APPLICATION KIT

The RUTGERS YOUNG SCHOLARS PROGRAM in DISCRETE MATHEMATICS

July 2 - July 27, 2012

"Our Twenty-second Summer"

"We're looking for students who are interested in math. Is that you?"

The Rutgers Young Scholars Program in Discrete Mathematics is sponsored by the Rutgers University Department of Mathematics and DIMACS, the Center for Discrete Mathematics and Theoretical Computer Science [DIMACS], and is supported by a grant from the National Security Agency (NSA).

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2012 READ FIRST

Do you …

… *like to work on challenging and puzzling problems*
… *like to look at a problem in many different ways*
… *get involved, make conjectures, and take risks*
… *enjoy the critical exchange of ideas with other students*
… *look forward to living in the math world for a month*
… *enjoy and feel compelled to do mathematics*
… *thrive on complexity and problems which have more than one answer*
… *like to reason out problems and play with concepts*
… *learn new material quickly and apply this learning to new situations*
… *concentrate on something you like for a long period of time*

Then this program is for you. Here is your opportunity to …

… *experiment with some interesting mathematical problems*
… *spend four weeks on a college campus doing mathematics with people like yourself*
… *meet a variety of professional mathematicians and learn about what they do*

Come to the Rutgers Young Scholars Program …

… *come join 30 high school students who share your fascination with mathematics.*

… *attend a variety of mathematical programs and activities … including sessions on discrete mathematics, discussions, and a robotics challenge.*

… *learn about careers in the mathematical sciences; these programs will involve industrial as well as academic mathematicians.*

The instructional staff includes well-known mathematicians. Teaching assistants who are graduate and undergraduate Rutgers students majoring in mathematics or computer science provide additional academic support. You will spend a lot of time with this staff; we hope that as a result you will understand what mathematicians do and why.

The focus of the program will be discrete mathematics, although the program will **not** be conducted like a course. Success will be measured in terms of your enthusiasm for the subject, not by the amount of material covered.

Your daily schedule will be full and will include various recreational and social activities.

> ***Remember, this will be an intensive program. We will expect you to participate actively in all of the activities. It will be fun and exciting, but it will require your concentrated effort. You can think of the program as "math camp," but you must remember that you will be doing math all day, though not in the evening. Be sure that you want to make this commitment!***

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2012 INSTRUCTIONS

COMPLETING APPLICATION MATERIALS

A completed application includes all of the following:

1. TYPED OR PRINTED (in black ink) responses to the questions on the Student Application Form;

2. a completed parent signature form;

3. a letter of recommendation from your teacher, who should mail the recommendation directly to our office, as well as a copy of your transcript of your high school record;

4. your solutions (partial or complete) to the 2012 Problems to Explore (at the end of these forms). *We recommend that you make a photocopy of your solutions prior to sending them in for safe-keeping and also in case we call to discuss your responses.*

These problems are challenging and most applicants will successfully solve only some of them; so don't get discouraged by what you haven't done … just show us what you have done, including any partial solutions. Please make sure to **show us all your work** and, wherever appropriate, **explain your reasoning.**

Applications will be reviewed on a first-come first-served basis until the program is filled. In some years, the program had more applicants than could be accommodated. We therefore encourage you to submit your completed application **as soon as possible**.

Partial scholarships are available. **You should not be discouraged from applying because of financial considerations**. Once you are accepted into the program, your parents will be asked if they wish to apply for financial assistance and, if so, will be asked to complete an application form and provide additional materials. Applications for financial assistance must be received by May 1, 2012, so **if you expect that your family will apply for a scholarship, please be sure to submit your solutions to the problems by April 9.**

MAILING INSTRUCTIONS

Please send the completed forms via First Class Mail. Because of the various weights of the enclosures, it would be wise to verify the correct postage to assure prompt delivery.

Materials should be submitted to the following address:

Rutgers Young Scholars Program
DIMACS – CORE Building
96 Frelinghuysen Road
Piscataway, New Jersey 08854-8018

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2012 STUDENT APPLICATION FORM

Student Name..Cell Phone (   )..

Student Address ....Home Phone (   ).......................................

City .... State............................Zip............................

Social Security Number .... Date of Birth .... Class of ☐ 2013 ☐ 2014 ☐ 2015

*** - ** - ****

Female .... Male ....   Email Address...

High School .... School phone (   )..

School Address..

City...State............................ Zip ....

Name of Teacher who will provide the Recommendation...

List the math courses you will have completed by July:...

....

List the math courses you plan to take next year:...

....

If you have previously attended a summer academic program, please name the program[s] and date[s]:

....

Optional (*used for statistical purposes*)

How would you best describe yourself (please check one):

☐Native American
☐Asian or Pacific Islander (including Indian subcontinent)
☐African American
☐Hispanic(including Puerto Rican)
☐White, Anglo, Caucasian
☐Other (Specify)

List any friends or relatives who have attended the Young Scholars Program:

Name:... Year..

How did you learn about the program? (You may check more than one)

☐YSP Website
☐Teacher Recommended
☐Guidance Counselor
☐Parents/Relatives
☐Former Program Student/Alumni
☐Brochure
☐Newspaper or Publication (name and section)..
☐Summer Program Guide (name and section)..
☐Other (please explain)..

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2012 **PARENT SIGNATURE FORM**

To the student:

Please print your name below and give this form to a parent or guardian to complete. Both parents may complete the form together, if they wish.

STUDENT'S NAME ____

To The Parent or Guardian

Your daughter/son is applying for admission to the Rutgers Young Scholars Program in Discrete Mathematics, a four-week, summer, residential program for mathematically talented high school students. The program will take place on the Rutgers University Busch Campus in Piscataway between the dates of July 2 - July 27, 2012, not including the weekends. Although Wednesday July 4 is a holiday, the program will take place as usual on that day.

The cost of the program will be $3,500; this will cover tuition, materials, and meals and lodging from Monday morning to Friday afternoon of each week. Partial scholarships are available. **No student should be discouraged from applying because of financial considerations**. Once your daughter/son is accepted into the program, you will be asked if you wish to apply for financial assistance and, if so, you will be asked to complete an application form and provide additional materials.

Please indicate here any special considerations (medical, physical, emotional, psychological, etc.) we should be aware of in terms of our responsibility to your child's education and general well-being for a month. This information will be kept strictly confidential. (You may use the back side of this page if you need additional space)

Special Considerations:____

Permission

"My daughter/son has permission to attend the Rutgers Young Scholars Program for the entire four week program from July 2 - July 27, 2012."

____

Your Name (please print) Relationship to Student

____

Home Address

____

City State Zip

____

Daytime telephone Evening telephone Cell phone

____

Email

Signature of parent or legal guardian..

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2012 RECOMMENDATION FORM

INSTRUCTIONS:

On this form, please tell us about the student you are recommending for the Rutgers Young Scholars Program in Discrete Mathematics.

Include information about the student's performance (including cumulative average and PSAT/SAT scores if available).

Please attach a transcript of the student's high school record.

Tell us about your student's abilities and interests, about the personal characteristics that make her or him most likely to benefit from and enjoy a four-week intensive exposure to a program in the mathematical sciences. Include specific examples drawn from your own experiences with the student.

You may write your remarks on the other side of this page and/or on a separate sheet of paper. Please be sure to write your student's full name in your recommendation.

STUDENT NAME:____

In view of this student's interest and ability in mathematics, I recommend that s/he be accepted to the Rutgers Young Scholars Program in Discrete Mathematics.

Signature of recommending teacher ____

Name (please print)____

City____State__________ Zip_____

School___________________________________School Phone Number (   )__________________

School Address____

City________________________State______Zip_________Email:______________________________

RETURN MATERIALS TO:

Rutgers University
Young Scholars Program
DIMACS – CORE Building
96 Frelinghuysen Road
Piscataway, New Jersey 08854-8018

The Rutgers Young Scholars Program in Discrete Mathematics is sponsored by the Rutgers University Department of Mathematics and DIMACS, the Center for Discrete Mathematics and Theoretical Computer Science, and is supported by a grant from the National Security Agency (NSA).

Problems to Explore
Rutgers Young Scholars Program
Summer 2012

PROBLEM 1: FORMING A START-UP COMPANY

Twenty people decide to start a new company, and each person agrees to contribute the same amount of money for the initial expenses of the business.

If they had started with five more people and each person, both old and new, had paid \$500 less than the initial amount, they would still have had the same amount of money for initial expenses.

How much did each of the twenty original people agree to contribute?

PROBLEM 2: NO CHANGE OF TEMPERATURE

At what temperature are the numerical readings in both Fahrenheit and Celsius (or centigrade) the same?

PROBLEM 3: A CHANGE OF SCALE

A car travels from A to B at a constant speed of v miles per hour and reaches B in one hour.

A second car starts at a point C on the same route, but 21 miles closer to B, and, travelling at a speed of v feet per second also reaches B in one hour.

What is the distance from A to B in miles?

PROBLEM 4: UNABLE TO MAKE CHANGE

What is the largest amount of change (quarters, dimes, nickels, and pennies) you could have and still not be able to make change for a dollar?

Explain your answer in detail.

PROBLEM 5: A PRIME CRYPTARITHM

Each * denotes a prime digit, that is, either 2, 3, 5, or 7 in the following multiplication. There is a unique solution.

$$\begin{array}{ccccc} & & * & * & * \\ & & & * & * \\ \hline & * & * & * & * \\ * & * & * & * & \\ \hline * & * & * & * & * \end{array}$$

PROBLEM 6: FIND THE RECTANGLES

(a) Find all rectangles with sides of integer length so that the area of the rectangle is 10 times its perimeter.

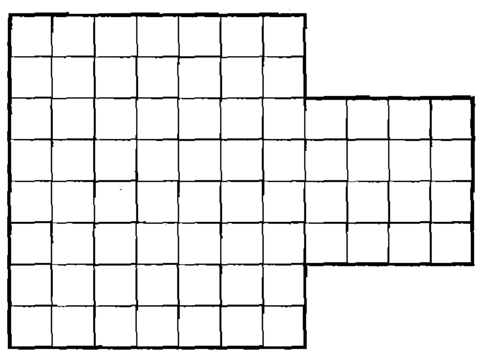
(b) Find all squares so that the perimeter of the square is ten times its area.

PROBLEM 7: ATTACKING QUEENS

Recall that in chess a queen attacks any square that is on a straight line — horizontally, vertically, or diagonally — from the square on which the queen stands. Regard the queen as also attacking the square which she occupies. What is the smallest number of queens required to attack all of the squares of a 6×6 chessboard?

PROBLEM 8: A DISSECTION PROBLEM

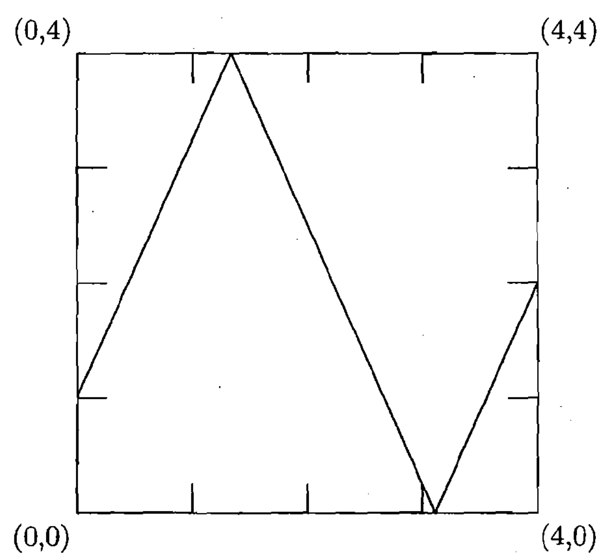
In the picture below is a 7×8 rectangle with a 4×4 square attached to the middle of one side of the rectangle having length 8. Dissect this figure into 6 congruent pieces.



PROBLEM 9: THE POOL SHOT

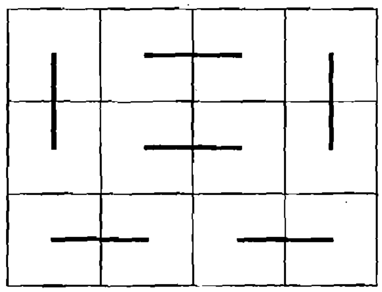
A pool player takes a shot on a 4×4 pool table. With the coordinate system and shot as indicated in the picture at the right, the shot begins at (0,1) and ends as (4,2). Where does the ball strike the top and bottom of the pool table?

Assume that the ball has no spin (or "English") so that the angle at which it leaves the edge of the pool table is the same as the angle at which it arrives at that edge.

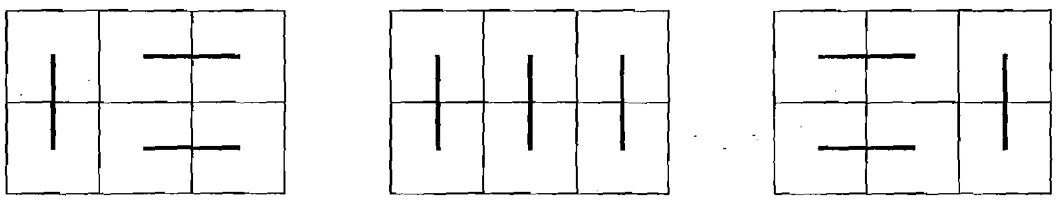


PROBLEM 10: COUNTING TILINGS OF AN $M \times N$ GRID WITH 1×2 TILES

The problem concerns covering an $m \times n$ grid with 1×2 tiles (dominoes) when m or n is an even integer. Below is given an example. Here a 3×4 grid is tiled with 1×2 tiles indicated by the dark lines between the centers of the grid squares.



This problem is about counting all possible ways of constructing such a tiling. It is easy to see that there are two ways to tile a 2×2 grid. It is not much more difficult to find the three ways of tiling a 2×3 grid that are shown below.



(a) How many tilings are there of a $2 \times m$ grid with 1×2 tiles? Either give the general rule or the specific answer for all m, $m \leq 8$.

(b) How many tilings are there of a 3×4 grid and of a 3×6 grid with 1×2 tiles?

PROBLEM 11: CHANGING POWER SUMS

Suppose that x_1, x_2, x_3, ..., x_k is a set of numbers.

We form the sums of the powers of x_1, x_2, x_3, ..., x_k up to the fourth power:

$\sigma_1 = x_1 + x_2 + x_3 + \cdots + x_k$, $\quad \sigma_2 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_k^2$,

$\sigma_3 = x_1^3 + x_2^3 + x_3^3 + \cdots + x_k^3$, $\quad \sigma_4 = x_1^4 + x_2^4 + x_3^4 + \cdots + x_k^4$.

Next we add 1 to each of the numbers x_1, x_2, x_3, ..., x_k and again form the sums of the powers of $x_1 + 1$, $x_2 + 1$, $x_3 + 1$, ..., $x_k + 1$ up to the fourth power:

$\tau_1 = (x_1 + 1) + (x_2 + 1) + (x_3 + 1) + \cdots + (x_k + 1)$,

$\tau_2 = (x_1 + 1)^2 + (x_2 + 1)^2 + (x_3 + 1)^2 + \cdots + (x_k + 1)^2$,

$\tau_3 = (x_1 + 1)^3 + (x_2 + 1)^3 + (x_3 + 1)^3 + \cdots + (x_k + 1)^3$,

$\tau_4 = (x_1 + 1)^4 + (x_2 + 1)^4 + (x_3 + 1)^4 + \cdots + (x_k + 1)^4$.

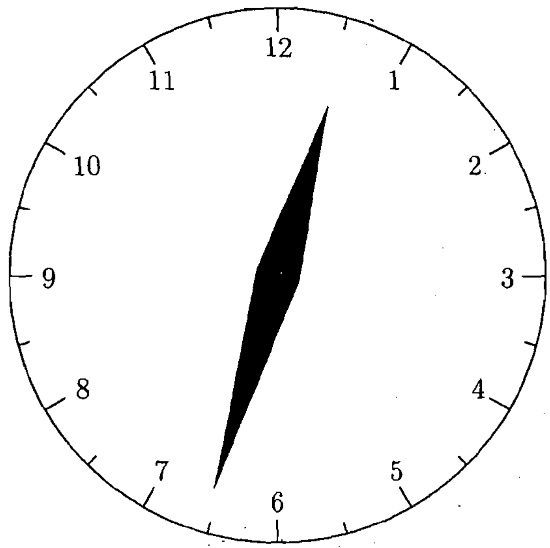
(a) Suppose that $\tau_1 = \sigma_1 + 5$, $\tau_2 = \sigma_2 + 171$, $\tau_3 = \sigma_3 + 4661$, and $\tau_4 = \sigma_4 + 119{,}019$. Find k, σ_1, σ_2, and σ_3.

(b) Under these conditions, if the numbers x_1, x_2, x_3, ..., x_k are all distinct prime numbers, find x_1, x_2, x_3, ..., x_k.

PROBLEM 12: WHAT TIME IS IT?

At 12 noon, of course, the hour hand on a clock and the minute hand on the clock both point to the marker above the 12 at the top of the clock, i.e., the hands are exactly vertical.

At some time shortly after 12:30, the hour hand and the minute hand point in exactly the opposite direction, or, in other words, the angle between the hands is 180°.



At exactly how many minutes after 12 noon does this occur?

PROBLEM 13: RUNNING LAPS

Two runners run at constant speed around an oval track. The first runner goes around the track every 60 seconds. The second runner, going in the opposite direction, meets the first runner every 35 seconds. How long does it take the second runner to go around the track?

PROBLEM 14: A POSITIVE INTEGER N WITH $N/2$ A PERFECT SQUARE AND $N/3$ A PERFECT CUBE

If $N = 30,233,088$, then the number $\frac{N}{2} = 15,116,544 = (3882)^2$ is a perfect square and $\frac{N}{3} = 10,077,696 = (216)^3$ is a perfect cube. However, there are smaller integers N with this property.

(a) Find the smallest positive integer N with $\frac{N}{2}$ a perfect square and $\frac{N}{3}$ a perfect cube.

(b) Find the smallest positive integer N with $\frac{N}{2}$ a perfect square, $\frac{N}{3}$ a perfect cube, and $\frac{N}{5}$ a perfect fifth power.

(c) Would it make sense in (b) to say instead $\frac{N}{2}$ a perfect square, $\frac{N}{3}$ a perfect cube, and $\frac{N}{4}$ a perfect fourth power? Explain your answer.

PROBLEM 15: DIOPHANTINE EQUATIONS

A Diophantine equation is an equation whose solutions are required to be integers. A well-known example is $x^2 + y^2 = z^2$, which has solutions $(x, y, z) = (3, 4, 5)$ and (5,12,13). Until fairly recently one of the most famous unsolved problems in mathematics (known as Fermat's Last Theorem) was to show that the equation $x^n + y^n = z^n$ has no integer solutions (x, y, z) with x, y, z non-zero and n an integer with $n \geq 3$. This was proved by Andrew Wiles in 1993.

(a) Find all solutions to the Diophantine equation $x^3 + y^4 = z^5$ in which the integers x, y, z are all powers of 2.

(b) Find two additional solutions to this equation.

(c) Similarly, find all solutions to the Diophantine equation $x^3 + y^3 = z^4$ with the integers x, y, z all powers of 2.

(d) Also, find two additional solutions to this equation.