STANDARD 5 — TOOLS AND TECHNOLOGY

K-12 Overview

> All students will regularly and routinely use calculators, computers, manipulatives, and other mathematical tools to enhance mathematical thinking, understanding, and power.

Descriptive Statement

Calculators, computers, manipulatives, and other mathematical tools need to be used by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Historically, people have developed and used manipulatives (such as fingers, base ten blocks, geoboards, and algebra tiles) and mathematical devices (such as protractors, coordinate systems, and calculators) to help them understand and develop mathematics. Students should explore both new and familiar concepts with calculators and computers, but should also become proficient in using technology as it is used by adults, that is, for assistance in solving real-world problems.

Meaning and Importance

Both mathematics and the way we do mathematics have changed dramatically in the last few decades. The presence of technology has made more mathematics accessible to us, has allowed us to solve mathematical problems never before solved, and has brought a new and extraordinarily higher level of proficiency with mathematical operations to all members of our society. At the same time, our resulting increased need to truly understand that mathematics has brought on a renewed interest in developing materials and approaches that provide concrete models and that can engage learners. Both of these new directions combine in this one standard which calls for the appropriate and effective use of tools and technology.

Manipulatives are concrete materials that are used for "modeling" or representing mathematical operations or concepts. In much the same way that children make and use models of race cars or the human skeleton so that they can study and learn about them, students can make and learn from models of two-digit numbers or multiplication. The difference between the two situations is that while the car and skeleton models are smaller versions of actual concrete things, the number models or multiplication models are concrete models of abstract concepts and operations.

When students use bundles of sticks and single sticks to represent tens and ones, or algebra tiles to represent polynomials, they are using manipulatives to "model" mathematical ideas. Technically, even when young children count on their fingers, they are concretely modeling numbers.

The mechanism by which concrete modeling aids children in constructing mathematical knowledge is still not completely understood. There is little doubt, however, that it does. There is a good deal of research which shows that the optimal presentation sequence for new mathematical content is *concrete-pictorial-abstract*. Activities with concrete materials should precede those which show pictured relationships and those should, in turn, precede formal work with symbols. Ultimately, students need to reach that final level of symbolic proficiency with many of the mathematical skills they master, but the meanings of those symbols must be firmly rooted in experiences with real objects. Otherwise, their performance of the symbolic operations will simply be rote repetitions of meaningless memorized procedures.

Calculators and **computers** provide still other benefits for students. In the 1994 *Position Statement on The Use of Technology in the Learning and Teaching of Mathematics*, the National Council of Teachers of Mathematics states:

> *Students are to learn how to use technology as a tool for processing information, visualizing and solving problems, exploring and testing conjectures, accessing data, and verifying their solutions. . . . In a mathematics setting, technology must be an instructional tool that is integrated into daily teaching practices, including the assessment of what students know and are able to do.*

The availability of technology requires that we re-evaluate our mathematics curricula. What we teach and how we teach it are now inextricably linked to these new tools. Their presence makes some traditional mathematics topics obsolete — we certainly do not still teach the square root extraction algorithm, but what about hours and hours of paper-and-pencil practice with the long division algorithm? How much is necessary? How much is adequate? How much will our students need that skill when they become adults? The presence of the technology also makes some traditional topics more important than ever — efficient calculator use requires a high level of estimation, place value, and mental math skills so that calculations can be quickly checked for reasonableness and accuracy. And the technology allows some topics to be dealt with that were never accessible to students previously — graphing calculators allow students to instantly see the graphs of complex functions that would have taken a whole class period to graph by hand, and computer-based geometry construction tools allow students to experiment with animation to see the effects of transformations of figures in three-dimensional space.

Stephen Willoughby, a former president of the National Council of Teachers of Mathematics, offers this analogy to those who might be reluctant to change the traditional curriculum:

> *When automobiles first appeared, there were undoubtedly many people who kept a spare horse in the garage lest the automobile fail, but very few of us still do today. Calculators, with and without batteries, have become so inexpensive and reliable that it is more efficient to keep an extra calculator handy than it is to learn to do well everything a calculator does better. Most of us no longer find the ability to shoe a horse or cinch a saddle to be essential skills. Is it not reasonable that in the near future we may feel the same way about multi-digit long division? (Willoughby, 1990, p. 62)*

The New Jersey State Board of Education, recognizing the need for students to develop appropriate and integrated technology skills, requires the use of calculators in the state's mathematics assessment program at the eighth and eleventh grade levels. To be adequately prepared to use calculators on those assessments, it is critical that the students have ample opportunity to use them as a regular and natural part of their mathematics classes and other testing.

We are only just beginning to explore the ways in which technology can be helpful to mathematics learners, but already there are tremendous opportunities at all levels. Two additional tools that have not yet been mentioned in this Overview possibly provide the most futuristic vision of what mathematics classrooms might soon look like. Calculator-Based Laboratories (CBLs) allow measurement probes to be connected to hand-held calculators to collect and analyze scientific data such as light, distance, voltage, temperature, and so on. This direct observation, collection, and display of data allows the student to focus more on the hypothesis, interpretation, and analysis phases of experiments. And the World-Wide Web, or graphical Internet, holds untold amounts of data and information. From geographic and census data, to current information about almost any mathematical or scientific subject, to rich sources of mathematical problems for K-12 students, the Internet will greatly expand the research capability of both teachers and students.

Strategies and teaching approaches which utilize technology have been shown to improve student attitudes, problem solving ability, ability to visualize mathematics, and overall performance. Here, possibly more than anywhere else in this *Framework*, we need to be open to new ideas and receptive to new approaches.

K-12 Development and Emphases

Many specific suggestions for appropriate uses of **calculators, computers,** and **manipulatives** are given in the following pages and, indeed, throughout this *Framework*. The point to be made here, though, is that the frequent, well-integrated use of these tools at all levels is essential. Young children find the use of concrete materials to model problem situations very natural. Indeed they find such modeling more natural than the formal work they do with number sentences and equations. Older students will realize that the adults around them use calculators and computers all the time to solve mathematical problems and will be prepared to do the same. Perhaps more challenging, though, is the task of getting the “reverse” to happen as well, so that technology is also used with *young children*, and the *older students’* learning is enhanced through the use of *concrete models*. Such opportunities do exist, however, and new approaches and tools are being created all the time.

IN SUMMARY, mathematical tools play an ever-more important role in today’s mathematics. Students who will be expected to be knowledgeable users of such tools when they leave school must see those tools as a regular and routine part of “doing mathematics” in school.

References

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Standard 5 — Tools and Technology — Grades K-2

Overview

This standard addresses the use of calculators, computers and manipulatives in the teaching and learning of mathematics. These tools of mathematics can and should play a vital role in the development of mathematical thought in students of all ages.

In the primary grades, **manipulatives** are the most natural of the three types of tools to use. Primary grade teachers have traditionally used many manipulative materials in their teaching of mathematics because they correctly perceived them to be of great value for young children. Typically, concrete materials are used to model mathematical concepts such as number or shape when those concepts are first introduced to the students.

Young children counting with lima beans, colored chips, linking cubes, smooth stones, or their fingers is a familiar sight in many New Jersey classrooms as they begin to master early counting skills and are introduced to addition and subtraction concepts. More sophisticated models should then be used, though, to begin to explore more sophisticated number concepts. Colored rods in graduated lengths give students a different sense of number than a set of discrete objects. Students should be able to see both a yellow rod **and** five colored chips as representative of the number five — the first being more of a measure model and the second a count model. Ice cream sticks and base ten blocks as well as chip trading activities help students begin to understand the very abstract concepts involved with place value and number base.

Attribute blocks, blocks with different shapes and colors, help students begin to classify and categorize objects and recognize their specific characteristics. Pattern blocks allow them to make patterns and geometric designs as they become familiar with the geometric properties of the shapes themselves. Geoboards allow students to explore the great variety of shapes that can be made and also to deal with issues of properties, attribute, and classification.

A great variety of different materials should be used to explore measurement. Paper clips, shoes, centimeter and decimeter rods, paper cutouts of handspans, and building blocks can all be used as non-standard units of length (even though some of them are really standard). Students place them down one after another to see how many paper clips long the desk is or how many handspans wide the doorway is. The transition can then be made to more standard measures and, following that, to rulers.

This list is, of course, not intended to be exhaustive. Many more suggestions for materials to use and ways to use them are given in the other sections of this *Framework.* The message in this section is a very simple one — concrete materials help children to construct mathematics that is meaningful to them.

Calculators have not been used traditionally in primary classrooms, but there are several appropriate uses for them. It is never too early for students to be introduced to the tool that most of the adults around them use whenever they deal with mathematics. In fact, many students now come to kindergarten having already played with a calculator at home or somewhere else. To ignore calculators completely at this level is to send the harmful message that the mathematics being done at school is different from the mathematics being done at home or at the grocery store.

The use of calculators at this level does not imply that students don't need to develop the arithmetic skills traditionally introduced at the primary level. They certainly do need to develop these skills. This Standard does not suggest that all traditional learning be replaced by calculator use; rather, it calls for the appropriate and effective use of calculators.

One of the most effective uses of the calculator with young children is the use of the constant feature of most calculators to count, forward or backward, or to *skip count*, forward or backward, by twos or threes or other numbers. This process allows children to anticipate what number will come next and then get confirmation of their guess when they see it appear in the display. Students can also greatly enhance their estimation ability through calculator use. *Range-finding games* ask students, for instance, to add a number to 34 that will give them an answer between 80 and 90. After the estimate is made, it is punched into the calculator to see whether or not it did the job. Calculators will prompt young students to be curious about mathematical topics that are not typically taught at their level. For example, when counting back by threes by entering 15 − 3 = = = . . . into the calculator, after the expected sequence of 12, 9, 6, 3, 0, the child will see −3, −6, −9, . . . A curious child will begin to ask questions about what those numbers are, but will also begin to develop an intuitive notion about negative numbers.

Computers are a valuable tool for primary children. As more and more computers find their way into primary classrooms, the software available for them will dramatically improve; however, there are already many good programs that can be used with kindergartners and first and second graders. *MathKeys* links on-screen manipulative materials to standard symbolic representations and to a writing tool for children to use. A number of different counting programs match objects on the screen to a standard symbolic representation of the number and the number is said aloud so that a young student can count along with the program. Many other new programs focus on money skills and help children recognize different coins and determine the values of sets of coins through simulated purchases.

Standard 5 — Tools and Technology — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Select and use calculators, software, manipulatives, and other tools based on their utility and limitations and on the problem situation.**

   - Students participate in races to complete a set of computation problems between some students who use calculators and others who use mental math. They try to determine what makes the calculator a useful tool in some circumstances (large numbers, harder operations) and not terribly useful in others (basic facts, easy numbers).
   - Students are regularly asked to make their own decisions about what is the right type of linear measuring device for a particular situation: mental estimation, colored rods, ruler, yard or meter stick, or tape measure. Different decisions are made in different circumstances: Estimation is fine when you are deciding whether you will fit through a small doorway, but accurate ruler measurement is important if you are cutting out a frame for a picture.
   - In problem solving situations, students are regularly provided with calculators, manipulatives, and other tools so that they may choose for themselves what will be useful to help solve the problem.

2. **Use physical objects and manipulatives to model problem situations, and to develop and explain mathematical concepts involving number, space, and data.**

   - Students use popsicle sticks to model multi-digit base-ten numbers and then use them to further model operations with the numbers.
   - Students use pipe cleaners and straws to make models of two-dimensional geometric shapes. They then compare, contrast, and sort all the shapes using whatever criteria they think are important, including number of corners, straight or curvy sides, number of sides, and so on.
   - Students work through the *Shapetown* lesson that is described in the First Four Standards of this *Framework.* Students in kindergarten are challenged to build towns with attribute blocks and loops based on a rule or pattern they make up.
   - Kindergarten students each use a cubic inch block to represent himself or herself in a bar graph that describes the favorite flavors of ice cream of all the students in the class. On a table in the front of the room, the teacher has placed mats that say *Vanilla, Chocolate*, and *Strawberry*. One by one, the students walk past the table, dropping their blocks on one of the piles that build up on the mats. When this concrete "bar graph" is complete, the children ask questions that can be answered with the data displayed: *What's the most favorite flavor in the class? What's the least favorite? Are there more people who like vanilla than*

chocolate?

3. **Use a variety of technologies to discover number patterns, demonstrate number sense, and visualize geometric objects and concepts.**

    - Students use the constant function on a calculator to count by ones, twos, tens, fourteens, and other numbers, both forward and backward. As they do so, they try to keep up with the calculator by saying the numbers orally as they come up in the display, and even trying to say them before they come up.
    - Students use a beginner's Logo to explore movement in two-dimensional space. They move the turtle on the computer screen forward and backward with simple commands and also turn the turtle through predetermined angles to the right and to the left with other commands. The turtle leaves a trail of where it's been on the screen so that its movements actually create a drawing of a figure. The students try to have the turtle draw a square, a different rectangle, and a triangle before progressing to harder tasks.
    - Students use a geoboard to make shapes that are composed of unit squares. One challenge they are given is to find as many shapes as possible that are made up of 10 unit squares.

4. **Use a variety of tools to measure mathematical and physical objects in the world around them.**

    - Young students develop meaning for rulers by first measuring with individual paper clips, then a paper clip chain, then taping the clip chain to a paper strip, then marking and numbering the ends of the clips on the strip, and last, removing the clip chain from the paper strip leaving just the marks and the numbers. This leaves the students with *paper clip rulers* with which they can measure the lengths of a variety of objects. The unit of measurement is, of course, a paper clip.
    - Students use a balance scale to determine the weights of a variety of classroom objects in terms of units that are other classroom objects; for example, *How many pennies does a math book weigh? How many paper clips does a pencil weigh?*
    - Students work through the *Will a Dinosaur Fit?* lesson that is described in the First Four Standards of this *Framework.* Second grade students measure the size of their classroom and other places in a variety of ways to determine whether dinosaurs they are studying would fit into them.
    - As part of the morning calendar routine, second graders check each of two thermometers — one Fahrenheit and one Celsius — and make daily recordings of the outside temperature. They record the temperatures in a chart and look for interesting patterns. They notice that, as the school year progresses and the temperatures change, whenever one of the temperatures goes up or down, so does the other.
    - Students regularly use both analog and digital stopwatches to practice timing events that are usually measured in seconds such as: the amount of time it takes a classmate to say the alphabet, how long a classmate goes without blinking, or how long the morning announcements take.

5. **Use technology to gather, analyze, and display mathematical data and information.**

- Students take a survey to determine every child's birth month and then use the *Graph Club* or *Primary Graphing and Probability Workshop* software to display the resulting data in graphs.
- Using a World Wide Web page that reports meteorological data (possibly http://www.rainorshine.com/weather/index/sites/njo/), students find the predicted high temperatures for a variety of cities in different regions around the country, write those numbers on a map of the United States, and then look for patterns and trends in different regions.
- Students use *Table Top* software to make a Venn diagram to show which of them have brothers, which have sisters and which have both (the intersection of the two sets). Students who have no siblings are shown outside the rings. Other attributes of the children are also used to make Venn diagrams.

References

Software

Graph Club. Tom Snyder Productions.

Logo. Many versions of Logo are commercially available.

Primary Graphing and Probability Workshop. Scott Foresman.

TableTop. TERC.

MathKeys. Minnesota Educational Computing Consortium (MECC).

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/
The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards.*

Standard 5 — Tools and Technology — Grades 3-4

Overview

This standard addresses the use of calculators, computers and manipulatives in the teaching and learning of mathematics. These tools of mathematics can and should play a vital role in the development of mathematical thought in students of all ages.

In grades 3 and 4, **manipulatives** have traditionally not been used as much as they have been in the primary grades. It is fairly common for teachers at this level to think that once initial notions of number and shape have been established with concrete materials in the lower grades, the materials are no longer necessary and a more symbolic approach is preferable. Research shows, however, that concrete materials and the modeling of mathematical operations and concepts is just as useful at these grade levels as it is for younger students. The content being modeled is, of course, different and so the models are different — but no less important.

Third- and fourth-graders can use square tiles to model one-digit multiplication arrays in a manner that makes the operation very meaningful for them, and later use base-ten blocks to model two-digit multiplication arrays. The added advantage to this kind of a model is the degree to which students who have used it can visualize what's happening with the factors in the problem and so can develop much better estimation and mental math skills than students who have simply learned the standard paper-and-pencil algorithms. The relationship between multi-digit multiplication and division is also clearly shown by such models.

Geometry models, both two- and three-dimensional, are an important part of learning about geometry and development of spatial sense in students of this age. Students should use geoboards to explore area and perimeter and to begin to develop procedures for finding the areas of irregular shapes. They can also use construction materials like pipe cleaners and straws to make three-dimensional geometric shapes like cubes and pyramids so that they can study them directly. Such models make it much easier to determine the number of faces or edges in a figure than two-dimensional drawings.

Third- and fourth-graders should also be in the habit of using a variety of materials to help them model problem situations in other areas of the mathematics curriculum. They might use different colored unifix cubes to represent all of the different double-decker ice cream cones that can be made with three different flavors of ice cream. They should be able to use a variety of measurement tools to measure and record the data in a science experiment. They might use coin tosses or dice throws to simulate real-world events that have a one-in-two chance or a one-in-six chance of happening.

This list is, of course, not intended to be exhaustive. Many more suggestions for materials to use and ways to use them are given in the other sections of this *Framework.* The message in this section is a very simple one — concrete materials help children construct mathematics that is meaningful to them.

There are several appropriate uses for **calculators** at these grade levels. It is never too early for students to be introduced to the tool that most of the adults around them will use whenever they deal with mathematics.

The use of calculators at this level does not imply that students don't need to develop arithmetic skills traditionally introduced at the primary level. They certainly do need to develop these skills. This Standard does not suggest that all traditional learning be replaced by calculator use; rather, it calls for the appropriate and effective use of calculators.

One of the most effective uses of the calculator with young children which can be continued in grade three is the use of the constant feature of most calculators to count, forward or backward, or to skip count. This process allows children to anticipate what number will come next and then get confirmation of their guess when they see it come up in the display. Students can greatly enhance their estimation ability through calculator use. *Range-finding games* ask students, for instance, to add a number to 342 that will give them an answer between 800 and 830. After the estimate is made, it is punched into the calculator to see whether or not it did the job.

Calculators will also prompt students to be curious about mathematical topics to which they are about to be introduced. For example, while routinely using calculators in problem solving activities, some students may notice that whenever they add, subtract, or multiply two whole numbers, they get a whole number for an answer. Sometimes that happens for division, too, but sometimes when they divide they get an answer like 3.5. *What does that mean?* These kinds of questions offer a great opportunity for some further exploration and investigation; for example, *Which problems give you answers like those? What happens when you solve those problems using pencil-and-paper?*

Computers are a valuable tool for students in third and fourth grade. As more and more computers find their way into these classrooms, the software available for them will dramatically improve; however, there are already many good programs that can be used with students of this age. *MathKeys* links on-screen manipulative materials to standard symbolic representations and to a writing tool for children. Logo can be used by students to explore computer programming and geometry concepts at the same time. *Tesselmania!* and other programs offer an opportunity to play with geometric transformations on the screen and produce striking designs. *The King's Rule* is a program that asks students to determine the rules that distinguish one set of numbers from another, fostering creative and inductive thinking. The World Wide Web can be an exciting and eye-opening tool for third-and fourth-graders as they retrieve and share information. Specifically, in these grades, they might look for state populations, meteorological data, and updates on current events.

Standard 5—Tools and Technology—Grades 3-4

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 3 and 4.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 3-4 will be such that all students:

1. **Select and use calculators, software, manipulatives, and other tools based on their utility and limitations and on the problem situation.**

   - Students participate in races between some students who use calculators and others who use mental math, each working to complete a set of computation problems involving newly learned arithmetic skills. They try to determine what makes the calculator a useful tool in some circumstances (large numbers, harder operations) and not terribly useful in others (basic facts, easy numbers).
   - Students work through the *Tiling a Floor* lesson that is described in the First Four Standards of this *Framework.* Third grade students test various shapes made of a variety of materials to determine which can be used to tessellate an area.
   - Students choose to use a computer spreadsheet on their classroom computer as a neat way to organize tables and charts, but they also use a full-function word processor when there is a good deal of text involved or when using different fonts and text formatting.
   - Students use base ten blocks rather than popsicle sticks when performing operations with large numbers because they can create models more efficiently and more quickly with them.

2. **Use physical objects and manipulatives to model problem situations, and to develop and explain mathematical concepts involving number, space, and data.**

   - Students use base ten blocks to demonstrate the operations of multiplication and division with multi-digit numbers using both repeated subtraction and partition methods.
   - Students work through the *Sharing Cookies* lesson that is described in the First Four Standards of this *Framework.* Fourth grade students use manipulatives to determine how to divide 8 cookies equally among 5 people.
   - Students use a variety of devices such as dice, coin flips, spinners, and decks of cards for generating random numbers and understand the essential equivalence of these devices.
   - Students use pipe cleaners and straws to build and study three-dimensional objects, finding it easier to discuss things like numbers of edges, faces, and vertices and the relationships among them if they have a physical model with which to work.
   - Students use geoboards to solve Farmer Brown's problem. She has 16 meters of fencing and wants to fence in the largest rectangular area possible for her dog to romp around in.
   - Students use colored rods or pattern blocks to develop early notions of fractions, using

different rods or blocks as the unit and discovering by trial-and-error the resulting fractional values of all of the other pieces.

3. **Use a variety of technologies to discover number patterns, demonstrate number sense, and visualize geometric objects and concepts.**

   - Students play the game *target practice* in the *New Jersey Calculator Handbook*. In it, one student enters a number into a calculator to be used as an operand, enters an operation (addition, subtraction, multiplication, or division) into the calculator by pressing the appropriate sign, and then specifies a "target range" for the answer. For instance, the student may enter: *82* x and specify the range as *2000-3000*. A second student must then enter a second operand into the calculator and press the equals key. If the answer is within the specified target range, the shot was a bull's eye.
   - Students play *The Biggest Product*, also from *The New Jersey Calculator Handbook*. In it, four cards are dealt face up from a shuffled deck of cards containing only the cards from ace to nine. The students who are playing then use their calculators to try to compose the multiplication problem that uses only the digits on the cards, each only once, that has the largest possible product. After several rounds, the students begin to notice a pattern in their answers and become much more efficient at finding the correct problems.
   - Students begin to use Logo to create geometric figures on the computer screen. They write routines that have the turtle's path describe a square, a rectangle, a triangle, and other standard polygons. As a challenge, they write a routine to have the turtle draw a simple house with windows and a roof.
   - Students solve the problems posed in *Logical Journey of the Zoombinis* by using logic and classification and categorization skills. In it, they create Zoombinis, little creatures that have specific characteristics that allow them to accomplish specified tasks.
   - After reading *Counting on Frank* by Rod Clement, students practice their estimation skills by using software of the same title.

4. **Use a variety of tools to measure mathematical and physical objects in the world around them.**

   - Students regularly use both analog and digital stopwatches to practice timing events that happen in short time periods such as: the amount of time it takes a classmate to recite the Pledge of Allegiance or count to 60, how long a classmate takes to run a 50 meter dash, or how long the morning announcements take. They begin to record the elapsed time in decimals that include tenths or hundredths of a second.
   - Students first estimate and then use a metric trundle wheel to measure long distances such as the distance from the cafeteria doors to the sandbox, the distance from the classroom door to the principal's office door, or the distance all the way around the school on the sidewalk.
   - Students read *Counting on Frank* by Rod Clement and repeat some of the estimates made by the boy in the book. *How many peas would it take to fill up the room? How long a line can a pen write?* They make up their own silly things to estimate, and devise ways to make the appropriate measures and estimates.

5. **Use technology to gather, analyze, and display mathematical data and information.**

- Students use the New Jersey State homepage http://www.state.nj.us on the World Wide Web to gather data about the latest reported populations for each of the municipalities in their county. They then enter the collected data into a simple spreadsheet and use its graphing function to produce a bar graph of all of the populations of the towns and cities. They highlight their own town to show where it stands in relationship to the others.
- Students use the *Graphing and Probability Workshop* or similar software to generate large amounts of random data. This software simulates a variety of probability experiments including up to 300 coin tosses, spinner spins, and dice rolls. Discussions focus on whether the simulated outcomes were as expected or were different from what was expected.
- There is always math help available at the Dr. Math World Wide Web site (dr.math@forum.swarthmore.edu). In Dr. Math's words, "Tell us what you know about your problem, and where you're stuck and think we might be able to help you. Dr. Math will reply to you via e-mail, so please be sure to send us the right address. K-12 questions usually include what people learn in the U.S. from the time they're five years old through when they're about eighteen."

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Software

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Logo. Many versions of Logo are commercially available.

MathKeys. Minnesota Educational Computing Consortium (MECC).

Tesselmania! Minnesota Educational Computing Consortium (MECC).

The King's Rule. Sunburst Communications.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/
The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 5 — Tools and Technology — Grades 5-6

Overview

This standard addresses the use of calculators, computers and manipulatives in the teaching and learning of mathematics. These tools of mathematics can and should play a vital role in the development of mathematical thought in students of all ages.

Traditionally, by grades 5 and 6, teachers are devoting relatively little time to student modeling with **manipulatives** since they have begun to concentrate on symbolic and abstract approaches to content. It is fairly common for teachers at this level to think that once initial notions of number and shape have been established with concrete materials in the lower grades, such materials are no longer necessary and a more symbolic approach is preferable. Research shows, however, that concrete materials and the modeling of mathematical operations and concepts is just as useful in these grades as it is for younger students. The content being modeled is, of course, different and so the models are different — but they are no less important.

Fifth- and sixth-graders can use a great variety of materials, including colored rods, base-ten blocks, pattern blocks, fraction strips and circles, and tangrams, to develop very rich notions of rational numbers and the operations associated with them. Initial fraction notions are well-modeled with colored rods. Students use different length rods to represent different units and then decide on the fractional or mixed number value of the other rods. Base-ten blocks, with the values 1, 0.1, 0.01, and 0.001 assigned to the different sizes, model operations with decimals just as well as the values normally associated with base-ten blocks model whole number computation. Pattern blocks and tangram pieces are comfortable and familiar tools with which to begin to explore notions of ratio and proportion.

Geometry models, both two- and three-dimensional, are an important part of learning about geometry and development of spatial sense in students of this age. Students can use geoboards to develop procedures for finding the areas of polygons or irregular shapes. They can also use construction materials like pipe cleaners and straws or cut-out cardboard faces to make complex three-dimensional geometric figures which can be studied directly. It is much easier to determine the number of faces or edges in a figure from such a model than from a two-dimensional drawing of the figure.

Fifth- and sixth-graders should also be in the habit of using a variety of materials to help them model problem situations in other areas of the mathematics curriculum. They might use two-colored counters to represent positive and negative integers in initial explorations of addition and subtraction of signed numbers. They should be able to use a variety of measurement tools to measure and record the data in a science experiment. They might use play money to concretely construct solutions to coin problems or to riddles that ask how much an individual actually profited or lost in some complex business dealing.

This list is, of course, not intended to be exhaustive. Many more suggestions for materials to use and ways to use them are given in the other sections of this *Framework*. The message in this section is a very simple one — concrete materials help children to construct mathematics that is meaningful to them.

There are many appropriate uses for **calculators** in these grade levels. In his article, *Using Calculators in* *the Middle Grades*, in *The New Jersey Calculator Handbook,* David Glatzer suggests that there are three major categories of calculator use in the middle grades:

To explore, develop, and extend concepts — for example, when the students use the square and square root keys to try to understand these functions and their relationship to each other.

As a problem solving tool — for example, to see how increasing each number in a set by 15 increases the mean of the set.

To learn and apply calculator-specific skills — for example, to learn how to use the memory function of a calculator to efficiently solve a multi-step problem.

These three categories provide a good framework for thinking about calculators at these grade levels. For another powerful example of the first category, consider using an ordinary four-function calculator to explore and begin to describe the relationships between common fractions and decimals. Entering 2/3 into the calculator by pressing *2*, then the division key, and then *3* gives a result of 0.6666667. Discussion of this result, attempts to create other similar results, and working out some of the problems by hand lead to discoveries about terminating and non-terminating decimals, repeating decimals, and fraction-decimal equivalence. Such explorations also should be used to highlight the limitations of the calculator, which does not always give the answer 1 when 1/3 is added three times.

Computers are a valuable resource for students in fifth and sixth grade, and the software tools available for them are more like adult tools than those available for younger children. The standard computer productivity tools — word processors, spreadsheets, graphing utilities, and databases — can all be used as powerful tools in problem solving situations, and students should begin to rely on them to help in finding and conveying problem solutions.

In terms of specific mathematics education software, there are many good choices. Logo, of course, can be used effectively by students at these grade levels to explore computer programming and geometry concepts at the same time. It is an ideal tool to learn about one of the critical cumulative progress indicators for Standard 14 (Discrete Mathematics) for grades 5-8: the use of iterative and recursive processes. *Oregon Trail II* is a very popular CD-ROM program that effectively integrates mathematics applications with social studies. *How the West Was One + Three x Four* also uses an old west theme to work on arithmetic operations. *Tesselmania!*, the *Teaching and Learning with Computers* series, *Elastic Lines*, and the *Geometry Workshop* all allow students to make geometric constructions on the computer screen and then transform them in a variety of ways in order to experiment with the effects of the transformations.

Graph Power, *Graphing and Probability Workshop*, *AppleWorks*, *TableTop*, *Graphers*, and *MacStat* are some of the many tools available that include database, spreadsheet, or graphing facilities written for students at this age. Many other valuable pieces of software are available.

The World Wide Web can be an exciting and eye-opening tool for fifth- and sixth-graders as they retrieve and share information. Specifically, in these grades, they might look for demographic data about geographic locations in which they are interested, summaries of the vote totals for different precincts in local elections, and home pages from other schools in this country and abroad.

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 5 and 6.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 5-6 will be such that all students:

1*. Select and use calculators, software, manipulatives, and other tools based on their utility and limitations and on the problem situation.

- In problem solving situations, students are no longer provided with instructions concerning which tool to use, but rather are expected to select the appropriate tool from the array of manipulatives, calculators, computers and other tools that are always available to them.
- Students work through the *Pizza Possibilities* lesson that is described in the First Four Standards of this *Framework.* They use a variety of manipulatives to help them visualize and solve the problem.
- Having used paper-and-pencil to develop an interesting geometric shape which tessellates the plane, the students go to the computer to use *Tesselmania!* to reproduce it and then to tile the computer screen with it. They color the printout from the program to produce a unique piece of artwork which is then posted in a class display.
- Students engage in the four activities of *Target Games: Estimation is Essential!* in *The New Jersey Calculator Handbook*. In these activities, students learn the role that estimation plays in effective calculator use and learn to identify reasonable and unreasonable answers in the calculator display.
- Students explore the rich source of problems at the Math Forum World Wide Web site at Swarthmore College (http://forum.swarthmore.edu).

2*. Use physical objects and manipulatives to model problem situations, and to develop and explain mathematical concepts involving number, space, and data.

- Using standard base ten blocks as a model, where the four blocks, as usual, represent one, ten, one hundred, and one thousand, students demonstrate their understanding of place value on an assessment by writing in their journals a verbal description (accompanied by drawings) of what a ten-thousand block and a one-hundredth block might look like.
- Students use gumdrops and toothpicks to build a variety of polyhedra. Using these models, they try to generalize a relationship among the faces, edges, and vertices that works for all solids. (There is one! It's called Euler's formula after its discoverer and is: $F + V = E + 2$.)
- Students read *How Much is a Million* by David Schwartz. The book describes how tall a stack of a million children standing on each other's shoulders would be, how long it would

* Activities are included here for Indicators 1, 2, 3, 4, and 5 which are also listed for grade 4, since the Standards specify that students demonstrate continual progress in these indicators.

take to count to a million, and so on. The students pick some object of their own and try to determine how big a space would be needed to contain a million of them. Typical objects to inquire about include blades of grass, pennies, and dollar bills.

3*. Use a variety of technologies to discover number patterns, demonstrate number sense, and visualize geometric objects and concepts.

- Students are asked to enter the number 6561 into their calculators and then to keep pressing the square root key to try to discover what it does. After more experimentation with the key, the students are asked to predict what would have to be entered into the calculator's display if they wanted to press the square root key 6 times and wind up with the number 4.

- Students work through the *Two-Toned Towers* lesson that is described in the First Four Standards of this *Framework.* Students use manipulatives to determine how many towers can be built which are 4 cubes tall and use no more than 2 colors, and then discuss the pattern that results when the length of the towers can be 4,5,6, or a larger number of cubes. They also relate their answers to the solution of the *Pizza Possibilities* lesson on the following page in the First Four Standards.

- Students use *Elastic Lines* or another version of an electronic geoboard to construct geometric figures and then transform them through rotations, reflections, and translations. Students create a figure and its transformed image on the screen and challenge each other to describe the specific transformation that created the image.

- Students play a computer golf game where they must hit a ball into a hole. The ball and the hole are both visible on the screen, but at opposite sides. Players specify an angular orientation (where 0° is straight up) and a number of units of length which will describe the path of the ball once it is struck. The object, just like in real golf, is to get the ball in the hole in as few strokes as possible. Good estimation of both angle measure and length are critical to success.

4*. Use a variety of tools to measure mathematical and physical objects in the world around them.

- Students divide into groups to make a scale model of their classroom by accurately measuring critical elements of the room, using a standard proportional relationship to convert the actual measurements to the model's measurements, and then measuring again to cut the modeling material (cardboard, balsa wood, or manila paper) to the correct size. Their model of the room should also contain models of the blackboard, the teacher's desk, some student desks, the shelves in the room, and so on. Each student group is responsible for different elements of the room.

- Students measure the volumes of several rectangular boxes by filling them with cubic inch blocks or cubic centimeter blocks. After some thought and discussion, they devise formulas to compute the volumes from direct measurement of the three appropriate dimensions.

- Students use ratios and proportions to determine the heights of objects that are too tall to easily measure directly. Measuring the heights of some known objects and the lengths of the

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*Activities are included here for Indicators 1, 2, 3, 4, and 5 which are also listed for grade 4, since the Standards specify that students demonstrate continual progress in these indicators.

shadows they cast, students determine the heights of the school building, the flagpole, and the tallest tree outside the school by measuring their shadows.

5*. Use technology to gather, analyze, and display mathematical data and information.

- Students use the New Jersey State homepage http://www.state.nj.us on the World Wide Web to gather data about the latest reported population for each county in the state and about the area of the counties. They enter the collected data into two adjacent columns in a spreadsheet and configure a third column to calculate the population density for each county (population / area). They highlight their own county in the printout of the spreadsheet to show where it stands in relationship to the others.
- Students use HyperStudio to create the reports they write about biographies of mathematicians, about how mathematics is used in real life, or about solutions to problems they've solved. The software allows them to create true multimedia presentations.
- Students measure various body parts such as height, length of forearm, length of thigh, length of hands, and arm span. They enter the data into a spreadsheet and produce various graphs as well as a statistical analysis of the class. They update their data every month and discuss the change, as it relates both to individuals and to the class.
- Students conduct a survey of the population of the entire school to determine the most popular of all of the choices for school lunches. After gathering the data, they enter it into a spreadsheet and use the program to graph it in a variety of ways — as a bar graph, a circle graph, and a pictograph. They discuss which of the graphs best illustrates their data and publish the one they choose in a report distributed to all of the students in the school.
- There is always math help available at the Dr. Math World Wide Web site (dr.math@forum.swarthmore.edu). In Dr. Math's words, "Tell us what you know about your problem, and where you're stuck and think we might be able to help you. Dr. Math will reply to you via e-mail, so please be sure to send us the right address. K-12 questions usually include what people learn in the U.S. from the time they're five years old through when they're about eighteen."

6. Use a variety of technologies to evaluate and validate problem solutions, and to investigate the properties of functions and their graphs.

- Students solve the problems posed in *Logical Journey of the Zoombinis* by using logic and classification and categorization skills. In it, they create Zoombinis, little creatures that have specific characteristics that allow them to accomplish specified tasks.
- Students use their knowledge of theoretical probability to predict the relative frequency of occurrence of each of the possible sums when rolling a pair of dice. They use simulation software like the *Graphing and Probability Workshop* to simulate the rolling of 300 pairs of dice. They examine the simulated frequencies and judge them to either be consistent or inconsistent with their predictions and reexamine their predictions if necessary.
- Students use the data they gathered earlier concerning the heights of objects and the lengths of the shadows they cast at the same time on a sunny day. They enter the data as ordered pairs (height, shadow) into a simple graphing program and notice that the resulting points all lie on a line. They use the line to predict the heights of objects whose shadows they can

measure.

- Students make a pattern using square tiles to build increasingly larger squares (a 1x1, a 2x2, a 3x3, and so on). They count the number of tiles it took to build each successive square and plot the resulting ordered pairs ((1,1), (2,4), (3,9), (4,16), . . .) on an x-y plane. The resulting parabola is a non-linear function which is easy to discuss.

7. Use computer spreadsheets and graphing programs to organize and display quantitative information and to investigate properties of functions.

- Students measure each of a variety of objects in both inches and centimeters. They enter the collected data into a spreadsheet as ordered pairs in two adjacent columns, measurements in inches followed by measurements in centimeters. They have the spreadsheet program graph the ordered pairs on an x-y plane. After they discover that all of the points lie on a line, they draw the line and use it to determine the customary measure of an object whose metric measure they know and vice versa.

- Students configure a simple spreadsheet to assist them in finding magic squares by automatically computing all of the sums. For example, they reserve a three-by-three array of cells for the magic square somewhere in the middle of the spreadsheet. In the cells that are at the end of the rows, they enter formulas that show the sums of the entries in the cells in each row, and enter similar formulas at the end of each column and diagonal. When proposed entries are placed in the magic square cells, their various sums are instantly provided in the adjacent cells, facilitating adjustment of the entries. The students then use their new tool to solve and create magic square puzzles.

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Geometry Workshop. Scott Foresman.

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Graphing and Probability Workshop. Scott Foresman.

How the West Was One + Three x Four. Sunburst Communications.

HyperStudio. Roger Wagner.

Logical Journey of the Zoombinis. Broderbund.

MacStat. Minnesota Educational Computing Consortium (MECC).

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Table Top. TERC.

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On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/
The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 5 — Tools and Technology — Grades 7-8

Overview

This standard addresses the use of calculators, computers and manipulatives in the teaching and learning of mathematics. These tools of mathematics can and should play a vital role in the development of mathematical thought in students of all ages.

Seventh- and eighth-graders can use a variety of **manipulatives** to enhance their mathematical understanding and problem solving ability. For example, new approaches to the teaching of elementary concepts of algebra incorporate concrete materials at many levels. Two-colored counters are used to represent positive and negative integers as students build a sense of operations with integers. Algebra tiles are used to represent variables and polynomials in operations involving literal expressions. Concrete approaches to equation solving are becoming more and more popular as students deal meaningfully with such mathematical constructs as equivalence, inequality, and balance.

In geometry, students can best understand issues of projection, perspective, and shadow by actually building concrete constructions out of blocks or cubes and viewing them from a variety of directions and in different ways. Slicing clay models of various three-dimensional figures convinces students of the resulting planar shape that is the cross-section. Using pipe cleaners and straws, students can build their own version of a Sierpinski tetrahedron.

Seventh- and eighth-graders should also be in the habit of using a variety of materials to help them model problem situations in other areas of the mathematics curriculum. They might use spinners or dice to simulate a variety of real-life events in a probability experiment. They should be able to use a variety of measurement tools to measure and record the data in a science experiment. They might use counters to represent rabbits as they simulate Fibonacci's famous question about rabbit populations.

This list is, of course, not intended to be exhaustive. Many more suggestions for materials to use and ways to use them are given in the other sections of the *Framework.* The message in this section is a very simple one — concrete materials help students to construct mathematics that is meaningful to them.

There are many appropriate uses for **calculators** in these grade levels as well. In his article, *Using Calculators in the Middle Grades*, in *The New Jersey Calculator Handbook*, David Glatzer suggests that there are three major categories of calculator use in the middle grades:

To explore, develop, and extend concepts — for example, when the students use the square and square root keys to try to understand these functions and their relationship to each other.

As a problem solving tool — for example, to see how increasing each number in a set by 15 increases the mean of the set.

To learn and apply calculator-specific skills — for example, to learn how to use the memory function of a calculator to efficiently solve a multi-step problem.

These three categories provide a good framework for thinking about calculators in the seventh and eighth grade. For another powerful example of the first category, consider the question of compounding interest.

When asked how much a bank account might accumulate after 10 years with an initial balance of $1000 and a simple annual interest rate of 6 percent, most students would first calculate the interest for the first year, add it to the initial balance to get the new balance, multiply that by 0.06 to get the interest for the second year, add that to the previous year's balance, and so forth. After discussion of that iteration, most seventh- and eighth-graders are able to understand that each year's balance is the product of the previous year's balance times *1.06*, so to find the balance after three years, one could simply use the formula: *$1000* x *1.06* x *1.06* x *1.06*. After still more discussion, most students will transform this into the standard formula, which is easy to apply with a calculator: *$1000* x 1.06^n. These concepts develop nicely in a classroom where all of the students have calculators and can do the computations easily and quickly. In a traditional classroom without calculators, the progression takes much longer and the resulting formula is much less believable to students.

Students at this level should also have some experience with graphing calculators. Although these tools will be most useful in the high school curriculum, middle school students should be exploring graphs of linear functions and other simple graphs and should be making use of the statistical capabilities of most graphing calculators. They should also be exploring the use of Calculator Based Laboratories (CBL) which enables them to gather data and display the data graphically in the viewing window.

Computers are also an essential resource for students in seventh and eighth grade, and the software tools available for them are more like adult tools than those available for younger children. The standard computer productivity tools — word processors, spreadsheets, graphing utilities, and databases — can all be used as powerful tools in problem solving situations, and students should begin to rely on them to help in finding and conveying solutions to problems.

In terms of specific mathematics education software, there are also many good choices. Logo, of course, can be used effectively by students at these levels to explore computer programming and geometry concepts at the same time. It is an ideal tool to learn about one of the critical cumulative progress indicators for Standard 14 (Discrete Mathematics) for grades 5-8: the use of iterative and recursive processes. *Oregon Trail II* is a very popular CD-ROM program that effectively integrates mathematics applications with social studies. *How the West Was One + Three x Four* also uses an old west theme to work on arithmetic operations.

A variety of computer golf games allow students to play a competitive game while sharpening their estimation ability with angle and length measure. The *Geometric Supposer and Pre-Supposer* series is one of the most popular geometry construction tools for students of this age. With it, students construct geometric figures on the screen, measure them, transform them, and identify a variety of geometric properties of their creations. Discovery-oriented lessons using these types of software are easy to create and very engaging and useful for students.

Graph Power, the *Graphing and Probability Workshop, AppleWorks*, *TableTop, Graphers*, and *MacStat* are some of the many tools available that include database, spreadsheet, or graphing facilities written for students at this age. Many other valuable pieces of software are available.

The World Wide Web can be an exciting and eye-opening tool for seventh- and eighth-graders as they retrieve and share information. Specifically, in these grades, they might look for good math problems from the Web bulletin boards, biographical data about famous mathematicians, and census data for local towns.

Standard 5 — Tools and Technology — Grades 7-8

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 7 and 8.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 7-8 will be such that all students:

1[*]. Select and use calculators, software, manipulatives, and other tools based on their utility and limitations and on the problem situation.

- In problem solving situations, students are no longer provided with instructions concerning which tool to use, but rather are expected to select the appropriate tool from the array of manipulatives, calculators, computers and other tools that are always available to them.
- Students continue with activities used in previous grades and use problems like those of *Target Games: Estimation is Essential!* in *The New Jersey Calculator Handbook,* in which students learn to identify reasonable and unreasonable answers in the calculator display.
- Students use a variety of materials to demonstrate their understandings of basic mathematical properties and relationships. For instance, they are able to use geoboards, dot paper, and *Geometer's Sketchpad* to demonstrate the Pythagorean Theorem.
- Students work through the *Rod Dogs* lesson that is described in The First Four Standards of this *Framework.* They use Cuisenaire rods to model the increase of the dimensions of an object by various scale factors, but when they realize that there are not enough rods to simulate the situation, they find other models which can be used.

2[*]. Use physical objects and manipulatives to model problem situations, and to develop and explain mathematical concepts involving number, space, and data.

- Students use two-colored counters to model signed numbers and integer operations. On the red side, a counter represents +1, on the white side, –1. As sets of counters are combined or separated to model the operations, students look for patterns in the answers so that they can write rules for completing the operations without counters.
- Students use the same counters and also red and white cubes, representing $+x$ and $-x$, to model and solve equations. By setting up counters and cubes to represent the initial equation and then removing equal sets from both sides, students model the essential elements of solving linear equations and develop the appropriate language with which to discuss those elements. Conversion to symbolic processes comes soon after mastery is achieved with the concrete objects.
- Students make three-inch cubes of clay and then experiment to see in how many different ways they can slice the cube with a plane to produce different cross-sections. Drawings of

[*] Activities are included here for Indicators 1, 2, 3, 4, and 5 which are also listed for grade 4, since the Standards specify that students demonstrate continual progress in these indicators.

the cross-sections and a description of the cuts that created them are displayed on a poster in the classroom.

3[*]. Use a variety of technologies to discover number patterns, demonstrate number sense, and visualize geometric objects and concepts.

- Students work on this seemingly simple problem from the New Jersey Department of Education's *Mathematics Instructional Guide*: *A copy machine makes 40 copies per minute. How long will it take to make 20,000 copies? A) 5 hours B) 8 hours 20 minutes C) 8 hours 33 minutes D) 10 hours.* Students immediately decide to use their calculators to solve the problem, but then have an interesting discussion regarding the calculator display of the answer. When 20,000 is divided by 40, the display shows 8.33333. *Which of the answer choices is that? Why?*
- Students use the *Geometer's Sketchpad* to create a triangle on the computer screen and simultaneously place on the screen the measures of each of the angles as well as the sum of the three angles. They notice that the sum of the angles is 180 degrees. They then click and drag one of the vertices around the screen to make a whole variety of other triangles. They notice that even though the measure of each of the angles changes in this process, the sum of 180 degrees never changes, thus intuitively demonstrating the triangle sum theorem.
- Students use videotape of a person walking to model integer multiplication. The videotape shows a person walking forward with a sign that says "forward" and then walking backward with a sign that says "backward." When run forward, the video shows the "forward" walker walking forward ($+ \cdot + = +$). When run backward, the video shows the "forward" walker walking backward ($- \cdot + = -$). The other two possibilities also work out correctly to show all of the forms of integer multiplication.

4[*]. Use a variety of tools to measure mathematical and physical objects in the world around them.

- Groups of students build toothpick bridges in a competition to see whose bridge can hold the most weight in the center of the span. Each group has the same materials with which to work and the bridges must all span the same distance. In the process of building the bridges, the students conduct a good deal of research into bridge designs and about factors that contribute to structural strength.
- Students work through the *Sketching Similarities* lesson that is described in the First Four Standard of this *Framework.* They use the *Geometer's Sketchpad* to measure the length of sides and the angles of similar figures to discover the geometric relationships between corresponding parts.
- Students explore the relationship between the height of a ramp and the length of time it takes a matchbox car to roll down it. The teacher provides stopwatches, long wooden boards, and meter sticks. The students use a spreadsheet program to enter their data relating height and time for several different heights, and use the spreadsheet's integrated graphing program to plot the ordered pairs. They then look for the relationship between the height and the time.
- Students are challenged to answer the question: *In how many ways can you measure a*

[*] Activities are included here for Indicators 1, 2, 3, 4, and 5 which are also listed for grade 4, since the Standards specify that students demonstrate continual progress in these indicators.

After the obvious spatial characteristics are named (volume, diameter, circumference, and so on), students get more creative and suggest bounceability, density, fraction of its

when dropped from one meter, and so on. When the list is complete, different groups of students select several of the possible characteristics and develop ways to measure them.

5*.

- rate of cooling of a cup of boiling water. The data is displayed in chart form and in a graph by the calculator after the experiment is performed.

  Students use *HyperStudio*
  mathematicians, about how mathematics is used in real life, or about solutions to problems they've solved. The software allows them to create true multimedia presentations.

  Students explore the great wealth of mathematical information available at the University of St. Andrews' History of Mathematics World Wide Web site (http://www.groups.dcs.st-

- Students gather data from their fellow students regarding the number of people in their

  histogram, showing the number of students with each size household, from the data on the calculator.

  Students decide to resolve the debate that two of them were having about which of their favorite baseball players was the better hitter. They find a great deal of numerical data on

  singles, doubles, triples, homeruns, and walks each batter had accumulated in his career. The class decides what weight to attribute to each type of hit and then computes a weighted

- There is always math help available at the Dr. Math World Wide Web site

  your problem, and where you're stuck and think we might be able to help you. Dr. Math will reply to you via e-mail, so please be sure to send us the right address. K-12 questions

  when they're about eighteen."

6.
properties of functions and their graphs.

- *Geometer's Sketchpad* to work out a solution to this problem from the New
  Mathematics Instructional Guide:
  sides of a square are increased by 20% and the other two sides are decreased by 10%. What is the percent of change in the area of the original square to the area of the newly
  In their solution
  attempts, they construct a square on the screen with known sides, and then a rectangle with

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uded here for Indicators 1, 2, 3, 4, and 5 which are also listed for grade 4, since the Standards specify that students demonstrate continual progress in these indicators.

the sides indicated in the parameters of the problem. The program calculates the areas of the two figures and the students are close to a solution.

- Students use their calculators to solve this problem from the New Jersey Department of Education's *Mathematics Instructional Guide*:

  A set of test scores in Mrs. Ditkof's class of 20 students is shown below.

  62 77 82 88 73 64 82 85 90 75
  74 81 85 89 96 69 74 98 91 85

  Determine the mean, median, mode, and range for the data.

  Suppose each student completes an extra-credit assignment worth 5 points, which is then added to his/her score. What is the mean of the set of scores now if each student received the extra five points? Explain how you calculated your answer.

- Students play *Green Globs and Graphing Equations*, a computer game in which they score points for writing the equations of lines that will pass through several green globs splattered on the x-y plane.

7. Use computer spreadsheets and graphing programs to organize and display quantitative information and to investigate properties of functions.

- Students use a simple spreadsheet/graphing program to solve this problem from the New Jersey Department of Education's *Mathematics Instructional Guide*:

  VOTING RESULTS

Class Colors	Number of Votes
red and white	10
green and gold	12
blue and orange	5
black and yellow	9

  Rather than using the tools the problem suggests (protractor, compass, and straight edge), the students enter the data into a spreadsheet and construct a circle graph from the spreadsheet.

- Students measure the temperatures of a variety of differently heated and cooled liquids in both Fahrenheit and Celsius. They then enter the collected data into a spreadsheet as ordered pairs in two adjacent columns, measurements in Fahrenheit followed by measurements in Celsius. They have the spreadsheet program graph the pairs on an x-y plane. After they discover that all of the points lie on a line, they draw the line and use it to determine the Fahrenheit temperature for a given Celsius temperature and vice versa.

- Students configure a spreadsheet to act as an order-processing form for a stationery store (or some other retail operation). They decide on the five items they'd like to sell, enter the prices they'll charge, and then program all of the surrounding cells to compute the prices for the quantities of items ordered, add the tax, and compute the final charge.

Items	Price	Quantity Ordered	Cost
Pencils	.05		
Pens	.29		
Paper Pads	.59		
Tape	.49		
Scissors	1.39		
		Tax:	
		Total:	

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Graphers. Sunburst Communications.

Graphing and Probability Workshop. Scott Foresman.

Green Globs and Graphing Equations. Sunburst Communications.

How the West Was One + Three x Four. Sunburst Communications.

HyperStudio. Roger Wagner.

Logo. Many versions of Logo are commercially available.

MacStat. Minnesota Educational Computing Consortium (MECC).

Oregon Trail II. Minnesota Educational Computing Consortium (MECC).

TableTop. TERC.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 5 — Tools and Technology — Grades 9-12

Overview

This standard addresses the use of calculators, computers and manipulatives in the teaching and learning of mathematics. These tools of mathematics can and should play a vital role in the development of mathematical thought in students of all ages.

High school students can use a variety of **manipulatives** to enhance their mathematical understanding and problem solving ability. For example, new approaches to the teaching of concepts of algebra incorporate concrete materials at many levels. Two-colored counters are used to represent positive and negative integers as students build a sense of operations with integers. Algebra tiles are used to represent variables and polynomials in operations involving literal expressions. Concrete approaches to equation solving are becoming more and more popular as students deal meaningfully with such mathematical constructs as equivalence, inequality, and balance.

In geometry, students can create solids of revolution by cutting plane figures out of cardboard, attaching a rubber band along an axis of rotation, winding it up, and then letting it unwind by itself, creating a vision of the solid as it does so. Students can use Miras to do reflective geometry — to find the center of a circle or a perpendicular bisector of a line segment. They might build models of a pyramid with a square base and a cube with the same size base to use in an investigation of the relationship of their volumes. Using pipe cleaners and straws, students can build their own version of a Sierpinski tetrahedron.

High school students should also be in the habit of using a variety of materials to help them model problem situations in other areas of the mathematics curriculum. They might use spinners or dice to simulate a variety of real-life events in a probability experiment. Suppose, for instance, they had statistics about the frequency of occurrence of a particular genetic trait in fruit flies and were interested in the probability that they would see it in a given population. A simulation using dice or a spinner might be a useful approach to the problem. They should be able to use a variety of measurement tools to measure and record the data in a science experiment. They might use counters to represent rabbits as they simulate Fibonacci's famous question about rabbit populations.

This list is, of course, not intended to be exhaustive. Many more suggestions for materials to use and ways to use them are given in the other sections of this *Framework.* The message in this section is a very simple one — concrete materials help students to construct mathematics that is meaningful to them.

There are many appropriate uses for **calculators** in these grade levels as well. In his article, "Technology and Mathematics Education: Trojan Horse or White Knight?" in *The New Jersey Calculator Handbook,* Ken Wolff suggests that the availability of calculators, especially graphing calculators, has presented a unique opportunity for secondary mathematics educators. He asserts: "Tedium can be replaced with excitement and wonder. Memorization and mimicry can be replaced with opportunities to explore and discover." Wolff offers some challenging problems for students to try with their calculators to illustrate the scope of what is now possible in secondary classrooms:

> *What happens when we continually square a number close to the value 1? Try continually taking the square root of a number. Does it matter what number you start with?*

happens? Can you explain why it happens? Replace the sine operator with the tangent and repeat the experiment.

He suggests that these are but a few of the problems that students will gladly try if they have a calculator, but would be very reluctant to do without one. Many secondary teachers have had similar experiences with

and how we teach it will dramatically change. Nowhere more than in these classrooms will the educational impact of this technology be felt. A sample unit on finding regression lines using graphing calculators can be *Framework.*

High school students should be using Calculator Based Laboratories (CBL) in conjunction with their

these activities should be coordinated with activities in their science classrooms.

Computers
them are very much like adult tools. The standard computer productivity tools — word processors, spreadsheets, graphing utilities, and databases — can all be used as powerful tools in problem solving

In terms of specific mathematics education software, there are also many good choices. The *Geometric* series, *Geometer's Sketchpad* *Cabri Geometry* are all popular geometry

measure them, transform them, and identify a variety of geometric properties of their creations. Discovery-oriented lessons using these types of software are easy to create and very engaging and useful for students.

Derive, , and *Mathematica*
and equations, solve a variety of equations, do two- and three-dimensional plotting, and much more. The programs offer significantly more power than the graphing calculators, but are also more expensive. They

There is also a good variety of algebra learning programs. *The Function Supposer* *Green Globs and Graphing Equations* *The Algebra Sketchbook* are all popular pieces of software that deal with functions

The World Wide Web can be an exciting and eye-opening tool for ninth- through twelfth-graders as they retrieve and share information. Specifically, in these grades, they might look for information about colleges

gathering and contributing local data.

Standard 5 — Tools and Technology — Grades 9-12

Indicators and Activities

The cumulative progress indicators for grade 12 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 9, 10, 11 and 12.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 9-12 will be such that all students:

1*. Select and use calculators, software, manipulatives, and other tools based on their utility and limitations and on the problem situation.

- Students have a variety of tools available to them in the well-equipped mathematics classroom: a bank of computers loaded with algebraic symbol manipulation and function-plotting programs, spreadsheet and graphing programs, and geometry construction programs; a set of graphing calculators for relevant explorations and computations; and manipulative materials related to the content studied. The students easily move from one type of tool to another, understanding both their strengths and limitations.
- Students work through the *Making Rectangles* lesson that is described in The First Four Standards of this *Framework.* They use algebra tiles organized in rectangular form to help them develop procedures for re-writing binomial expressions as multiplication problems (factoring).
- Students can use *algebra tiles*, *Hands-On Equations* materials, and a variety of equation-manipulating software to simplify and solve equations. They understand and can demonstrate the relationship between various manipulations of tiles or pawns and the corresponding symbolic actions in the software solution procedure.
- Students use both graphing calculator techniques and paper-and-pencil techniques for solving systems of equations. Depending on the complexity of the system and on the degree of accuracy needed in the answer, they may try to locate the intersection of two graphs by tracing and zooming on the calculator screen, by calculating the solution with matrices, or by using a simple addition or substitution paper-and-pencil method.

2*. Use physical objects and manipulatives to model problem situations, and to develop and explain mathematical concepts involving number, space, and data.

- Students use a process described in *Algebra in a Technological World* to construct cones from a circular piece of paper by cutting a wedge-shaped sector from it and then taping together the edges. They then try to find the cone constructed in this manner that has the largest volume. A similar activity is described in The *Ice Cones* lesson in The First Four Standards of this *Framework.*

* Activities are included here for Indicators 1, 2, 3, 5, and 7, which are also listed for grade 8, since the Standards specify that students demonstrate continued programs in their indicators.

- Students use molds to make cones of clay and then experiment to see in how many different ways they can slice the cone with a plane to produce different cross-sections. Drawings of the cross sections and a description of the cuts that created them are displayed on a poster in the classroom.
- In one of the units of the *Interactive Mathematics Program*, students read *The Pit and the Pendulum* by Edgar Allan Poe and then work in groups to investigate the properties and behavior of pendulums. The ultimate goal, after a good deal of measurement and statistical manipulations of their data, is to determine how much time the prisoner in the story has to escape from the 30-foot, razor-sharp, descending pendulum.

3*. Use a variety of technologies to discover number patterns, demonstrate number sense, and visualize geometric objects and concepts.

- Students play *Green Globs and Graphing Equations*, a computer game in which they score points for writing the equations of functions that will pass through several green globs splattered on the x-y plane. As they gain experience with the game, their ability to hit the targets with more and more creative functions improves.
- Students use a set of spherical materials like the *Lenart Sphere* to study a non-Euclidean geometry. With these materials, students make geometric constructions on the surface of a sphere to realize that, in some geometries, a triangle *can* have three right angles, and to find the spherical equivalent of the line that is the shortest distance between two points.
- Students use calculators to investigate interesting number patterns. For example, they try to determine why this old trick always works: *Enter any three-digit number into the calculator. Without clearing the display, enter the same three digit number again so that you have a six-digit number. Divide the number by 7. Then divide the result by 11. Then divide that result by 13. What is in the display?*

5*. Use technology to gather, analyze, and display mathematical data and information.

- Students use a simulation program to check their predictions regarding the answer to this problem from the New Jersey Department of Education's *Mathematics Instructional Guide*: *Two standard dice are rolled. What is the probability that the sum of the two numbers rolled will be less than 5? A) 1/3 B) 1/6 C) 1/9 D) 1/12*. After determining the probability theoretically, they use a simulation program for 1000 rolls of two dice and check the outcome data to see if their predicted probability was in the right ballpark.
- Students use HyperStudio to create the reports they write about biographies of mathematicians, about how mathematics is used in real life, or about solutions to problems they've solved. The software allows them to create true multimedia presentations.
- Students explore the great wealth of mathematical information available at the University of St. Andrews' History of Mathematics World Wide Web site (http://www.groups.dcs.st-and.ac.uk/~history/).
- Students use *Algebra Animator* software to simulate and manipulate the motion of a variety

* Activities are included here for Indicators 1, 2, 3, 5, and 7, which are also listed for grade 8, since the Standards specify that students demonstrate continued progress in these indicators.

of objects such as cars, projectiles, and even planets. They gather data about the motion and directly visualize both the functions that describe the motion and their graphs.

- Working in small groups, students use a distance probe connected to a graphing calculator to collect data about the rate of approach of a classmate walking toward the calculator. After the walk is finished, the calculator plots the student's position relative to the calculator as a function of time. The group then presents the finished graph to the rest of the class and challenges them to describe the walk that was taken: *What rate of progress was made? Was it steady progress? Where did the student stop? Was there ever any backward walking?*
- Students explore the rich links suggested on the Cornell University Math and Science Gateway World Wide Web Site (http://www.tc.cornell.edu/Edu/MathSciGateway).
- There is always math help available at the Dr. Math World Wide Web site (dr.math@forum.swarthmore.edu). In Dr. Math's words, "Tell us what you know about your problem, and where you're stuck and think we might be able to help you. Dr. Math will reply to you via e-mail, so please be sure to send us the right address. K-12 questions usually include what people learn in the U.S. from the time they're five years old through when they're about eighteen."

7[*]. Use computer spreadsheets and graphing programs to organize and display quantitative information and to investigate properties of functions.

- Students work through the *Building Parabolas* lesson that is described in The First Four Standards of this *Framework.* They use both the *Green Globs* software and their graphing calculators to investigate how the various coefficients affect the graph of parabolas.
- Students use calculators, a spreadsheet, and an integrated plotter to work on this problem from *Algebra in a Technological World*:

  A new professional team is in the process of determining the optimal price for a special ticket package for its first season. A survey of potential fans reveals how much they are willing to pay for a four-game package. The data from the survey are displayed below.

Price of the Four-Game Package	Number of Packages That Could Be Sold at That Price
$96.25	5,000
90.00	10,000
81.25	15,000
56.25	25,000
50.00	27,016
40.00	30,000
21.25	35,000

On the basis of the foregoing data, find a relationship that describes the price of a package

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as a function of the number sold (in thousands). Then determine the selling price which will maximize the revenue, and its number of packages likely to be sold at this price.

- Students investigate the growth of the world's population by researching estimates of the level of population at various times in history and plotting the corresponding ordered pairs in a piece of software called *Data Models*. They then use the software tool to find a line or curve of best fit and use the resulting graph to predict the population in the year 2100. As a last step, they find the predictions made by several social scientists and compare them to their own.
- Students work on a lesson from *The New Jersey Calculator Handbook* which uses graphing calculators to focus on the linear functional relationship between circumference and diameter. The students measure everyday circular objects to collect a sample of diameters and circumferences. They then enter their data into a calculator which plots a scattergram for them and finds a line of best fit. The slope of the line is, of course, an approximation of π.
- Students use the *Geometry Inventor* for constructions which illustrate a proof of the Pythagorean Theorem. With the construction tool, they create a right triangle in the center of the computer screen, and a square on each of the legs. They then make a table of the areas of the three squares. As they manipulate the triangle to adjust the relationship among the lengths of the legs, they notice that the basic additive relationship of the areas of the three squares remains the same.
- Students use *The Geometer's Sketchpad* to create an initial polygon and then apply a series of complex transformations to it resulting in a whole sequence of transformed polygons spread out across the screen. The results are often striking colorful images that the students can preserve as evidence of the connections between geometry and modern artistic design.

8. Use calculators and computers effectively and efficiently in applying mathematical concepts and principles to various types of problems.

- Students quickly determine the appropriate window for finding the intersection of two functions by playing with the zoom and range functions on a graphing calculator.
- Students solve a variety of on-line trigonometry problems posted on the *Trigonometry Explorer* World Wide Web site (http://www.cogtech.com/EXPLORE).
- Having just conducted a science experiment where they collected data about the rates of cooling of a liquid in three different containers, the students quickly and efficiently enter the data into a computer spreadsheet and generate broken-line graphs to represent the three different settings.
- Students solve the following problem by writing a function that describes the volume of the box, plotting the function on a graphing calculator, and searching visually for the peak of the graph. *An open-topped box is made from a six-inch square piece of paper by cutting a square out of each corner, folding up the sides and taping them together. What size square should be cut out of the corners to maximize the volume of the box that is formed?*

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Lenart Sphere. Key Curriculum Press.

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Software

Algebra Animator. Logal.

The Algebra Sketchbook. Sunburst Communications.

Cabri Geometry. IBM.

Data Models. Sunburst Communications.

Derive. Soft Warehouse.

The Function Supposer. Sunburst Communications.

Geometer's Sketchpad. Key Curriculum Press.

Geometric Pre-Supposer. Sunburst Communications.

Geometric Supposer. Sunburst Communications.

Geometry Inventor. Logal.

Green Globs and Graphing Equations. Sunburst Communications.

HyperStudio. Roger Wagner.

Maple. Brooks/Cole Publishing Co.

Mathematica. Wolfran Research.

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