The RUTGERS YOUNG SCHOLARS PROGRAM in DISCRETE MATHEMATICS

July 6 - July 31, 2009

"Our Nineteenth Summer"

"We're looking for students who are interested in math. Is that you?"

The Rutgers Young Scholars Program in Discrete Mathematics is sponsored by the Rutgers University Center for Mathematics, Science, and Computer Education, which conducts the program, and the Center for Discrete Mathematics and Theoretical Computer Science [DIMACS] -- a consortium of Rutgers and Princeton Universities, AT&T Laboratories, Bell Labs, Bellcore, and NEC Research Institute.
Do you …

... like to work on challenging and puzzling problems
... like to look at a problem in many different ways
... get involved, make conjectures, and take risks
... enjoy the critical exchange of ideas with other students
... look forward to living in the math world for a month
... enjoy and feel compelled to do mathematics
... thrive on complexity and problems which have more than one answer
... like to reason out problems and play with concepts
... learn new material quickly and apply this learning to new situations
... concentrate on something you like for a long period of time

Then this program is for you. Here is your opportunity to …

... experiment with some interesting mathematical problems
... spend four weeks on a college campus doing mathematics with people like yourself
... meet a variety of professional mathematicians and learn about what they do

Come to the Rutgers Young Scholars Program …

... come join 30 high school students who share your fascination with mathematics.

... attend a variety of mathematical programs and activities … including sessions on discrete mathematics, discussions, hands-on computer activities, and field trips.

... learn about careers in the mathematical sciences; these programs will involve industrial as well as academic mathematicians.

The instructional staff includes well-known mathematicians. Teaching assistants who are graduate and undergraduate Rutgers students majoring in mathematics or computer science provide additional academic support. You will spend a lot of time with this staff; we hope that as a result you will understand what mathematicians do and why.

The focus of the program will be discrete mathematics, although the program will not be conducted like a course. Success will be measured in terms of your enthusiasm for the subject, not by the amount of material covered.

Your daily schedule will be full and will include various recreational and social activities.

Remember, this will be an intensive program. We will expect you to participate actively in all of the activities. It will be fun and exciting, but it will require your concentrated effort. Be sure that you want to make this commitment!
COMPLETING APPLICATION MATERIALS

A completed application includes all of the following:

1. TYPED OR PRINTED (in black ink) responses to the questions on the Student Application Form;

2. a completed parent signature form;

3. a completed financial assistance form if applying for financial assistance;

4. a letter of recommendation from your teacher, who should mail the recommendation directly to our office, as well as a copy of your transcript of your high school record;

5. you will also be expected to submit your solutions to the 2009 Problems to Explore (partial or complete). Please make sure to show all work when working on your solutions and send that in with your solutions. We recommend that you make a photocopy of your solutions prior to sending them in for safe-keeping and also in case we call to discuss your responses.

MAILING INSTRUCTIONS

Please send the completed forms via First Class Mail. Because of the various weights of the enclosures, it would be wise to verify the correct postage to assure prompt delivery.

Applications will be reviewed on a first-come first-serve basis until the program is filled. All materials should be returned to the following address as soon as possible.

RETURN MATERIALS TO: Rutgers University
Young Scholars Program
SERC Building, Room 225
118 Frelinghuysen Road
Piscataway, New Jersey 08854-8019
Student Name..................................................................................................................................................

Student Address ....................................................................Phone (     ) .....................................................

City ...................................................................................... State............................Zip ..............................

Social Security Number ........................................ Date of Birth .............. Class of □ 2010 □ 2011 □ 2012

Female ...... Male ......... Email Address...........................................................................................

High School ........................................................................... School phone (    )..........................

School Address...............................................................................................................................................

City..................................................................................................State............................. Zip ...................

Name of Teacher who will provide the Recommendation.................................................................

List the math courses you will have completed by July:........................................................................

........................................................................................................................................................................

List the math courses you plan to take next year:...............................................................................

........................................................................................................................................................................

If you have previously attended a summer academic program, please name the program[s] and date[s]:

........................................................................................................................................................................

Optional (used for statistical purposes)

How would you best describe yourself (please check one):

□ Native American    □ Hispanic (including Puerto Rican)

□ Asian or Pacific Islander (including Indian subcontinent)    □ White, Anglo, Caucasian

□ African American □ Other (Specify)

List any friends or relatives who have attended the Young Scholars Program:

Name:........................................................................................................... Year..........................

How did you learn about the program? (You may check more than one)

□ YSP Website    □ Parents/Relatives

□ Teacher Recommended □ Former Program Student/Alumni

□ Guidance Counselor □ Brochure

□ Newspaper or Publication (name and section)....................................................................................

□ Summer Program Guide (name and section)........................................................................................

□ Other (please explain)............................................................................................................................
To The Candidate

Please print your name below and give this form to a parent or guardian to complete. Both parents may complete the form together, if they wish.

CANDIDATE’S NAME

To The Parent or Guardian

Your daughter/son is applying for admission to the Rutgers Young Scholars Program in Discrete Mathematics, a four-week, summer, residential program for mathematically talented high school students. The program will take place on the Rutgers University Busch Campus in Piscataway between the dates of July 6 - July 31, 2009.

The cost of the program will be $3,500; this will cover tuition, materials, meals, and lodging. A few partial scholarships are available. **No student should be discouraged from applying because of financial considerations.** If applying for financial assistance please complete the enclosed application form for financial assistance.

Please indicate here any special considerations (medical, physical, emotional, psychological, etc.) we should be aware of in terms of our responsibility to your child’s education and general well-being for a month. This information will be kept strictly confidential. (You may use the back side of this page if you need additional space)

Special Considerations:________________________________________________________________________
___________________________________________________________________________________________

Permission

"My daughter/son has permission to attend the Rutgers Young Scholars Program for the entire four week program from July 6 - July 31, 2009"

Your Name (please print) Relationship to Candidate

Home Address

City State Zip

Daytime telephone Evening telephone

Email

Signature of parent or legal guardian.................................................................................................................................
INSTRUCTIONS:

On this form, please tell us about the student you are recommending for the Rutgers Young Scholars Program in Discrete Mathematics.

Include information about the student's performance (including cumulative average and PSAT/SAT scores if available) and attach a transcript of the student's high school record.

Tell us about your student's abilities and interests, about the personal characteristics that make her or him most likely to benefit from and enjoy a four-week intensive exposure to a program in the mathematical sciences. Include specific examples drawn from your own experiences with the student.

Please write your remarks on the other side of this page and continue on a separate sheet of paper. Please be sure to write your student's full name in your recommendation.

STUDENT NAME:

In view of this student's interest and ability in mathematics, I recommend that s/he be accepted to the Rutgers Young Scholars Program in Discrete Mathematics.

Signature of recommending teacher ____________________________________________

Name (please print)____________________________________________________________

City___________________________________________________State__________ Zip_____

School_____________________________________School Phone Number (     )__________________

School Address________________________________________________________________________

City________________________State______Zip_________Email:______________________________

RETURN MATERIALS TO: Rutgers University
Young Scholars Program
SERC Building, Room 225
118 Frelinghuysen Road
Piscataway, New Jersey   08854-8019

The Rutgers Young Scholars Program in Discrete Mathematics is sponsored by the Rutgers University Center for Mathematics, Science, and Computer Education, which conducts the program, and the Center for Discrete Mathematics and Theoretical Computer Science [DIMACS].
Problems to Explore
Rutgers Young Scholars Program
Summer 2009

PROBLEM 1: CLASS WEIGHT

Mr. Smith's sixth grade class has 24 students. The average weight of the girls is 96 pounds, the average weight of the boys is 108 pounds, and the average weight of the class is 103 pounds. How many girls and how many boys are in the class?

PROBLEM 2: DIVIDING THE MONEY

In a class action suit, the plaintiffs were collectively awarded $36,000,000. The money was to be equally divided among those eligible.

Before the money was distributed, it was discovered that 6 additional people were eligible for compensation. Again, dividing the money equally, this reduced the amount that would be paid to each person by $10,000.

In the end, how many people received money and how much did each receive?

PROBLEM 3: THE PEDESTRIAN AND THE BUSES

Each half hour a bus makes a 30 minute trip from A to B and starting at the same time another bus makes the 30 minute trip from B to A.

A pedestrian leaves A, following the bus route from A to B, at the same time the bus does. After walking for 25 minutes, the pedestrian meets a bus going from B to A.

How long after this will the pedestrian be overtaken by a bus going from A to B?

Assume that the average rate of each bus over the whole trip is the same as the average rate on that portion of the trip from the time the bus leaves the terminal until it meets the pedestrian. Likewise, make a similar assumption about the average rate of the pedestrian.

PROBLEM 4: TRIANGLES IN A DECAGON

A triangle is formed using three of the vertices of a regular decagon. How many such triangles can be drawn having no edge in common with the decagon?

For example, the triangle on the left is of this type, while the triangle on the right is not since it has a common edge with the decagon.
PROBLEM 5: SOME SUMS OF SIX DIGIT INTEGERS

Suppose that a, b, c are distinct non-zero digits.
(a) Find a formula, depending on a, b, c, for the sum of all six digit integers whose only digits are a, b, c.
(b) Find a formula for the sum of all six digit integers having as digits exactly 2 a’s, 2 b’s, 2 c’s.
(c) Find the sum of all six digit integers with three non-zero digits, each occurring twice.

PROBLEM 6: SHH!

In the addition at the left each letter stands for one of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. Assume that different letters represent different digits. Find all possible additions having this pattern.

Once one solution has been found, an additional solution can be found by interchanging the digits in positions E and K, since these letters do not occur elsewhere. Thus, in listing solutions, assume that E > K.

PROBLEM 7: ATTACKING QUEENS

Recall that in chess a queen attacks any square that is on a straight line — horizontally, vertically, or diagonally — from the square on which the queen stands. Regard the queen as also attacking the square which she occupies. What is the smallest number of queens required to attack all of the squares of a 6x6 chessboard?

PROBLEM 8: ADDITION BY GEOMETRIC DISSECTION

Verify that \(9^2 + 2^2 = 7^2 + 6^2\) by cutting a 9x9 square into four pieces that together with a 2x2 square can be rearranged to give a 7x7 square and a 6x6 square.

PROBLEM 9: MANY DIFFERENT WEIGHTS

A set of five different positive integer weights has the property that the collective weight of any pair is different from the collective weight of any other pair.
(a) If the weights of the pairs all lie between 100 and 110, what are the individual weights?
(b) If the weights of the pairs all lie between 101 and 111, what are the individual weights?
Give all possible answers.
For example, a set of four different weights is 10, 12, 13, 14 and the collective weights of the pairs are: 22, 23, 24, 25, 26, 27. Thus, the weights of the pairs all lie between 22 and 27. The analogous problem for four weights would be to use that the weights of the pairs all lie between 22 and 27 to reconstruct the individual weights 10, 12, 13, 14.
PROBLEM 10: A THIRD AS A FRACTION USING ALL NON-ZERO DIGITS

Using each of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly once form two numbers $x$ and $y$ so that $x/y = 1/3$.

(Suggestion: In the beginning it may be useful to recall that an integer is divisible by 3 or 9 if and only if the sum of its digits is divisible by 3 or 9, respectively.)

---

PROBLEM 11: POLYGONS WITH ANGLES IN SEQUENCE

The pentagon below looks at first like a regular pentagon, but closer inspection shows that it is slightly skewed. In fact the interior angles, starting from the top and going around in the counterclockwise direction, are: $106^\circ, 107^\circ, 108^\circ, 109^\circ, 110^\circ$.

Thus, going from the smallest angle to the largest, the degree measures of the angles form a sequence of consecutive integers.

(a) For which other integers $n \geq 3$ is there a polygon with $n$ sides, with all interior angles less than $180^\circ$, so that the degree sequence of the angle measures, from smallest to largest, consists of consecutive integers?

(b) For which other integers $n \geq 3$ is there a polygon with $n$ sides, with all interior angles less than $180^\circ$, so that the degree sequence of the angle measures, from smallest to largest, consists either of consecutive even integers, such as $104, 106, 108, 110, 112$, or consecutive odd integers, such as $95, 97, 99, 101$?

---

PROBLEM 12: ALL ARRANGEMENTS OF 1, 2, 3, 4 IN SEQUENCE

Number four separate squares of paper 1, 2, 3, 4 and place the squares in a row in that order. The object is, by interchanging adjacent squares, to obtain every arrangement of 1, 2, 3, 4 exactly once, and, on the final step, to return to the starting configuration 1, 2, 3, 4.

For example, if there are instead three pieces of paper numbered 1, 2, 3, then all possible arrangements of 1, 2, 3 are: 123, 132, 213, 231, 312, 321. The following sequence of moves shows how it is possible, by interchanging adjacent numbers, to pass through each arrangement exactly once and then return to the initial position:

123, 213, 231, 321, 312, 132, 123.

Is it possible, starting with 1234 and interchanging adjacent numbers, to pass through every arrangement of 1, 2, 3, 4 exactly once and then return to the initial position 1234?

If this is possible, verify it as in the example above by listing the arrangements in the order in which each occurs. If it is not possible, explain why.

---

PROBLEM 13: WHICH EQUILATERAL TRIANGLE?

A point within an equilateral triangle is at distance 5, 7, 8 from each of the vertices. What is the length of a side of the triangle?
PROBLEM 14: COUNTING CERTAIN SEQUENCES OF X'S, Y'S, AND Z'S

We consider certain sequences of the letters X, Y, Z. All of the sequences will have the property that the same letter does not occur in succession. Thus, for example, XYZZX is not allowed because of the consecutive Z's.

(a) Assume, in addition, the sequences have the property that in every subsequence of four successive letters, each of the letters X, Y, Z occurs at least once. In other words, none of the sequences XYXY, XZXZ, YXYX, YZYZ, ZXZX, ZYZY occurs as a consecutive subsequence.

The problem is to count all sequences of this type of a given length. Once those sequences that begin with XY have been counted, it is easy to count all of the others. Thus, we can assume that all of the sequences begin with XY.

For example, all of the sequences of this type having length 5 are: XYXZX, XYXZY, XYZXY, XYZXZ, XYZYX.

How many sequences of this type have length 12?

(b) The problem is basically the same, except now we insist only that in every successive subsequence of five letters, all of the letters X, Y, Z occur at least once.

Now, for example, all of the sequences of this type having length 5 are: XYXYZ, XYXZX, XYXZY, XYZXY, XYZXZ, XYZYX, XYZYZ.

How many sequences of this type have length 12?

In each case, explain fully why the answer is correct. A good guess is good, but it is not enough.

PROBLEM 15: CROSSWORD PUZZLE PATTERNS

A crossword puzzle pattern is produced by darkening certain squares of an n x n grid so that:

1. All words have length at most three.
2. The undarkened squares are connected in the sense that one can go from any undarkened square to any other by proceeding in a series of steps, each step horizontal or vertical, through other undarkened squares.

The problem is to determine the smallest number of squares one must darken to achieve such a pattern.

As an example consider the following three patterns on a 5 x 5 grid. The first pattern fails to satisfy condition (1), since there is a word of length 4. In the second example, since all words have length at most 3, condition (1) holds, but condition (2) fails. In the third example, both conditions (1) and (2) hold. Note that in the third example, six squares have been darkened. It is possible to show, on a 5 x 5 grid, that one cannot obtain a crossword puzzle pattern that satisfies (1) and (2) by darkening only five squares. Thus, the minimum number of squares that must be colored to achieve a pattern satisfying conditions (1) and (2) is six.

What is the minimum number of squares that must be darkened to achieve a pattern satisfying conditions (1) and (2):

(a) On a 6 x 6 grid?   (b) On a 7 x 7 grid?   (c) On an 8 x 8 grid?

Explain why your answer is a minimum.