Abstracts:

Plenary Lectures:

Amir Ali Ahmadi
Title: On local minima in polynomial optimization

Amitabha Basu
Title: Complexity of branch-and-bound and cutting plane algorithms

Abstract: We present some results on the theoretical complexity of branch-and-bound (BB) and cutting plane (CP) algorithms. In the first part of the talk, we study the relative efficiency of BB and CP, when both are based on variable disjunctions. In the second part of the talk, we will discuss the conjecture that the split closure has polynomial complexity in fixed dimension, which has remained open for a while now even in 2 dimensions. We settle it affirmatively in two dimensions, and complement it with a polynomial time pure cutting plane algorithm for 2 dimensional IPs based on split cuts.

Dan Bienstock
Title: Solving nonconvex problems using disjunctions

In this talk we present a number of results and techniques used to solve nonconvex optimization problems, based on ideas originating from disjunctive programming. The topics we consider range from training of neural networks, to polynomial optimization problems. The talk will focus on theory and if time allows we will also discuss numerical results.

Santanu Dey
Title: Convexification of substructures in quadratically constrained quadratic program

Abstract:
An important approach to solving non-convex quadratically constrained quadratic program (QCQP) to global optimality is to use convex relaxations and branch-and-bound algorithms. In our first result, we show that the exact convex hull of the solutions of a general quadratic equation intersected with any polytope is second-order cone representable. The proof is constructive and relies on the discovery of an interesting property of quadratic functions, which may be of independent interest: A set defined by a single quadratic equation is either (1) the boundary of a convex set, or (2) the boundary of union of two convex sets or (3) it has the property that through every point on the surface, there exists a straight line that is entirely contained in the surface. We next study sets defined for matrix variables that satisfy rank-1 constraint together with
different choices of linear side constraints. We identify different conditions on the linear side constraints, under which the convex hull of the rank-1 set is polyhedral or second-order cone representable. Finally, we present results from comprehensive set of computational experiments and show that our convexification results together with discretization significantly help in improving dual bounds for the generalized pooling problem. This is joint work with Asteroide Santana and Burak Kocuk.

Jean-Bernard Lasserre  
Title:  

Leo Liberti  
Title: Random projections for quadratic programming  

Random projections map a set of points in a high dimensional space to a lower dimensional one while approximately preserving all pairwise Euclidean distances. Although random projections are usually applied to numerical data, we show they can be successfully applied to quadratic programming formulations over a set of linear inequality constraints. Instead of solving the higher-dimensional original problem, we solve the projected problem more efficiently. We also show how to retrieve a feasible solution of the original problem from the lower-dimensional solution of the projected problem. We then show that the retrieved solution can be used to bound the optimal objective function value of the original problem from below and above, and prove that the lower and upper bounds are not too far apart. We then discuss a set of computational results on randomly generated instances, as well as a variant of Markowitz’ portfolio problem. This is joint work with C. D’Ambrosio, P.-L. Poirion, K. Vu.

Nick Sahinidis (M/T)  
Title: Spectral relaxations and branching strategies for global optimization of mixed-integer quadratic programs  

Abstract: We consider the global optimization of nonconvex quadratic programs and mixed-integer quadratic programs. We present a family of convex quadratic relaxations which are derived by convexifying nonconvex quadratic functions through perturbations of the quadratic matrix. We investigate the theoretical properties of these quadratic relaxations and show that they are equivalent to some particular semidefinite programs. We also introduce novel branching variable selection strategies which can be used in conjunction with the quadratic relaxations investigated in this paper. We integrate the proposed relaxation and branching techniques into the global optimization solver BARON, and test our implementation by conducting numerical experiments on a large collection of problems. Results demonstrate that the proposed implementation leads to very significant reductions in BARON’s computational times to solve the test problems. This is joint work with Carlos Nohra and Arvind Raghunathan.
Juan-Pablo Vielma  
Title: Modeling Power of Mixed Integer Convex Optimization Problems and Their Effective Solution with Julia and JuMP

More than 50 years of development have made mixed integer linear programming (MILP) an extremely successful tool. MILP’s modeling flexibility allows it describe a wide range of business, engineering and scientific problems, and, while MILP is NP-hard, many of these problems are routinely solved in practice thanks to state-of-the-art solvers that nearly double their machine-independent speeds every year. Inspired by this success, the last decade has seen a surge of activity on the solution and application of mixed integer convex programming (MICP), which extends MILP’s versatility by allowing the use of convex constraints in addition to linear inequalities.

In this talk we cover various recent developments concerning theory, algorithms and computation for MICP. Solvers for MICP can be significantly more effective than those for more general non-convex optimization, so one of the questions we cover in this talk is what classes of non-convex constraints can be modeled through MICP. We also cover various topics concerning the modeling and computational solution of MICP problems using the Julia programming language and the JuMP modeling language for optimization. In particular, we show how mixed integer optimal control problems where the variables are polynomials can be easily modeled and solved by seamlessly combining several Julia packages and JuMP extensions with the Julia-written MICP solver Pajarito.jl. Finally, we introduce Hypatia.jl: a Julia-based interior point solver for general non-symmetric conic programming.

Robert Weismantel (M-W)  
Title: Integer optimization from the perspective of determinants

Abstract: For an integer optimization problem (IP), one important data parameter is the maximum absolute value among all square submatrices of the constraint matrix. We present recent developments about this topic. In particular, we show that almost all problems (IP) with constant data parameter can be solved in polynomial time.

Contributed Talks

Akshay Gupta  
Title: Mixed-integer conic optimization: strong subadditive duals and extended formulations

A mixed-integer conic program (MICP) optimizes a linear function over the set of mixed-integer points in the nonnegative orthant satisfying the conic inequality constraints $\text{preceq}_KB$, where $A: \mathbb{R}^n \to \mathbb{E}$ is a linear map, $\mathbb{E}$ is a Euclidean space (either $\mathbb{R}^m$ or $\mathbb{R}^{m\times m}$), and $K$ is a nonempty closed convex pointed full-dimensional cone. Commonly occurring special cases are MILP, MISOCP, and MISDP. Strong duals for MILPs were established in the 1970’s and these duals are infinite-dimensional optimization problems in the space of...
subadditive functionals. More recently, Moran, Dey, and Vielma (2012) generalized these results to MICP and established strong duality under some technical conditions. This dual imposes subadditivity over the domain of the functionals, which is taken to be the entire space $E$. Our first main result is to show that strong duality persists even if the domain of the functional is any set that is closed under sum decomposition over the polyhedral cone formed by the generators of $A$. This reduces the size of the subadditive dual and in particular, our dual is an LP when the primal is pure integer, which is a generalization of classical results on linear IPs. Our second result is to establish a new strong dual for MICP that has subadditivity constraints and some other linear and conic constraints. Thus, it dispenses with the directional-derivative constraints that the Moran et al. dual has for continuous variables and which are computationally intractable. Our third result is to use our strong duals to construct extended formulations for the integer hull of MICP. This generalizes the extension for linear IPs that was derived by Lasserre (2004,2005) using connections with integer Farkas lemma and subadditive duality. This is based on joint work with Temitayo Ajayi and Andrew Schaefer (Rice University) and Amin Khademi (Clemson University).

**Aleksandr Kazachkov**
Title: Sparse Cutting Planes For Quadratically-Constrained Quadratic Programs

Quadratically-constrained quadratic programs (QCQPs) are an active and challenging research topic in optimization. Their remarkable expressiveness has made these problems a cornerstone in the development of theoretical and practical improvements in non-convex optimization. While modern computational methods, especially those associated to semidefinite programming (SDP), are able to provide strong bounds, they typically rely on computationally-expensive computations hindering their applicability in medium-to-large-scale problems. In this work, we develop a computationally-efficient method that emulates the SDP-based approximations of nonconvex QCQPs via a cutting plane algorithm. These cuts are required to be sparse, in order to ensure attractive numerical properties, and efficiently computable. We present a novel connection between such sparse cut generation and the sparse principal component analysis (PCA) problem in statistics, which allows us to achieve these two goals. We show extensive computational results advocating for the use of our approach. Based on joint work with Santanu S. Dey, Andrea Lodi, and Gonzalo Muñoz.

**Giacomo Nannicini**
Title: Some ideas for the simplex method on a quantum computer