the map is steep (red or yellow) but unburned (green) the priority value
    would be 3 or 4. Unburned regions, however, require little rehabilitation, so reduce this priority to 1.

Feedback #9 Rehabilitation Priorities
Apply the rule to this problem to assign rehabilitation priorities to each of the rehabilitation subregions.

Region	A	B	C	D	E	F	G	H	I	J
Severity	3	1	4	1	1	4	4	3	1	3
Steepness	4	1	3	3	1	4	3	1	1	4
Total	**7**	**2**	**7**	**4**	**2**	**8**	**7**	**4**	**2**	**7**

Region	A	B	C	D	E	F	G	H	I	J
Priority	7	2	7	4	2	8	7	4	2	7

Feedback #10 Rehabilitation Times

Rehabilitation rates:

High Priority regions – 80 acres per day
Medium Priority regions – 120 acres per day
Low Priority regions – 160 acres per day

Region	A	B	C	D	E	F	G	H	I	J
Priority	7	2	7	4	2	8	7	4	2	7
GSP cm^2	21.93	15.16	16.83	13.81	13.36	18.74	13.73	10.39	8.18	20.70
Sq. Yds	7641115	5282230	5864111	4811847	4655052	6529617	4783972	3620209	2850174	7212544
Acreage	1579	1091	1212	994	961	1349	988	748	589	1490
Completion Time (days)	20.0	7.0	15.5	8.5	6.0	17.0	12.5	6.5	4.0	19.0

Summary:

Region	A	B	C	D	E	F	G	H	I	J
Priority	7	2	7	4	2	8	7	4	2	7
Completion Time (days)	20.0	7.0	15.5	8.5	6.0	17.0	12.5	6.5	4.0	19.0

Feedback #11 Practice Schedule

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Crew 1	C			B	A			G						
Crew 2	E						F			D				

Note: At 6 days Crew 2 can work on either F or G which have the same priority and completion time. At 7 days Crew 1 has finished their region so moves to work on the remaining medium priority subregion in Crew 2's region. In this schedule F and G could be switched.

Feedback # 12 Final Schedule

Crew 1: C (15.5 days), A (20 days) and B (7 days). This crew is always working in its region and completes all of its crew regions.

Crew 2: F (17 days), D (8.5 days), H (6.5 days) and E (6 day). This crew does all of its own region and also subregion H which is in crew 3's region.

Crew 3: G(12.5 days), J (19 days), and I (4 days). This crew is always working in its region. Crew 2 is used to do Crew 3's region H.

Daily Schedule:
Crew 1: **C** (days 0 to 15.5), **A** (days 15.5 to 35.5), **B** (days 35.5 to 42.5).
Crew 2: **F** (days 0 to 17), **D**(days 17 to 25.5), **H** (days 25.5 to 32), **E** (days 32 to 38)
Crew 3: **G** (days 0 to 12.5), **J** (days 12.5 to 31.5), **I** (days 31.5 to 35.5)

Total times:

Crew 1 – 42.5 days
Crew 2 – 38 days
Crew 3 – 35.5 days

References

All websites were checked August 2007.

Information on Voronoi Diagrams.

Center for Discrete Mathematics and Theoretical Computer Science.
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O'Rourke, Joseph, *Computational Geometry in C*, (2nd edition), Chapter 5 "Voronoi Diagrams", Cambridge University Press, 1998.

Information on Scheduling Algorithms.

Tannenbaum, Peter, *Excursions in Modern Mathematics*. (5th edition), Chapter 8 "The Mathematics of Scheduling." p. 315-355. Prentice Hall 2004

Information on Guided Design.

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Information about particular forest fires.

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BAER Burn Severity map for the Hayman Fire, June 2002, Pike-San Isabel National Forests in Colorado.   http://www.uppersouthplatte.net/resource/haymfire/intnsmap.html

"Sign on San Diego" website, http://www.signonsandiego.com/news/fires/

Appendices

Appendix A. Maps

Plain Map

Campground
Severity/Slope Sample Sites
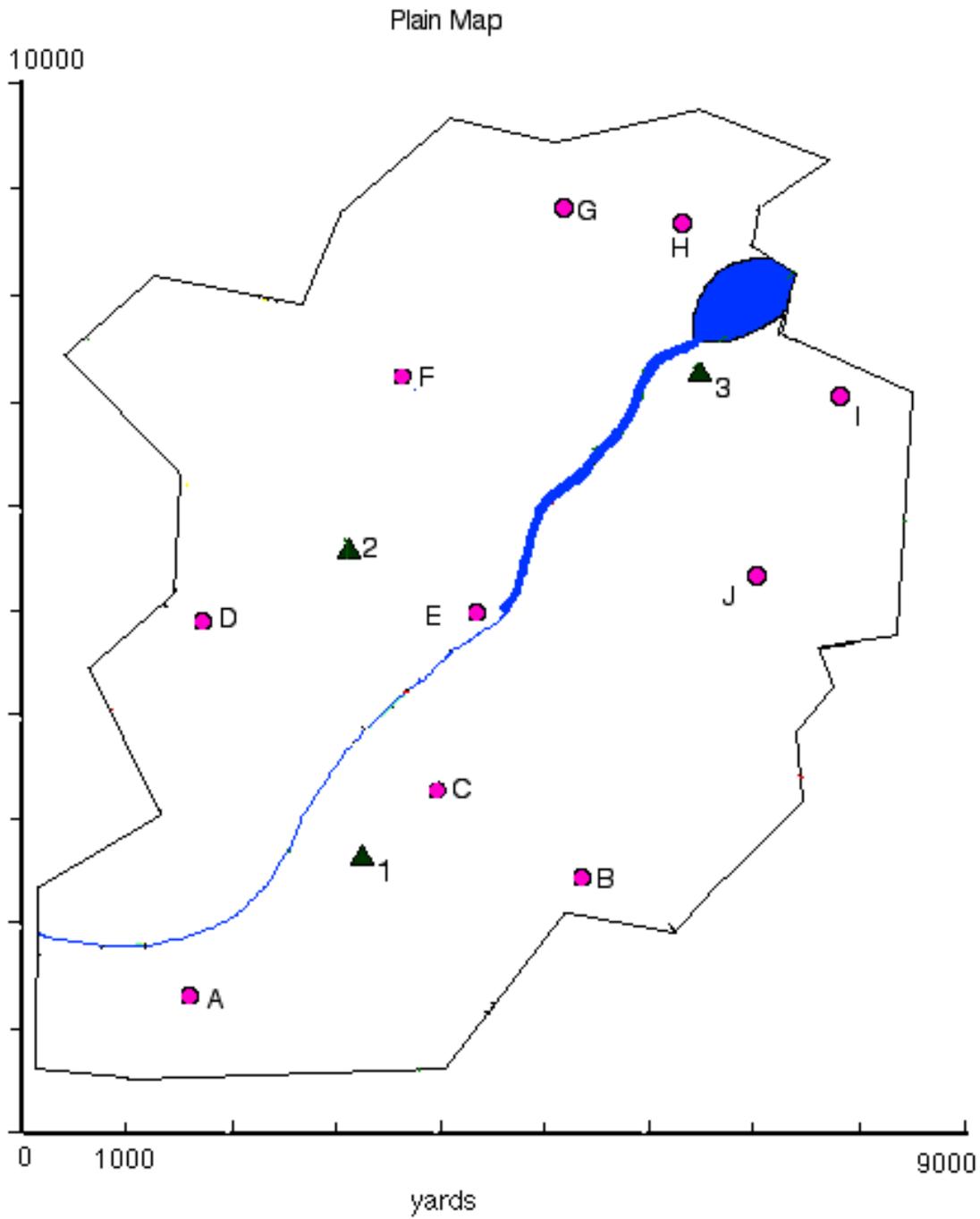
Burn Severity Map
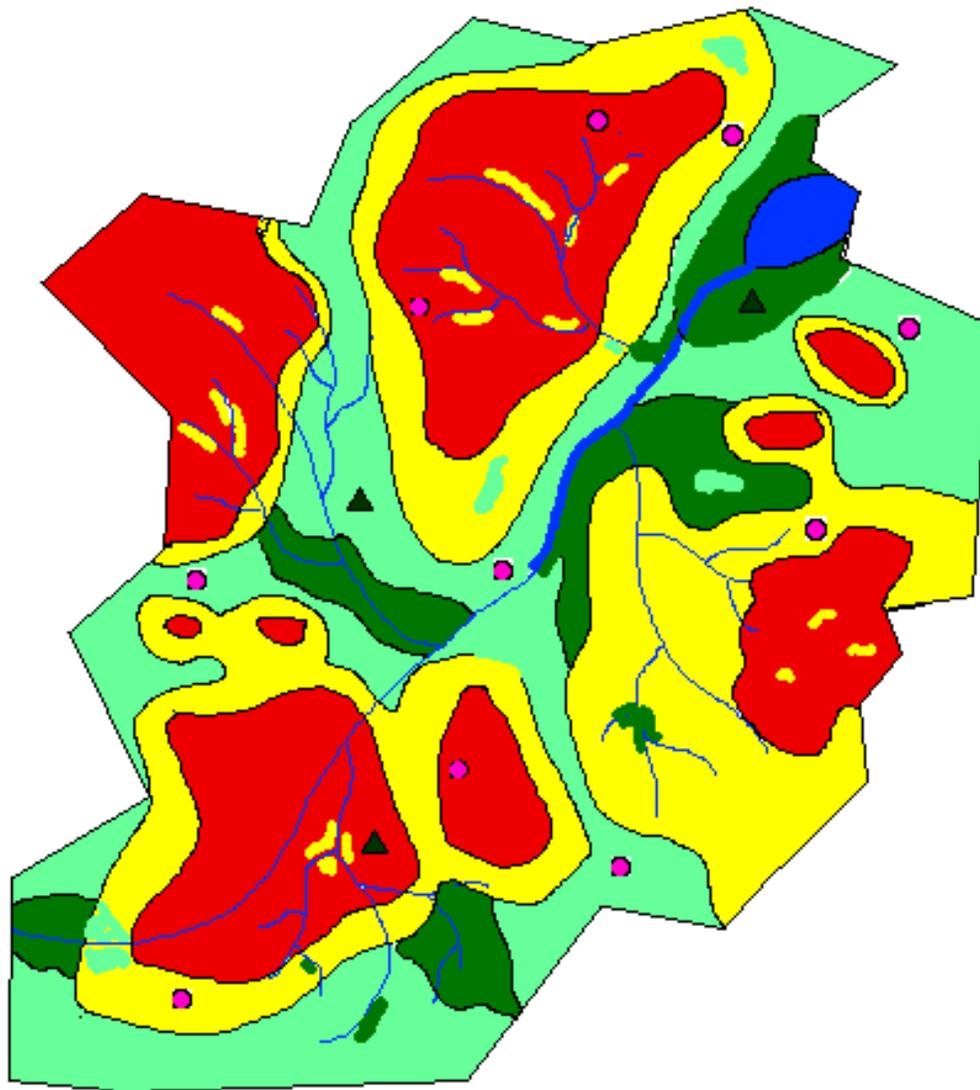
■ (blue)	Water	▲	Campground	
■ (dark green)	Unburned	●	Severity/Slope Sample Sites	
■ (light green)	Low			
■ (yellow)	Moderate			
■ (red)	High			

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Slope Map

Water
Mild Slope
Moderate Slope
Steep Slope

Campground
Severity/Slope Sample Sites

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Appendix B. Geometer's Sketchpad

Bounded Voronoi Diagrams on Geometer's Sketchpad

A basic understanding of Geometer's Sketchpad and Voronoi diagrams is assumed.

Process
Creating a Voronoi diagram on Geometer's Sketchpad (GSP) can be a challenging process without practice. Before formalizing the definitions and theory, let's look at the process we will use to make the diagrams in GSP.   This process depends on two facts from geometry:
1. Points on the perpendicular bisector of a segment are the same distance from the endpoints of the segment.
2. The center of the circumcircle of a triangle is the intersection of the perpendicular bisectors of the edges of the triangle. Hence, the center of the circumcircle is the same distance from the vertices of the triangle.

Consequently, the Voronoi diagram will consist of portions of the perpendicular bisectors of the segments connecting the Voronoi sites. We can form the perpendicular bisectors by triangulating the Voronoi sites and connecting the centers of the circumcircles of these triangles with the midpoints of the edges of their respective triangles.

To create the Voronoi diagram in GSP, first create the convex hull of the Voronoi sites. In the diagram below, A, B, C and D are the Voronoi sites. There are two possible triangulations, but only one will work for us. We show one possibility here.
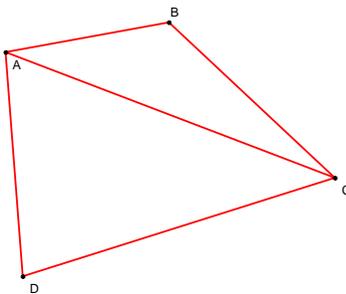
Figure A.

Next we draw the circumcircle of each of the triangles in the triangulation, as shown below. We then check to be sure that we have what is called the *Delauney Triangulation*. This is done by checking *The Empty Circle Property*. In the diagram below we notice that the Voronoi site B is in the circumcircle of triangle ACD. Whenever a Voronoi site is in the interior of one of the circumcircles, the Empty Circle Property is <u>not</u> satisfied.
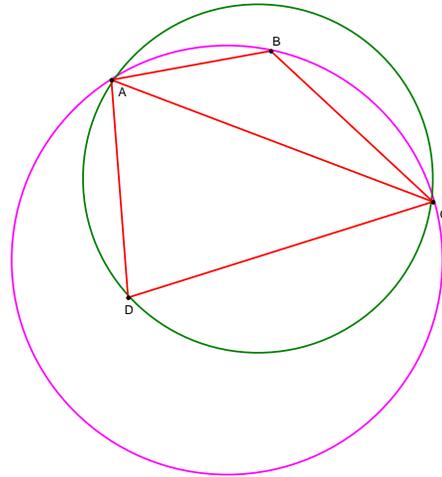
Figure B.

Since the Empty Circle Property is not satisfied, we do not have the Delauney Triangulation. Therefore, we erase the shared segment AC and insert the segment BD. We now have a different triangulation of the four Voronoi sites.

Figure C.

We repeat the process, drawing the circumcircles of the two new triangles.

Figure D.

The interior of these two circles does not contain any Voronoi sites and therefore, the *Empty Circle Property* is satisfied and so this is the *Delauney Triangulation*. Recall that all the points on the perpendicular bisector of a segment are equidistant from the endpoints of that segment. In

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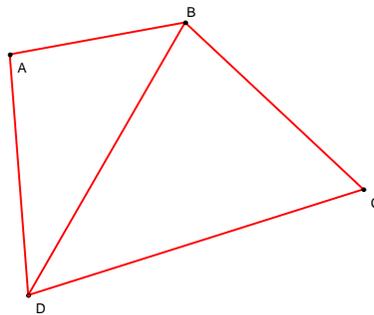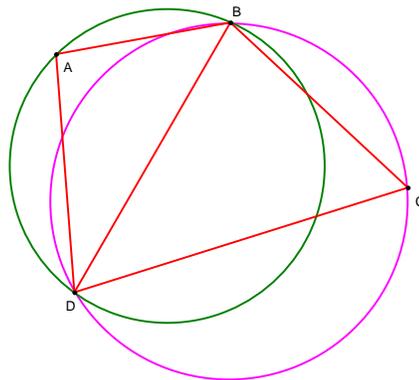this case the endpoints are the Voronoi sites. Thus, the Voronoi diagram will consist of segments and rays that are subsets of the perpendicular bisectors of the edges of the Delaunay triangles. Drawing rays from the center of a circumcircle through the midpoints of each segment on the respective triangle will create the perpendicular bisectors. The blue rays and segments form the Voronoi Diagram.
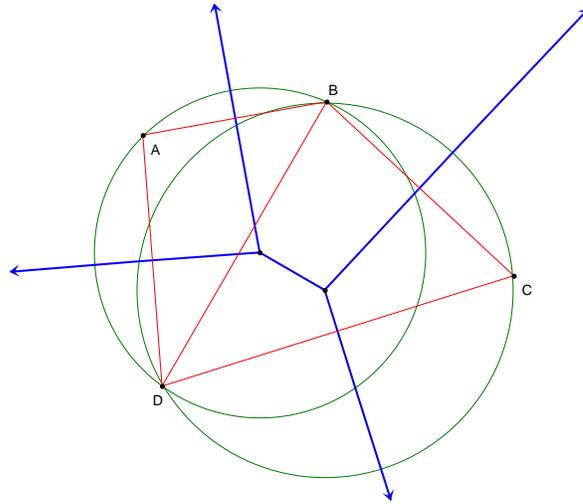
Figure E.

We end this Appendix with a summary of the key definition and theorems used in creating the Voronoi Diagram.

Background

Definition: The *Delaunay Triangulation* is the straight-line dual graph of a Voronoi diagram, where no four sites are on the same circle.

It is helpful to know that the Delaunay triangulation is a partitioning of the convex hull into "fat" triangles. "Fat" triangles have larger minimum angles. Comparing the two triangulations in Figure A. and Figure C., we see that Figure C. has what intuitively appear to be "fatter" triangles.

Properties

1. The boundary of the Delaunay triangulation is the convex hull of the sites.

2. Empty Circle Property: The interior of the circumcircle of each triangle of the Delaunay triangulation contains no sites.

3. The center of each circumcircle is a Voronoi vertex.

Warning: When you are finished, if you want to delete the work that you have done, use the **Edit-Undo** command repeatedly. Selecting all and deleting will delete the rectangles and pictures that were already in the document.

Practice

We will start with a simple problem to become acquainted with the basics of drawing a Voronoi diagram.

- Open the Geometer's Sketchpad Voronoi document that accompanied this module. We will start on the first page "Becoming Acquainted"
    1. Use the **Point** tool to construct three points inside the rectangle. These are the sites.
    2. Use the **Segment** tool to make the triangle containing these three points. Make the segments dashed. This is the Delaunay triangulation.
    3. On the **Custom Tools** menu, choose the **Perpendicular Bisector** tool. Use this tool to make the perpendicular bisector of each side. (See note at end of section on Custom Tool use.)
    4. Use the **Point** tool to construct the intersection of the bisectors. This is the Voronoi vertex.
    5. Use the **Segment** tool to construct the Voronoi edges. Make the edges thick. Recall that the edges are segments along the perpendicular bisectors. The edges should start at the vertex and extend to the edge of the rectangle.
    6. Select the three perpendicular lines, click **Edit-Action Buttons-Hide/Show**. This creates a button to easily hide or show the perpendicular lines. To change the label of the button, click the black tab next to the button, this selects the button. Click **Edit-Properties-Label**. Type the label that you want.
    7. Repeat the previous step, this time creating a button to hide the Delaunay triangulation.
    8. Hint: This action button can be done with any set of objects. It may be helpful to create buttons to hide the sites and the Voronoi diagram. For more information consult the Reference Manual that came with GSP.
    9. Once you are comfortable with the three-site Voronoi, move on to the next page, but do not delete the work you have just done because we will use it in a little bit.

- We want to try this process again with four sites because it is a little more complicated. Use the page called "Learn with Four"
    1. Use the **Point** tool to construct four points inside the rectangle. These are the sites.
    2. Use the **Segment** tool to make the Delaunay triangulation.
    3. Verify that you have drawn the Delaunay triangulation. Use the **Custom Tool-Circumcircle** to check the empty circle property. (See note the section on Custom Tool Use.) Make the centers of the circles green.
    4. Create an action button to hide the circumcircles, and also one to hide the Delaunay triangulation. Then hide the circumcircles. Do not hide the centers, these are the Voronoi vertices.
    5. Use the **Perpendicular Bisector** tool to make the perpendicular bisector of each side of the Delaunay Triangulation.
    6. Create an action button to hide the perpendicular bisectors.
    7. Use the **Segment** tool to construct the Voronoi edges. Recall that the edges are segments along the perpendicular bisectors. The edges should start at the vertex and extend to the edge of the rectangle.
    8. Hide the perpendicular bisectors.

- Now we want to find the area of each of the Voronoi regions that we created. Return to the "Becoming Acquainted" page where you created the three-site Voronoi diagram and we will learn to find areas.
    1. Select the three sites and click **Display-Show Label**. If the labels are not A, B, and C change the label so that they are A, B, and C.
    2. Begin in the region labeled A. Select all the points that are on the boundary of region A in order.
    3. Select **Construct-Quadrilateral Interior**. Region A should now be shaded.
    4. Select **Measure-Area**. The area measurement should appear somewhere on the picture. Drag this measurement so that it is within region A.
    5. Note: the measurement is given in square centimeters. Since this measurement is made within the program, it may not be the same as the measurement you obtain using pencil and paper.
- Congratulations! You have now made Voronoi Diagrams on Geometer's Sketchpad and found the area of Voronoi regions. Continue to practice, possibly trying it with more sites. Good Luck!

NOW RETURN TO INSTRUCTION 5.

Application to Fire Problem

Begin on the "Plain Map" page.

- Step One: Crew Regions
  Use the campsites as the Voronoi sites. Follow the instructions for a three site Voronoi diagram. Since this is a nested Voronoi diagram, we want to make the points and lines specific colors and line widths to keep our confusion level to a minimum. For crew regions: green points, dashed black Delaunay triangulation, thick black Voronoi diagram.
- Step Two: Rehabilitation Regions
  Within each crew region, use the sample sites as the Voronoi sites. Follow the instructions for either three or four site Voronoi diagrams. The colors and line widths for these regions are pink points, dashed blue Delaunay triangulation, thin blue Voronoi diagram.
- Step Three: Go to the "Solution #1" page and verify that your Voronoi diagram is the same.
- Step Four: Find the area of each of the Rehabilitation Regions. The page called "Area Map" can be used here. This page has miscellaneous points and segments removed to make the area process easier.
- Step Five: Go to the "Solution #2" page and verify that your areas are the same. You notice that there is a scale marking, this is to help in the conversion from square centimeters to square yards to acres.

Custom Tool Use
The use of custom tools is a convenient way to do the same action repeatedly. To use a custom tool, you must provide the tool with some information before it will create the desired result. Access to the custom tools is via the triangle and 3 dots marker on the bottom of the toolbar on the left side of the GSP worksheet.

1. Circumcircle - This tool will create the circumcircle for three points. Under the Custom Tools first select "circumcircle". Then, move the cursor to the page and choose the three points that you want the circumcircle of, and the tool will create the circumcircle.
2. Perpendicular Bisector - This tool will create the perpendicular bisector of a line segment. Under the Custom Tools first select "perpendicular bisector". Then move the cursor to the line segment for which you want the perpendicular bisector. The line will be highlighted and the bisector will appear. Click the line, and the bisector is created.

Appendix C. Other Applications

The following are other potential applications that combine scheduling and Voronoi diagrams.

1. *City-wide clean up day.* School students could clean up the neighborhoods that are nearest to their school. Priority could be placed on cleaning up areas that are nearest to lakes, parks, or tourist areas.

2. *Door-to-door political campaigns.* Workers could be sent out to canvas areas nearest to the voting areas. Priority could placed on campaigning in areas which have higher rates of voter participation or in areas in which the politician is lagging in the polls.

3. *Security of national airspace.* Airspace over the United States is currently divided into 20 en-route zones, each managed by a center that monitors long-range jet flights. These are similar to but are not Voronoi diagrams, perhaps due to geographic issues such as needing to have clear channels of communication that could be blocked by major mountain ranges. In addition, major airports have approach control centers. When major airports are close enough, these control centers serve as sites for Voronoi diagrams. Between the en-route zones and the control center zones, something akin to a nested Voronoi diagram is made. In a crisis, we might want to relay urgent information to pilots or deploy Air Force planes, with priority going to areas with suspected imminent risk, the nation's capital, or major urban areas.

Appendix D. Further Explorations

Ambitious instructors teaching classes with more focus on modeling could consider having the students create their own scheduling algorithm. Numerous issues arise that have not been included in the algorithm in this module. These include:

1. Incurring a cost if the crew has to travel outside its region but not if it works in its own region. In this module we rounded the days up to the next 0.5 to provide a buffer. If there are many regions, this may provide more buffer than is really necessary.
2. Reducing the amount of moving a crew does by including adjacency of regions in the algorithm.
3. Further investigating rehabilitation times. For example, unburned regions may require some rehabilitation if burns in nearby areas have caused damage to that region, but they probably don't require as much time as if there were a burn in the area.
4. In our algorithm, when there are options, crews work on smaller subregions. This is because they can complete this more quickly and move on to other subregions in their crew regions, lessening the likelihood that other crews will have to come in to help. The drawback is that an outside crew may then be assigned to a larger subregion and might not be able to finish that in one working day. What happens if the associated crew selects a large subregion instead of a small subregion?

One of the drawbacks of the new method for dividing the forest is that the Voronoi regions are awkwardly shaped compared to the grids. Instructors familiar with the taxicab distance could work with that rather than with a Euclidean distance. In addition, the forest could be divided into smaller grids to begin with, where each small grid is associated with one of the Voronoi sites at which the steepness and severity samples are taken. This is how the crew regions were determined in the original method.

Appendix E Additional Examples

We include three more examples of the scheduling algorithm, each using the same map shown below.   These can assist instructors in understanding how the algorithm is applied or be used to give students additional practice in implementing the algorithm.
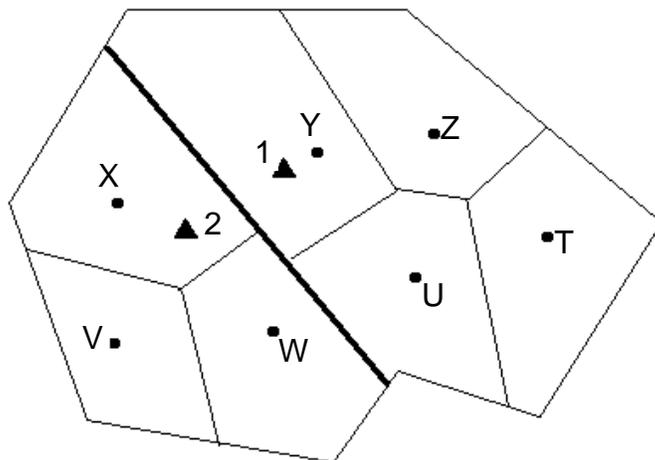
Example 1.

	T	U	V	W	X	Y	Z
Priority	4	6	4	2	8	8	8
Time	3	4	5	6	7	10	2

Example 2.

	T	U	V	W	X	Y	Z
Priority	2	6	4	2	8	8	8
Time	6	5	3	4	5	7	6

Example 3.

	T	U	V	W	X	Y	Z
Priority	6	7	4	2	8	8	8
Time	3	4	5	4	10	7	2

Answers.

Example 1.

Day:	1	3	5	7	9	11	13	15	17	19	21
Crew 1	Z	Y					T				
Crew 2	X			U		V		W			

Example 2.

Day:	1	3	5	7	9	11	13	15	17	19
Crew 1	Z			U			T			
Crew 2	X		Y				V	W		

Note: At 11 days, crew 1 could work on the medium priority area V which is in crew 2's region. Since, however, crew 2 can start V within one day (at 12 days), crew 1 is <u>not</u> assigned to V.

Example 3.

Day:	1	3	5	7	9	11	13	15	17	19
Crew 1	Z	Y				U		V		
Crew 2	X					T	W			

Note: At 13 days, the only open regions are V and W, both in crew 2's region. Crew 2 can work on either V or W, but Crew 1 cannot work on W since it is low priority. By the last rule in the algorithm, we assign Crew 1 to V and Crew 2 to W. Without that last rule, Crew 2 could be assigned both V and W while Crew 1 was moved to another forest.