VOTING, ELECTIONS and SELECTION

Follow-up Session

Leadership Program in Discrete Mathematics

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LEADERSHIP PROGRAM IN DISCRETE MATHEMATICS

Instructor's Notes

Revised October 2001

Follow-up Session – Voting, Elections and Selection

Materials Needed **Allocated Time**

Activity # 1 - Generating Ways of Selecting a Winner……15 minutes

Activity # 2 - Using Preference Charts…….……..55 minutes

Activity # 3 – An Interesting Example…….15 minutes

Activity # 4 – Voting Paradoxes…….….10 minutes

Activity # 5 – Comparing Voting Methods……..……30 minutes

- Video: FAPP's video: "Impossible Dream – Elections"

Activity #6 – Some Real-life Presidential Election Problems…….…15 minutes

……..……..TOTAL WORKSHOP TIME: 140* minutes

* In addition, 10 minutes are allocated for a break in this 2 ½ hour workshop.

Activity # 1 – Generating Ways of Selecting a Winner

(Allocated Time = 15 minutes, 10 minutes for part A and 5 minutes for part B)

A. **Use TSPs #1-3 to introduce the topic and to ask the participants to generate two "lists": A list of classroom situations where group decision-making is required, and a list of ways to "vote" or achieve group consensus**.

TSP #2 notes that the major question is how to take a number of individual preferences and combine them to form a single group preference – which could be called the winner. It also outlines different examples of situations where individual preferences are combined to select winners. For example, selection of Academy Award winners, Emmy winners, college football rankings, presidential winners, sport winners (e.g. gymnastics) etc. We need to add TSPs that show how these work in practice – not here but at the appropriate places, like the 2000 Olympics TSP for sequential runoffs.

Working in small heterogeneous groups (to which participants are assigned), participants generate different ways of selecting winners, preferably in classroom situations; for starters, they could be asked to consider how they would have their students decide which of three books they would like to have the teacher read to them. The goal (unstated) should be for each group to generate three or four different ways of selecting a winner.

B. Each small group reports to the larger group, with the workshop leader recording the different suggested scenarios on one overhead transparency, and the various methods on another transparency.

Activity #2 – Using Preference Charts

(Allocated time = 55 minutes, 10 minutes for part A, 15 minutes each for parts B and C, 10 minutes for part D, and 5 minutes for part E)

A. Tell the participants that we will be going to a concert at a future follow-up session, and that we need to decide what type of music we shall be listening to. Use TSP #4 to have them choose from the following list of choices:

A	Musicals
B	Jazz
C	Oldies (Rock & Roll)

Ask each participant to list the three types of music in the order, most to least (no ties), in which they prefer the various types. For example, the participant who best likes Jazz, then Musicals, but detests Oldies would make the list – it would actually be B, A, C – I've always found that C gets a clear majority; we should find a better example.

C
A

B

Tell the participants that we will want to know how many people had chosen each ordering of the selections A, B, and C. However, before doing that, ask the participants to systematically list all of the permutations of A, B, and C (How many are there?). On TSP #5, generate a table something like the following: We don't use the word "permutations" in any of our workshops, and shouldn't use it here (or below) either.

Note: the slides will use this ordering of the 6 permutations of ABC, so it is a good idea to make your table in this order at this point.

A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

Now record the group's musical choices using TSP #6. Mention that this is called a "preference chart" and conduct a brief discussion of how to read and use them.

Superimpose TSP #7 on TSP #6 to draw the distinction between the Plurality method and Majority Rule. This doesn't work well because of the footers on the TSP pages; it would be best to cut the footer off of TSP #6. *In the latter, a candidate wins only if he/she gets more than half the votes; indeed, if a majority is required, there may be no winner. But a Plurality winner only needs the greatest number of first place votes, which may be less than 50%.* Determine the winner of the group's musical selections using each of these methods.

If in fact one of the choices gets a majority of the votes, you may want to add a number of five votes to one of the columns because, you can tell the participants, it's not too interesting when there is a majority winner. (You can tell the class, humorously of course, that the teacher gets five votes.) It would be better to add extra votes to a number of columns – perhaps add enough so that the winner has only 40% of the total – and divide the extra votes between the four other columns.

Use TSPs #8-9 (superimposed on TSP #6) to introduce Run-Off and the Borda Count method. Determine the winner of the group's musical selections using each of these methods.

At this point, review the transparency that contains the methods that participants proposed, and check off on that list those methods that have already been discussed.

A major point to make is that if each voter provides complete ranking information, then it is possible to simulate many of the methods without having to go back to the voter several times with questions like "Who would you have voted for if your first choice dropped out of the race?"

B. Distribute Handout #1 containing examples which give interesting results using Plurality, Runoff and Borda Count. Ask~~Invite~~ them to determine, from the preference charts given, who would win under these methods.

When they are done, take a few minutes to review the problems, using TSP #10 and TSP #11. Discuss why B wins in the Borda count, and what are the features of using a Borda count method. Give examples of how Borda counts are used in real examples, using TSPs (with information that we have to locate). Note that a Borda count makes it possible for everyone's second choice to be the winner.

C. Use TSPs #12-13 to introduce Sequential Run-Off and to work through an example of this method. Note that you can have ties, and that you can only break them if an agreed-upon method was selected beforehand. Also note that if you use two different methods you can have different winners – so it is important to decide beforehand which method you will use.

Note: Point out why the music example does not suffice for an example pf Sequential Run-Off - with 3 candidates, it is identical with the regular Run-Off method.

Distribute Handout #2 (which is the same as TSP #14) and invite the participants to determine, from the preference chart given, who would win using the Sequential Run-Off method. Review the result using TSP #15; note that the result is a tie between candidates A and C. Voting methods do not always lead to definitive results; if a specific voting method is to be used in a given situation, it is important to specify in advance how ties will be broken – failure to do so is a recipe for conflict. This should be mentioned earlier.

Use TSPs # 16 to illustrate how Sequential Run-Off is used to select the sites for the Olympic games. Note that because a preference chart is not used, they have to conduct a new ballot after each candidate is eliminated. This is different from the type of Sequential Run-Off that we have been discussing because a voter can (and often does) vote for a different candidate than the one he/she supported in the previous ballot, even if his/her candidate wasn't eliminated. There should be similar examples for Borda count, which is often used with variations – e.g., 10 points for first place, 6 points for second place, etc.

D. Use TSPs #17 to introduce the Condorcet method. As you work through the example, illustrate the use of a directed graph to keep track of the results. Since participants have not seen directed graphs previously, this idea has to be introduced here – also it seems better to draw an arrow from A to B if B beats A, although it is counter to the usual practice, because it is easier to see the winner if all arrows are pointing in that direction.

Distribute Handout #3 and invite the participants to determine, from the preference chart given, who would win using the Condorcet method. Before they begin ask the participants how many one-on-one contests they have to consider, and elicit the

answer that it is "4 choose 2", or 6. At each table have one pair work on the three matches that involve A, and another pair work on the three matches that don't involve A. (Otherwise, this will take too long!) Review the result with the entire group.

Note: Point out that the Condorcet method does not always yield a winner.

It is likely that they have devised each of these methods (or variations of them) in their groups. *Note that some of the methods that they devise may not naturally lend themselves to preference charts (for example, approval voting). If that happens, then now would be the time to make the point that these preference charts don't say it all, especially since Approval Voting is the next method discussed*.

E. Use TSPs #19-20 to discuss Approval Voting. Before taking the vote, ask each person to make sure that their votes now are consistent with their earlier preference – that is, if they now approve one choice and disapprove another, then the first should have been higher in their preference. After the vote, ask them whether the information in this table can be extracted from the preference chart. The answer is "No" because some people may have approved only their first choice, some may have approved their first two choices, and some may have approved all three choices. We need to give some examples of where approval voting is used.

[Time for a 5-10 minute break]

Activity #3 – An Interesting Example

(Allocate time = 15 minutes, 10 minutes for part A and 5 minutes for part B)

A. Distribute Handout #4 and put up TSP #21 with Malkevitch's example with 5 methods giving different winners (but don't tell then this, let it be a surprise!). Work through Plurality and Run-Off with the class. Then assign one of Borda Count, Sequential Run-Off or Condorcet to each group to work through. When this is done, record the results on TSP #21, regroup and compare notes. The additional copy of the Preference Chart included as TSP #22 may be helpful in reviewing this problem.

B. Motivate a discussion about which method is best for the given problem, which answer is the most reasonable/unreasonable, and why. There is no right or best answer to this question. Continue this discussion into the next activity.

Activity #4 – Voting Paradoxes

(Allocated time = 10 minutes, 5 minutes each for parts A and B)

A. To motivate a discussion in the direction of Arrow's Impossibility Theorem, ask them to discuss what shortcomings and advantages they saw in the various methods. Supposing that they had to choose or design a voting scheme in some situation, ask them what characteristics they feel is reasonable for a voting scheme.

B. Put up TSP #23 and ask them if this is one characteristic they feel is reasonable for a voting system. Show TSP #24 and work through the example given toshow that the Plurality method does not have this characteristic. This is important because that's the way our presidential elections work state by state. Work through the example on TSP #25 to show that the Borda Count method also fails this property. TSP #26 describes another characteristic that you would like in a voting system. TSP #27 shows that the Run-Off method fails this in a dramatic way.

Summarize by showing TSP #28 with four criteria (The last two they have already seen) and then mentioning the six voting schemes discussed all fail on at least one count according to these criteria. Should they try to design the "Ultimate Fair Voting Scheme?" TSP #29 says "No!"

Note: Do not spend much time on this. It will be discussed further in the video and it is not the point of the workshop! Tell them not to feel like they have to get this point before moving on.

Activity #5 - Comparing Voting Methods

(Allocated time = 30 minutes)

A. Show the FAPP: Impossible Dream-Elections video.

The video includes some good observations about the different systems, and they should be incorporated into these notes so that the video is reinforcing those ideas rather than just introducing them.

Activity #6 – Some Real-life Presidential Election Problems

(Allocated time = 15 minutes, 10 minutes for part A and 5 minutes for part B)

I did not have time to do this activity!

A. Use TSP #30 to review with them the information about how our Presidency is won, just so everyone gets the idea. They should all know that the last time the Electoral College elected a president who did not receive a plurality in the popular vote was 2000; they probably don't know that this happened in four other elections – in 1800, 1824, 1876, and 1888 – you can mention this here and indicate that you will discuss these elections later if there is time, and otherwise they can read the information in the Resource Book.

Distribute Handout #5 and note that we will assume that all of the electoral votes for Maine will go to the same candidate. Then let them work on this example. Surprisingly, you need less than 24% even in this small example.

The following is presented anti-climactically – it comes across as just a bunch of facts; we need to think of a better way of organizing this information and incorporating it into the workshop.

B. If you have time, you can have fun with the anecdotes from TSP #32-33 and discuss the results of the 2000 Presidential election using TSP #34. (There will probably be some helpful social studies teacher who you can encourage to elaborate.)

You can discuss how you only need 270/538 electoral votes (about half) to win, and to get that many votes, you need merely a majority (about half) in those states, and

thus you can claim victory with roughly 25% of the popular vote, even if your opponent gets the other 75%. Things are even worse, because apportionment of electoral votes is only approximate, and you can actually win with 21.6% of the popular vote, your opponent getting the remaining 78.4%.

In 1876, Rutherford B. Hayes won 185 electoral votes with popular vote 4,036,298. His opponent, Samuel J. Tilden, won 184 with popular vote 4,300,590. This caused some complaints, but they didn't change the system.

In 1888, Benjamin Harrison won 233 electoral votes with popular vote 5,439,853. His opponent, Grover Cleveland, won 168 electoral votes with popular vote 5,540,309. Harrison won the presidency. Four years later, with the same candidates, Cleveland won with a popular and electoral landslide.

There are also instances of electors not voting with their states: In 1956, an Alabama elector cast a vote for Jones and Talmadge, a third party ticket. In 1976, a Washington State elector voted for Ronald Reagan, in a race between Carter and Ford. In 1988, a West Virginia elector voted for Bentson as President and Dukakis as Vice President.

Supplementary Notes

1. Other characteristics of voting have been identified. For example, Sincere Voting is voting in a manner consistent with one's preference schedule. In Sincere Voting, one always votes for the most preferred. Strategic Voting, however can include voting insincerely on a ballot on an attempt to achieve a more preferable outcome than could have resulted by voting sincerely.

2. A few political cartoons from the 2000 Presidential Election have been included as Supplementary Transparencies. Although amusing, I thought that these cartoons were now very dated.

VOTING, ELECTIONS and SELECTION

TRANSPARENCIES

Follow-up Session

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Obtaining Group Preferences

There are many situations where a group of people must arrive at a group decision that is based on individual preferences.

- Academy Awards
- Class President
- Where to go on a family vacation
- College Football rankings

Many methods have been devised to determine, as fairly as possible, what the group should do based upon what the individuals wish to do.

NOW IT IS YOUR TURN!

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Make a List of:

- Classroom situations where group decision-making is required

And a List of:

- Methods for arriving at a group decision

Rank the following:

A: Musicals

B: Jazz

C: Oldies (Rock & Roll)

How many arrangements are there of A, B and C?

A Preference Chart

Our music preferences:

A	A	B	B	C	C
		A	C	A	B
B	C	C	A	B	A
C	B				

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An Important Distinction

Majority Rule:
The candidate who has more than 50% of the first-place votes is the winner.

Plurality:
The candidate who has the largest number of first-place votes is the winner.

Run-Off:

The top two first-place vote getters are determined, and then a new election is held between them.

Borda Count:

In a race with N candidates,
the Borda method awards
N points for a first-place ranking,
N-1 points for a second-place ranking,
etc….
and 1 point for a last-place ranking.
The winner is the candidate with the highest point count.

Handout #1 Using Preference Charts

1.

A	B	C
C	C	B
B	A	A
12	7	6

Majority Rule:

Plurality:

Run-off:

Borda Count:

2.

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

Majority Rule:

Plurality:

Run-off:

Borda Count:

Sequential Run-Off:

This method eliminates the candidate with the fewest first-place votes, reassigns those votes in a new election, and repeats this process until only one candidate, the winner, is left.

Consider the following preferences:

A	A	B	C	D	E	E
B	C	C	B	C	D	D
C	D	D	A	B	A	C
D	B	E	E	E	B	B
E	E	A	D	A	C	A
12	11	9	7	5	5	8

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If candidate D is eliminated:

A	A	B	C		E	E
B	C	C	B	C		
C			A	B	A	C
	B	E	E	E	B	B
E	E	A		A	C	A
12	11	9	7	5	5	8

And then candidate B is eliminated:

A	A		C		E	E
	C	C		C		
C			A		A	C
		E	E	E		
E	E	A		A	C	A
12	11	9	7	5	5	8

And then candidate E is eliminated:

A	A		C			
	C	C		C		
C			A		A	C
		A		A	C	A
12	11	9	7	5	5	8

Candidate C is the Sequential Run-Off Winner.

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Handout #2 Sequential Run-Off

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

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Handout #2 Sequential Run-Off

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

If candidate D is eliminated:

A	B	C	C	
B	A	B	B	B
C	C	A		C
			A	A
15	9	11	8	5

And then candidate B is eliminated:

A		C	C	
	A			
C	C	A		C
			A	A
15	9	11	8	5

There is a tie between Candidates A and C.

Voting for the Site of the 2000 Olympics

Here are the details of the voting in 1993 by the 89 members of the IOC on the site of the 2000 games. Note: One member did not vote in the third and fourth rounds.

	Beijing	Berlin	Istanbul	Manchester	Sydney
1	32	9	7	11	30
2	37	9	---	13	30
3	40	---	---	11	37
4	43	---	---	---	45

How is this method different from a sequential runoff?

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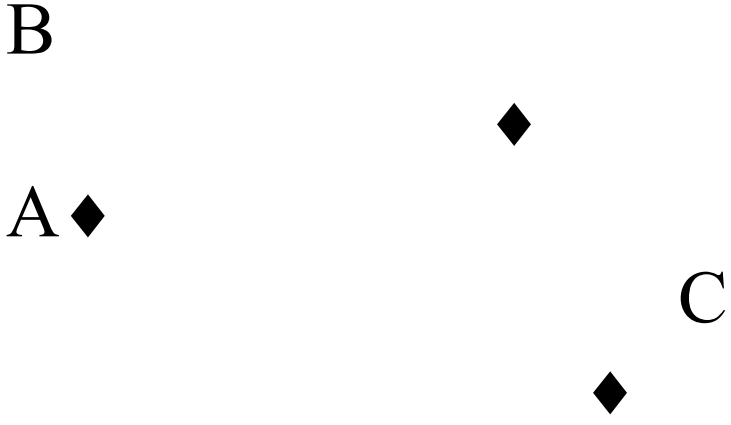
Condorcet:

The Condorcet method awards the election to the choice that can beat each of the other choices in one-on-one contests.

Consider the following preferences:

A C B	B C A	C B A
12	7	6



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Handout #3 The Condorcet Method

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

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Approval Voting:

Each voter gives one vote to as many candidates as he or she wishes.

The winner is the candidate with the most "approval votes."

Approval Voting does not use preference charts.

How many would be willing to go to a concert involving:

A	B	C
Musicals	Jazz	Oldies (Rock & Roll)

Is this information on the preference chart?

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Advantages of Approval Voting

- It gives voters
  more flexible options

- It helps elect
  the strongest candidate

- It reduces
  negative campaigning

- It increases voter turnout

- It gives minority candidates
  their proper due

- It is very practical

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Handout #4 An Interesting Example

B	A	D	C	E	E
C	E	A	D	A	D
E	C	E	E	C	C
D	D	C	A	D	A
A	B	B	B	B	B
18	12	10	9	4	2

Plurality: _____

Run-Off: _____

Borda Count: _____

Sequential Run-Off: _____

Condorcet: _____

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An Interesting Example

B	A	D	C	E	E
C	E	A	D	A	D
E	C	E	E	C	C
D	D	C	A	D	A
A	B	B	B	B	B
18	12	10	9	4	2

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DOES THIS SOUND REASONABLE TO YOU?

If candidate X is the winner of an election, and one or more of the *other* candidates is removed and the ballots recounted, then X should still be the winner of the election.

Consider the following Preferences:

A	B	C
B	C	B
C	A	A
20	18	16

Who is the Plurality Winner?

If we remove the “irrelevant” choice C from the ballot, then who is the Plurality winner?

Does this contradict what you said earlier?

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Consider these College Football Rankings by 90 Sportswriters:

A	B
C	D
B	A
D	C
50	40

Who is the Borda Count winner?

If C is removed due to a recruiting violation, then who is the Borda Count winner?

Does this contradict what you said earlier?

DOES THIS SOUND REASONABLE TO YOU?

If candidate X is the winner of an election, and, in a reelection, all the voters who change their preferences do so in a way that is always favorable to X, then X should still be the winner of the election.

Consider the following Preferences:

A	C	B	B
B	A	C	A
C	B	A	C
38	30	25	7

Who is the Run-Off Winner?

Suppose that the 7 voters whose preference is B-A-C are aware in advance of the likely outcome of the election and decide to switch their support to A, and vote A-B-C.

A	C	B	A
B	A	C	B
C	B	A	C
38	30	25	7

Now who is the Run-Off winner?

Does this contradict what you said earlier?

Fairness Criteria

Majority Criterion: If there is a candidate that is the first choice of a majority of the voters, then that candidate should be the winner of the election.

Condorcet Criterion: If there is a candidate that wins in a one-to-one comparison with any other candidate, then that candidate should be the winner of the election.

Monotonicity Criterion: If candidate X is the winner of an election and, in a reelection, all the voters who change their preferences do so in a way that is always favorable to X, then X should still be the winner of the election.

Independence of Irrelevant Alternatives: If candidate X is the winner of an election, and one or more of the other candidates is removed and the ballots recounted, then X should still be the winner of the election.

Kenneth Arrow's Impossibility Theorem (1952):

It is not possible to have a preferential voting method that satisfies all of the fairness criteria above.

That is, there is no preferential voting system that is completely fair.

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Presidential Elections

The Electoral College selects the President of the United States.
The electors, equal in number to each state's delegation in Congress, typically award their votes on a winner-take-all basis.

Exceptions: Maine and Nebraska pick two electors by statewide popular vote and the remainder by the popular vote in each Congressional district.

The Electoral College can elect a candidate who does not win the most popular votes.

When did this last happen? How many times has this happened in the 54 elections since 1789?

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Handout #5 Electoral Votes

Suppose that an election was held in just these 8 states and that in each state the winner of the popular vote is awarded all of electoral votes for that state. The winner of the election is the candidate with the most electoral votes.

Question: What is the smallest percentage of the *popular* vote that a candidate needs to win the Presidency in these 8 states?

State	**Electoral Votes**	**Population (in tens of thousands)**
Maine	4	249
Maryland	10	480
Massachusetts	12	603
Michigan	18	933
Minnesota	10	439
Mississippi	7	259
Missouri	11	514
Montana	3	80
Totals	**75**	**3557**

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Some Surprising Presidential Facts

1800: Thomas Jefferson and Aaron Burr each won 73 electoral votes. The House of Representatives chose Thomas Jefferson. Aaron Burr became Vice President.

1824: Andrew Jackson received 99 electoral votes, while John Quincy Adams had 84, William Crawford had 41, and Henry Clay had 37. None of the candidates achieved an electoral majority.

The House of Representatives chose John Quincy Adams although Andrew Jackson received the largest popular vote with 42%.

1876: Rutherford B. Hayes edged out Samuel J. Tilden by a single electoral vote, though he lost to Mr. Tilden at the polls. This result was confirmed (by a single vote) by a 15-member bipartisan commission.

1888: Benjamin Harrison received 233 electoral votes to Grover Cleveland's 168, although Grover Cleveland captured a plurality in the popular vote. Four years later, Cleveland beat Harrison, becoming the only President elected to non-consecutive terms.

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2000 Presidential Election

	Electoral Vote	Popular Vote
Gore	267	50,158,094 (48%)
Bush	271	49,820,518 (48%)
Others	0	3,835,594 (4%)

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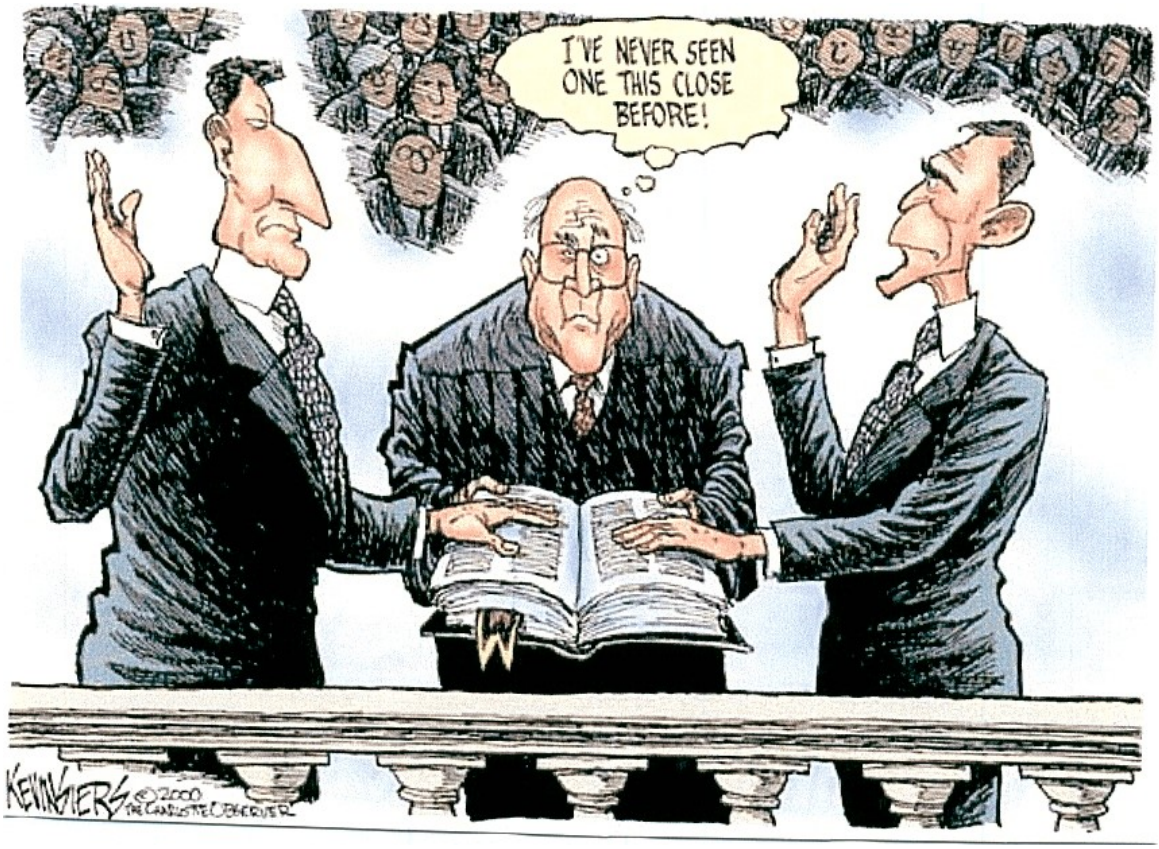
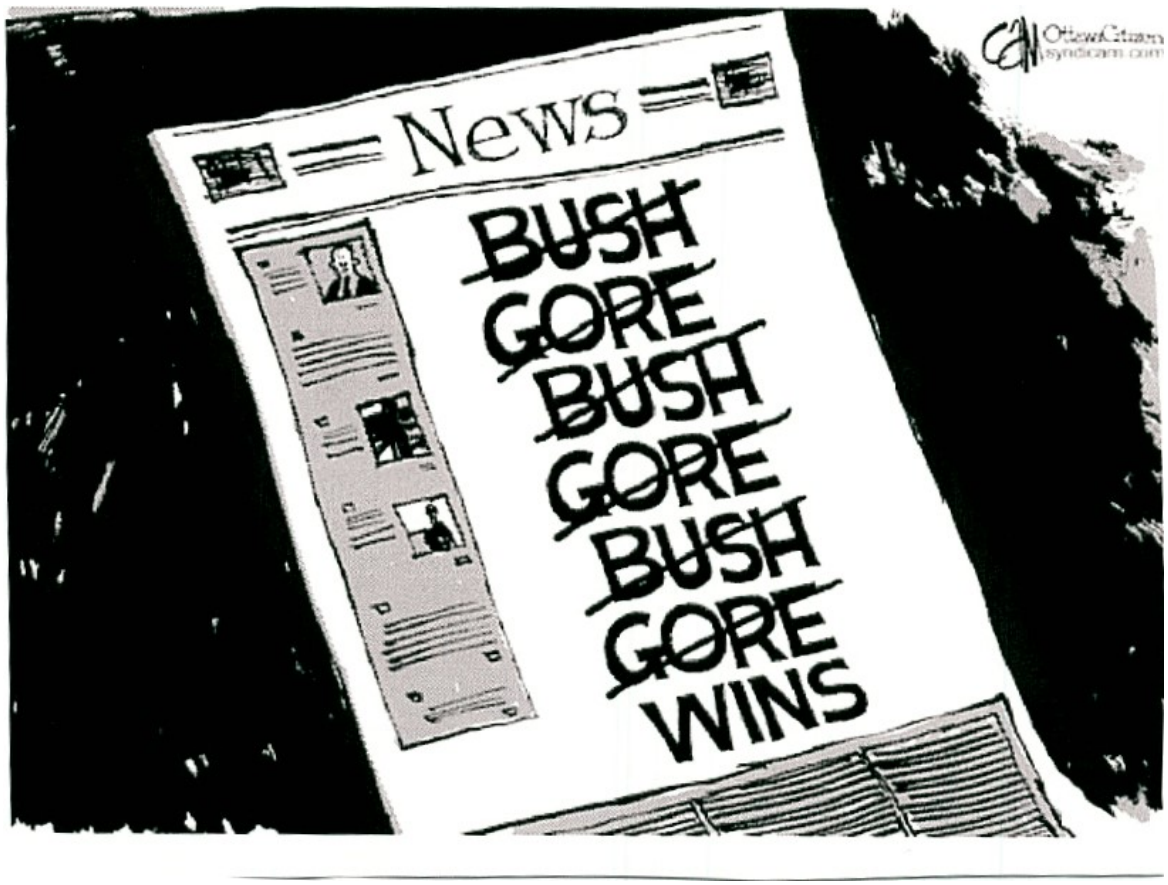

I'VE NEVER SEEN ONE THIS CLOSE BEFORE!
KEVIN SIERS ©2000 THE CHARLOTTE OBSERVER

©2000 THE AUGUSTA CHRONICLE
GORE DEFEATS BUSH
W
BUSH DEFEATS GORE
AL


News
BUSH
GORE
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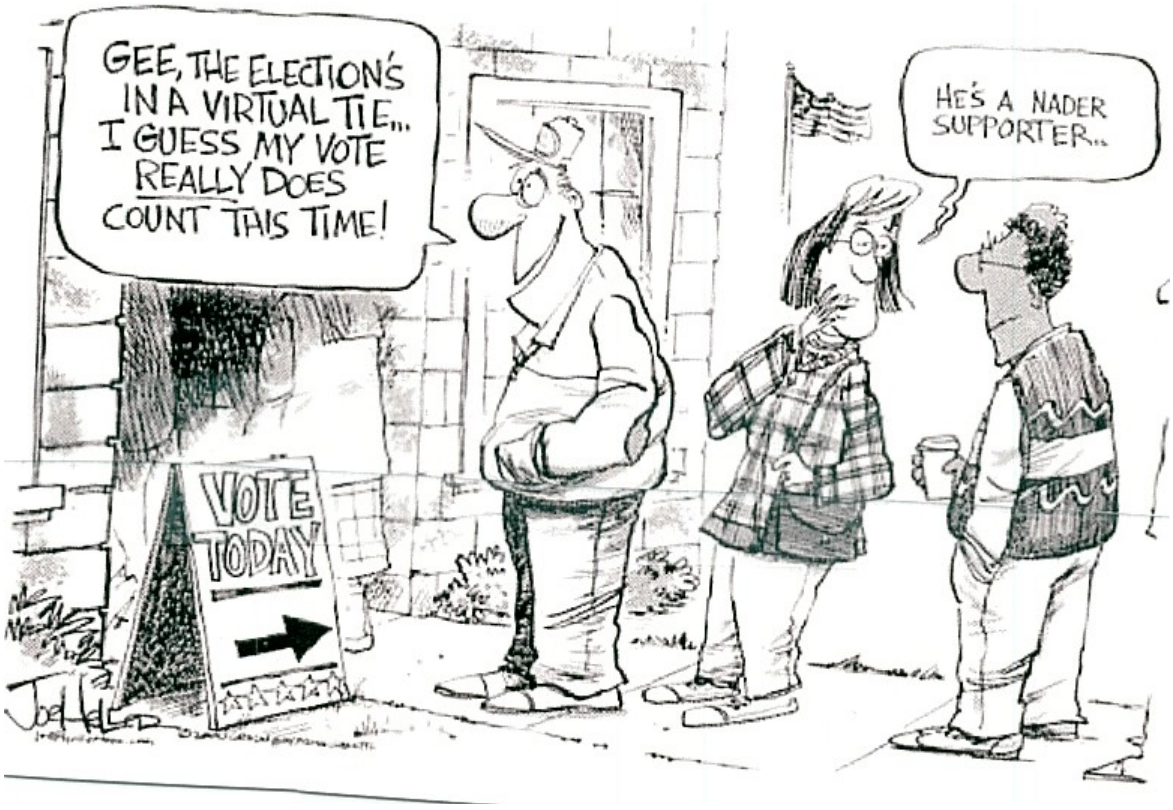
WINS

PRESIDENT OF THE UNITED


GEE, THE ELECTION'S IN A VIRTUAL TIE... I GUESS MY VOTE REALLY DOES COUNT THIS TIME!
HE'S A NADER SUPPORTER...
VOTE TODAY

MARLETTE
COMPLAINTS
FLA.

VOTING, ELECTIONS and SELECTION

HAND-OUTS

Follow-up Session

Leadership Program in Discrete Mathematics

Handout #1 Using Preference Charts

For each of the preference charts shown below, determine which option will be chosen under the Majority Rule, Plurality, Run-off, and Borda Count selection methods.

Majority Rule:
The candidate who has more than 50% of the first-place votes is the winner.

Plurality Winner:
The candidate who has the largest number of first-place votes is the winner.

Run-Off:
The top two first-place vote getters are determined, and then a new election is held between them.

Borda Count:
In a race with N candidates, the Borda method awards N points for a first-place ranking, N-1 points for a second-place ranking, etc…. and 1 point for a last-place ranking.
The winner is the candidate with the highest point count.

1.

A	B	C
C	C	B
B	A	A
12	7	6

2.

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

Handout #2 Sequential Run-Off

For the preference chart shown below, determine which option will be chosen under the Sequential Run-Off method.

Sequential Run-Off:
This method eliminates the candidate with the fewest first-place votes, reassigns those votes in a new election, and repeats this process until only one candidate, the winner, is left.

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

Handout #3 The Condorcet Method

For the preference chart shown below, determine which option will be chosen under the Condorcet method.

Condorcet:
The Condorcet method awards the election to the choice that can beat each of the other choices in one-on-one contests.

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

Handout #4 An Interesting Example

B	A	D	C	E	E
C	E	A	D	A	D
E	C	E	E	C	C
D	D	C	A	D	A
A	B	B	B	B	B
18	12	10	9	4	2

For each of the following methods, decide who wins the election, based upon the preference schedule shown above.

Plurality: The candidate who has the largest number of first-place votes is the winner.

The Plurality Winner is _____.

Run-Off: The top two first-place vote getters are determined. Then a new election is held between them.

The winner by the Run-Off method is _____.

Borda Count: In a race between *N* candidates, the Borda method awards *N* points for a first-place ranking, *N*-1 points for a second-place ranking, etc…., and 1 point for a last-place ranking. The winner is the candidate with the highest point count.

The winner by the Borda Count method is _____.

Sequential Run-Off: This method eliminates the candidate with the fewest first-place votes, reassigns those votes in a new election, and repeats this process until only one candidate, the winner, is left.

The winner by the Sequential Run-Off method is _____.

Condorcet: The Condorcet method awards the election to the choice that can beat each of the other choices in one-on-one contests.

The winner by the Condorcet method is _____.

Handout #5 Electoral Votes

In the United States, in real-life, Presidential elections are carried out as follows: Popular elections are held in each state, and the plurality winner for that state wins all the electoral votes apportioned to that state. These electoral votes are apportioned to each state according to its population, bigger states getting (roughly) proportionally more votes. The winner of the Presidency is the candidate who gets the most *electoral* votes.

This has some strange consequences, not the least of which is that a candidate can win the Presidency with fewer popular votes than an opponent.

Listed below are the 8 states that begin with the letter "M". Suppose that an election was held in just these 8 states and that in each state the winner of the popular vote is awarded all of electoral votes for that state. The winner of the election is the candidate with the most electoral votes.

Question: What is the smallest percentage of the *popular* vote that a candidate needs to win the Presidency in these 8 states?

State	**Electoral Votes**	**Population (in tens of thousands)**
Maine	4	249
Maryland	10	480
Massachusetts	12	603
Michigan	18	933
Minnesota	10	439
Mississippi	7	259
Missouri	11	514
Montana	3	80
Totals	**75**	**3557**

VOTING ELECTIONS and SELECTION

RESOURCE MATERIALS

Follow-up Session

Leadership Program in Discrete Mathematics

Some Voting Methods

Majority Rule:
The candidate who has more than 50% of the first-place votes is the winner.

Plurality:
The candidate who has the largest number of first-place votes is the winner.

Run-Off:
The top two first-place vote getters are determined, and then a new election is held between them.

Borda Count:
In a race with N candidates, the Borda method awards
N points for a first-place ranking,
N-1 points for a second-place ranking,
etc….
and 1 point for a last-place ranking.
The winner is the candidate with the highest point count.

Sequential Run-Off:
This method eliminates the candidate with the fewest first-place votes, reassigns those votes in a new election, and repeats this process until only one candidate, the winner, is left.

The Condorcet Method:
The Condorcet method awards the election to the choice that can beat each of the other choices in one-on-one contests.

Approval Voting:
Each voter gives one vote to as many candidates as he or she wishes.
The winner is the candidate with the most "approval votes."

Resources on Voting, Elections and Selection

An excellent source for classroom material on voting:

Is Democracy Fair? The Mathematics of Voting and Apportionment, Leslie Johnson Nielson and Michael de Villiers, Key Curriculum Press, 1997, ISBN 1-55953-277-7

Web-based Resources on Mathematics and Elections:

http://mathforum.com/t2t/faq/election.taco
The Math Forum section on Mathematics and Elections includes references as well as links to many resources on this topic. Some of the past Math Forum's Problems of the Week can also be accessed such as A Voting Problems, Approval Method of Voting and The 2000 Olympics.

http://illuminations.nctm.org/lessonplans/9-12/vote/index.html
This NCTM Publication-Based Lesson Plan outines activities in which students explore alternative voting methods. Three Student Activity Sheets are included

http://www.igc.org/cvd/
The Center for Voting and Democracy site includes a link to information on Instant RunOff Voting including a guide to using IRV in School Elections and a practical example in which the Muppets use IRV to choose a new CEO.

Some books with elections materials in them:

For All Practical Purposes: Introduction to Contemporary Mathematics, COMAP, WH Freeman, 1999.

Excursions in Modern Mathematics, Peter Tannenbaum/Robert Arnold, Prentice-Hall, 2000.

Discrete Mathematics Through Aplications, Nancy Crisler, Patience Fisher, Gary Froelich, WH Freeman, 2000.

The video is available from COMAP:

For All Practical Purposes's video: "Impossible Dream – Elections"

1-800-772-6627
57 Bedford Street
Lexington MA 02173

Obtaining Group Preferences

There are many situations where we would like to obtain a group consensus based upon individual preferences.

- Academy Awards
- Class President
- Where to go on a family vacation
- College Football rankings

Many methods have been devised to determine, as fairly as possible, what the group should do based upon what the individuals wish to do.

NOW IT IS YOUR TURN!

Make a List of:

- Classroom situations where group decision-making is required

And a List of:

- Methods for achieving group consensus

Rank the following:

A: Musicals
B: Jazz
C: Oldies (Rock & Roll)

A Preference Chart

Our music preferences:

A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

Handout #1 Using Preference Charts

For each of the preference charts shown below, determine which option will be chosen under the Majority Rule, Plurality, Run-off, and Borda Count selection methods.

Majority Winner:
The candidate who has more than 50% of the first-place votes is the winner.

Plurality Winner:
The candidate who has the largest number of first-place votes is the winner.

Run-Off:
The top two first-place vote getters are determined, and then a new election is held between them.

Borda Count:
In a race with N candidates, the Borda method awards N points for a first-place ranking, N-1 points for a second-place ranking, etc…. and 1 point for a last-place ranking.
The winner is the candidate with the highest point count.

1.

A	B	C
C	C	B
B	A	A
12	7	6

2.

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

Handout #2 Sequential Run-Off

For the preference chart shown below, determine which option will be chosen under the Sequential Run-Off method.

Sequential Run-Off:
This method eliminates the candidate with the fewest first-place votes, reassigns those votes in a new election, and repeats this process until only one candidate, the winner, is left..

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

Handout #3 The Condorcet Method

For the preference chart shown below, determine which option will be chosen under the Condorcet method.

Condorcet:
The Condorcet method awards the election to the choice that can beat each of the other choices in one-on-one contests.

A	B	C	C	D
B	A	B	B	B
C	C	A	D	C
D	D	D	A	A
15	9	11	8	5

Handout #4 An Interesting Example

B	A	D	C	E	E
C	E	A	D	A	
					D
E	C	E	E	C	C
D	D	C	A	D	
					A
A	B	B	B	B	B
			9	4	2
18	12	10			

For each of the following methods, decide who wins the election, based upon the preference schedule shown above.

Plurality: The candidate who has the largest number of first-place votes is the winner.

The Plurality Winner is _____.

Run-Off: The top two first-place vote getters are determined. Then a new election is held between them.

The winner by the Run-Off method is _____.

Borda Count: In a race between *N* candidates, the Borda method awards *N* points for a first-place ranking, *N*-1 points for a second-place ranking, etc…., and 1 point for a last-place ranking. The winner is the candidate with the highest point count.

The winner by the Borda Count method is _____.

Sequential Run-Off: This method eliminates the candidate with the fewest first-place votes, reassigns those votes in a new election, and repeats this process until only one candidate, the winner, is left.

The winner by the Sequential Run-Off method is _____.

Condorcet: The Condorcet method awards the election to the choice that can beat each of the other choices in one-on-one contests.

The winner by the Condorcet method is _____.

Handout #5 Electoral Votes

In the United States, in real-life, Presidential elections are carried out as follows: Popular elections are held in each state, and the plurality winner for that state wins all the electoral votes apportioned to that state. These electoral votes are apportioned to each state according to its population, bigger states getting (roughly) proportionally more votes. The winner of the Presidency is the candidate who gets the most *electoral* votes.

This has some strange consequences, not the least of which is that a candidate can win the Presidency with fewer popular votes than an opponent.

Listed below are the 8 states that begin with the letter "M". Suppose that an election was held in just these 8 states and that in each state the winner of the popular vote is awarded all of electoral votes for that state. The winner of the election is the candidate with the most electoral votes.

Question: What is the smallest percentage of the *popular* vote that a candidate needs to win the Presidency in these 8 states?

State	**Electoral Votes**	**Population (in tens of thousands)**
Maine	4	249
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Massachusetts	12	603
Michigan	18	933
Minnesota	10	439
Mississippi	7	259
Missouri	11	514
Montana	3	80
Totals	**75**	**3557**

Sequential Run-Off:

This method eliminates the candidate with the fewest first-place votes, reassigns those votes in a new election, and repeats this process until only one candidate, the winner, is left.

Consider the following preferences:

A	A	B	C	D	E	E
B	C	C	B	C	D	D
C	D	D	A	B	A	C
D	B	E	E	E	B	B
E	E	A	D	A	C	A
12	11	9	7	5	5	8

If candidate D is eliminated:

A	A	B	C		E	E
B	C	C	B	C		
C			A	B	A	C
	B	E	E	E	B	B
E	E	A		A	C	A
12	11	9	7	5	5	8

And then candidate B is eliminated:

A	A		C		E	E
	C	C		C		
C			A		A	C
		E	E	E		
E	E	A		A	C	A
12	11	9	7	5	5	8

And then candidate E is eliminated:

A	A		C			
	C	C		C		
C			A		A	C
		A		A	C	A
12	11	9	7	5	5	8

Candidate C is the Sequential Run-Off Winner.

Voting for the Site of the 1996 Olympics

Cities may submit bids to become hosts for the summer or winter Olympic Games. The International Olympic Committee then uses Plurality with Elimination (a sort of Sequential Run-Off) to select the site.

Here are the details of the voting in 1990 by the 86 members of the IOC on the site of the 1996 games.

	Athens	Atlanta	Belgrade	Manchester	Melbourne	Toronto
1	23	19	7	11	12	14
2	23	20	---	5	21	17
3	26	26	---	---	16	18
4	34	30	---	---	---	22
5	35	51	---	---	---	---

How is this method different from Sequential Run-Off?

Voting for the Site of the 2000 Olympics

Cities may submit bids to become hosts for the summer or winter Olympic Games. The International Olympic Committee then uses Plurality with Elimination (a sort of Sequential Run-Off) to select the site.

Here are the details of the voting in 1993 by the 89 members of the IOC on the site of the 2000 games. Note: One member did not vote in the third and fourth rounds.

	Beijing	Berlin	Istanbul	Manchester	Sydney
1	32	9	7	11	30
2	37	9	---	13	30
3	40	---	---	11	37
4	43	---	---	---	45

How is this method different from Sequential Run-off?.

Approval Voting

Each voter gives one vote to as many candidates as he or she wishes.
The winner is the candidate with the most "approval votes."

Advantages of Approval Voting

- It gives voters more flexible options
- It helps elect the strongest candidate
- It reduces negative campaigning
- It increases voter turnout
- It gives minority candidates their proper due
- It is very practical

DOES THIS SOUND REASONABLE TO YOU?

If candidate X is the winner of an election, and one or more of the *other* candidates is removed and the ballots recounted, then X should still be the winner of the election.

Consider the following Preferences:

A	B	C
B	C	B
C	A	A
20	18	16

Who is the Plurality Winner?

If we remove the "irrelevant" choice C from the ballot, then who is the Plurality winner?

Does this contradict what you said earlier?

Consider these College Football Rankings by 90 Sportswriters:

A	B
C	D
B	A
D	C
50	40

Who is the Borda Count winner?

If C is removed due to a recruiting violation, then who is the Borda Count winner?

Does this contradict what you said earlier?

DOES THIS SOUND REASONABLE TO YOU?

If candidate X is the winner of an election, and, in a reelection, all the voters who change their preferences do so in a way that is always favorable to X, then X should still be the winner of the election.

Consider the following Preferences:

A	C	B	B
B	A	C	A
C	B	A	C
38	30	25	7

Who is the Run-Off Winner?

Suppose that the 7 voters whose preference is B-A-C are aware in advance of the likely outcome of the election and decide to switch their support to A, and vote A-B-C.

A	C	B	A
B	A	C	B
C	B	A	C
38	30	25	7

Now who is the Run-Off winner?

Does this contradict what you said earlier?

Fairness Criteria

Majority Criterion: If there is a candidate that is the first choice of a majority of the voters, then that candidate should be the winner of the election.

Condorcet Criterion: If there is a candidate that wins in a one-to-one comparison with any other candidate, then that candidate should be the winner of the election.

Monotonicity Criterion: If candidate X is the winner of an election and, in a reelection, all the voters who change their preferences do so in a way that is always favorable to X, then X should still be the winner of the election.

Independence of Irrelevant Alternatives: If candidate X is the winner of an election, and one or more of the other candidates is removed and the ballots recounted, then X should still be the winner of the election.

Kenneth Arrow's Impossibility Theorem (1952):

It is not possible to have a preferential voting method that satisfies all the fairness criteria above.

That is, there is no preferential voting system that is completely fair.

Presidential Elections

The Electoral College selects the President of the United States.

The electors, equal in number to each state's delegation in Congress, typically award their votes on a winner-take-all basis.

Exceptions: Maine and Nebraska pick two electors by statewide popular vote and the remainder by the popular vote in each Congressional district.

The Electoral College sometimes elects a candidate who did not win the most popular votes.

Some Surprising Presidential Facts

1800: Thomas Jefferson and Aaron Burr each won 73 electoral votes.
The House of Representatives chose Thomas Jefferson. Aaron Burr became Vice President.

1824: Andrew Jackson received 99 electoral votes, while John Quincy Adams had 84, William Crawford had 41, and Henry Clay had 37. None of the candidates achieved an electoral majority.

The House of Representatives chose John Quincy Adams although Andrew Jackson received the largest popular vote with 42%.

1876: Rutherford B. Hayes edged out Samuel J. Tilden by a single electoral vote, though he lost to Mr. Tilden at the polls. This result was confirmed (by a single vote) by a 15-member bipartisan commission.

1888: Benjamin Harrison received 233 electoral votes to Grover Cleveland's 168, although Grover Cleveland captured a plurality in the popular vote. Four years later, Cleveland beat Harrison, becoming the only President elected to non-consecutive terms.

2000 Presidential Election

	Electoral Vote	Popular Vote
Gore	267	50,158,094 (48%)
Bush	271	49,820,518 (48%)
Others	0	3,835,594 (4%)

1992 Presidential Election
Study by Steven Brams and Samuel Merrill

Based on data collected in a University of Michigan study.

1658 voters were asked to rate each of three candidates on a "feeling thermometer," giving each candidate a score from 0 to 100 degrees.

Brams/Merrill decided that a voter approved the candidate he/she felt warmest about and also approved the candidate he/she felt second warmest about if either (a) that candidate was rated over 50 degrees whereas the third candidate was not, or (b) all three candidates were rated over 50 but the second candidate's rating was closer to the first candidate's rating than to the third.

Conclusion: For this group of voters, the percentage who actually voted for the three candidates were:

Clinton – 47.8% Bush – 34.00% Perot – 18,2%

The **approval votes** for the candidates were:

Clinton – 61.9% Bush – 47.8% Perot – 39.7%

Campaign strategies would have shifted if approval voting were used.