APPLICATION KIT

The RUTGERS YOUNG SCHOLARS PROGRAM in DISCRETE MATHEMATICS

July 1 - July 26, 2013

"Our Twenty-third Summer"

"We're looking for students who are interested in math. Is that you?"

The Rutgers Young Scholars Program in Discrete Mathematics is sponsored by the Rutgers University Department of Mathematics and DIMACS, the Center for Discrete Mathematics and Theoretical Computer Science [DIMACS], and is supported by a grant from the National Security Agency (NSA).

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2013 READ FIRST

Do you …

… *like to work on challenging and puzzling problems*
… *like to look at a problem in many different ways*
… *get involved, make conjectures, and take risks*
… *enjoy the critical exchange of ideas with other students*
… *look forward to living in the math world for a month*
… *enjoy and feel compelled to do mathematics*
… *thrive on complexity and problems which have more than one answer*
… *like to reason out problems and play with concepts*
… *learn new material quickly and apply this learning to new situations*
… *concentrate on something you like for a long period of time*

Then this program is for you. Here is your opportunity to …

… *experiment with some interesting mathematical problems*
… *spend four weeks on a college campus doing mathematics with people like yourself*
… *meet a variety of professional mathematicians and learn about what they do*

Come to the Rutgers Young Scholars Program …

… *come join 30 high school students who share your fascination with mathematics.*

… *attend a variety of mathematical programs and activities … including sessions on discrete mathematics, discussions, and a robotics challenge.*

… *learn about careers in the mathematical sciences; these programs will involve industrial as well as academic mathematicians.*

The instructional staff includes well-known mathematicians. Teaching assistants who are graduate and undergraduate Rutgers students majoring in mathematics or computer science provide additional academic support. You will spend a lot of time with this staff; we hope that as a result you will understand what mathematicians do and why.

The focus of the program will be discrete mathematics, although the program will **not** be conducted like a course. Success will be measured in terms of your enthusiasm for the subject, not by the amount of material covered.

Your daily schedule will be full and will include various recreational and social activities.

Remember, this will be an intensive program. We will expect you to participate actively in all of the activities. It will be fun and exciting, but it will require your concentrated effort. You can think of the program as "math camp," but you must remember that you will be doing math all day, though not in the evening. Be sure that you want to make this commitment!

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2013 INSTRUCTIONS

COMPLETING APPLICATION MATERIALS

A completed application includes all of the following:

1. TYPED OR PRINTED (in black ink) responses to the questions on the Student Application Form;

2. a completed parent signature form;

3. a letter of recommendation from your teacher, who should mail the recommendation directly to our office, as well as a copy of your transcript of your high school record;

4. your solutions (partial or complete) to the 2013 Problems to Explore (at the end of these forms). *We recommend that you make a photocopy of your solutions prior to sending them in for safe-keeping and also in case we call to discuss your responses.*

   These problems are challenging and most applicants will successfully solve only some of them; so don't get discouraged by what you haven't done … just show us what you have done, including any partial solutions. Please make sure to **show us all your work** and, wherever appropriate, **explain your reasoning.**

Applications will be reviewed on a first-come first-served basis until the program is filled. In some years, the program had more applicants than could be accommodated. We therefore encourage you to submit your completed application **as soon as possible**.

Partial scholarships are available. **You should not be discouraged from applying because of financial considerations**. Once you are accepted into the program, your parents will be asked if they wish to apply for financial assistance and, if so, will be asked to complete an application form and provide additional materials. Applications for financial assistance must be received by May 1, 2013, so **if you expect that your family will apply for a scholarship, please be sure to submit your solutions to the problems by April 8.**

MAILING INSTRUCTIONS

Please send the completed forms via First Class Mail. Because of the various weights of the enclosures, it would be wise to verify the correct postage to assure prompt delivery.

Materials should be submitted to the following address:

Rutgers Young Scholars Program
DIMACS – CORE Building
96 Frelinghuysen Road
Piscataway, New Jersey 08854-8018

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2013 STUDENT APPLICATION FORM

Student Name...Cell Phone (   )...

Student Address ....Home Phone (   ).......................................

City .... State.............................Zip.............................

Social Security Number .... Date of Birth .... Class of ☐ 2014 ☐ 2015 ☐ 2016

*** - ** - ****

Female .... Male ....   Email Address..

High School .... School phone ( )..

School Address...

City..State............................ Zip ....

Name of Teacher who will provide the Recommendation..

List the math courses you will have completed by July:..

....

List the math courses you plan to take next year:...

....

If you have previously attended a summer academic program, please name the program[s] and date[s]:

....

Optional (*used for statistical purposes*)
How would you best describe yourself (please check one):

☐Native American
☐Asian or Pacific Islander (including Indian subcontinent)
☐African American
☐Hispanic(including Puerto Rican)
☐White, Anglo, Caucasian
☐Other (Specify)

List any friends or relatives who have attended the Young Scholars Program:

Name:.. Year..

How did you learn about the program? (You may check more than one)

☐YSP Website
☐Teacher Recommended
☐Guidance Counselor
☐Parents/Relatives
☐Former Program Student/Alumni
☐Brochure
☐Newspaper or Publication (name and section)...

☐Summer Program Guide (name and section)..

☐Other (please explain)..

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2013 PARENT SIGNATURE FORM

To the student:

Please print your name below and give this form to a parent or guardian to complete. Both parents may complete the form together, if they wish.

STUDENT'S NAME ______________________________

To The Parent or Guardian

Your daughter/son is applying for admission to the Rutgers Young Scholars Program in Discrete Mathematics, a four-week, summer, residential program for mathematically talented high school students. The program will take place on the Rutgers University Busch Campus in Piscataway between the dates of July 1 - July 26, 2013, not including the weekends. Although Thursday July 4 is a holiday, the program will take place as usual on that day.

The cost of the program will be $3,500; this will cover tuition, materials, and meals and lodging from Monday morning to Friday afternoon of each week. Partial scholarships are available. **No student should be discouraged from applying because of financial considerations**. Once your daughter/son is accepted into the program, you will be asked if you wish to apply for financial assistance and, if so, you will be asked to complete an application form and provide additional materials.

Please indicate here any special considerations (medical, physical, emotional, psychological, etc.) we should be aware of in terms of our responsibility to your child's education and general well-being for a month. This information will be kept strictly confidential. (You may use the back side of this page if you need additional space)

Special Considerations:______________________________

Permission

"My daughter/son has permission to attend the Rutgers Young Scholars Program for the entire four week program from July 1 - July 26, 2013."

____

Your Name (please print) Relationship to Student

____

Home Address

____

City State Zip

____

Daytime telephone Evening telephone Cell phone

____

Email

Signature of parent or legal guardian...

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2013 **RECOMMENDATION FORM**

INSTRUCTIONS:

Please tell us about the student you are recommending for the Rutgers Young Scholars Program in Discrete Mathematics.

Include information about the student's performance (including cumulative average and PSAT/SAT scores if available).

Please attach a transcript of the student's high school record.

Tell us about your student's abilities and interests, about the personal characteristics that make her or him most likely to benefit from and enjoy a four-week intensive exposure to a program in the mathematical sciences. Include specific examples drawn from your own experiences with the student.

You may write your remarks on the other side of this page and/or on a separate sheet of paper. Please be sure to write your student's full name in your recommendation.

STUDENT NAME:____

In view of this student's interest and ability in mathematics, I recommend that s/he be accepted to the Rutgers Young Scholars Program in Discrete Mathematics.

Signature of recommending teacher ______________________________________

Name (please print)____

Email address: ____Cell phone: ________

School_______________________________School Phone Number (   )_______________

School Address____

City____State_____Zip________

RETURN MATERIALS TO:

Rutgers University
Young Scholars Program
DIMACS – CORE Building
96 Frelinghuysen Road
Piscataway, New Jersey 08854-8018

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Problems to Explore
Rutgers Young Scholars Program
Summer 2013

PROBLEM 1: AUTOMOBILE TRIP

A man goes on a 100 mile automobile trip. For the first hour he averages 50 miles per hour. He decides that he wants to average 60 mile per hour for the whole trip.

What must his average speed be for the remainder of the trip?

PROBLEM 2: POWER OUTAGE

When the power returns after a power outage, an analog (traditional round) electric clock continues measuring time from the moment the power returns. On the other hand, a digital clock returns to 12:00 and measures the amount of time that elapses after that.

When a man leaves his house at 8:00 in the morning, his watch, round electric clock, and digital clock all agree. When he returns in the afternoon, his watch gives the time as 3:20, his round electric clock reads 2:40, and his digital clock shows 1:30.

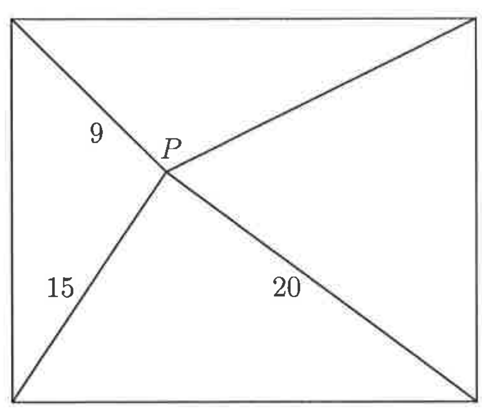
At what time did the power go off, and when did the power come back on?

PROBLEM 3: SWIMMING LAPS

Two swimmers, starting from opposite ends of the pool at the same time, swim laps. It takes the first swimmer 30 seconds to go from one end of the pool to the other and takes the other swimmer 45 seconds. If they swim for 12 minutes, how many time do they pass each other? (Count passes if the swimmers are going in the same direction, opposite directions, or turning around.)

PROBLEM 4: DISTANCES FROM THREE CORNERS OF A RECTANGLE

The distance of the point P from each of the three corners of the rectangle is as given. What is the distance of P from the fourth corner?



PROBLEM 5: TWO BICYCLISTS

The distance from the town of Poole to the town of Robards is 18 miles. Two bicyclists leave Poole at the same time, both heading to Robards, where each will turn around and return to Poole. Cyclist A has average speed that is 4 miles per hour faster than cyclist B.

Cyclist A gets to Robards and turns around heading back to Poole. He meets cyclist B 3 miles outside of Robards.

What is the average speed of each cyclist?

PROBLEM 6: SPANISH ADDITION

$$\begin{array}{r} \mathrm{T\ R\ E\ S} \\ \mathrm{S\ E\ I\ S} \\ \hline \mathrm{N\ U\ E\ V\ E} \end{array}$$

The addition at the left is correct in Spanish, since *tres* is three, *seis* is six, and *nueve* is nine. It will still be correct if the letters are replaced by some the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. Assume that different letters represent different digits. Find all possible additions having this pattern.

PROBLEM 7: ATTACKING QUEENS

Recall that in chess a queen attacks any square that is on a straight line — horizontally, vertically, or diagonally — from the square on which the queen stands. Regard the queen as also attacking the square which she occupies. What is the smallest number of queens required to attack all of the squares of a 6×6 chessboard?

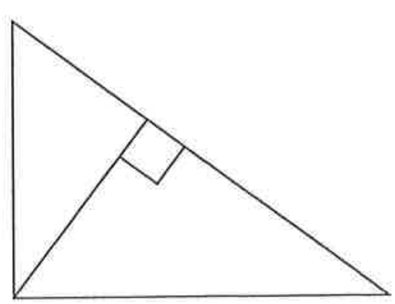
PROBLEM 8: TRIPLED VOLUMES

A 1×2 rectangle becomes a 2×3 rectangle if the length of each side is increased by 1. Then, the area triples from 2 to 6.

Find the dimensions of all rectangular boxes with edges of integer length so that if the length of each edge increases by 1, the volume triples.

PROBLEM 9: TRIANGLES FROM PERIMETER AND HEIGHT

A right triangle has perimeter 390 and height 60 measured by a perpindicular from the right angle to the hypotenuse, as shown.



Find the lengths of its sides.

PROBLEM 10: CHANGING SQUARES

The square, S, of a non-zero positive integer remains a perfect square if it is increased by 455 and if it is increased by 2640.

Find all possibilities for the square, S.

PROBLEM 11: DISSECTING TRAPEZOIDS

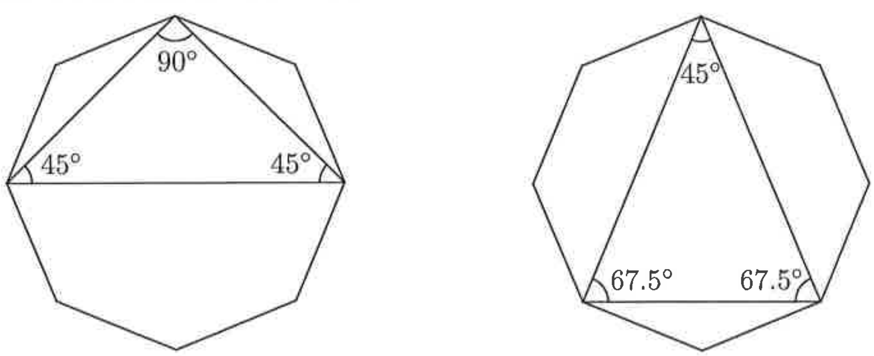
Two trapezoids are shown below.

The first is composed of three equilateral triangles, as shown below. The second is found by cutting a square into two triangles and placing a triangle congruent to one of the two triangles on the side of the square, also as shown.

Dissect each trapezoid into four congruent trapezoids.

PROBLEM 12: TRIANGLES IN A NONAGON

There are two different ways to form a triangle using three of the vertices of a regular octagon so that the octagon and triangle have no common edge. The first triangle has angles 90°, 45°, 45°, as shown, and the second angles 45°, 67.5°, 67.5°, also shown.

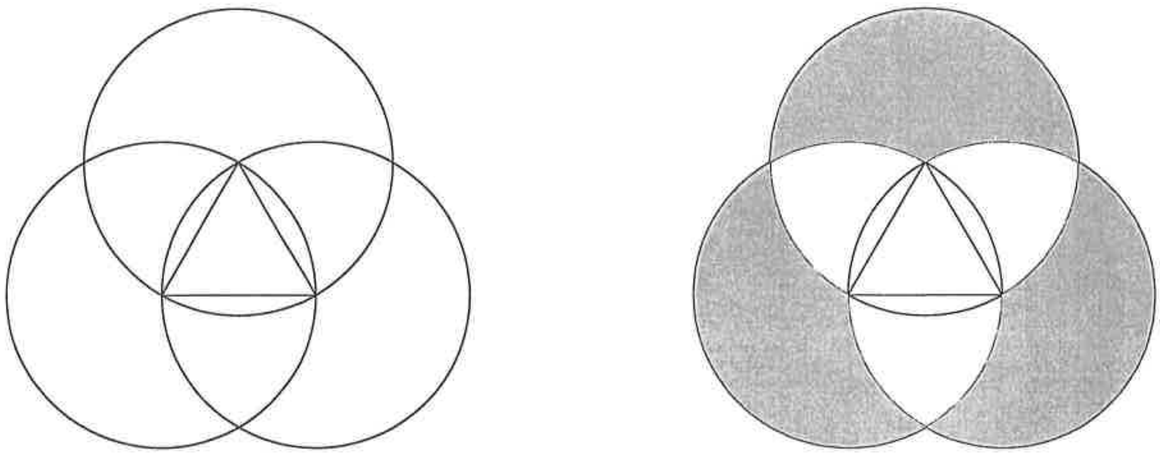


Determine all of the different ways to form a triangle using three of the vertices of a regular nonagon so that the triangle and nonagon have no common edge, and find the angles of these triangles, as above. (Recall that a regular nonagon is a nine sided polygon with all edges of the same length and all internal angles of the same size.)

PROBLEM 13: A SHADED AREA

The central triangle is equilateral with sides of length 1. Each of the circles has radius 1 and has center on one of the vertices of the equilateral triangle.

In the picture on the right, some of the area has been shaded. Find the shaded area.



PROBLEM 14: DIOPHANTINE EQUATIONS AND THE FIBONACCI SEQUENCE

A Diophantine equation is an equation whose solutions are required to be integers. A well-known example is $x^2 + y^2 = z^2$, which has solutions $(x, y, z) = (3, 4, 5)$ and (5,12,13). Until fairly recently one of the most famous unsolved problems in mathematics (known as Fermat's Last Theorem) was to show that the equation $x^n + y^n = z^n$ has no integer solutions (x, y, z) with x, y, z non-zero and n an integer with $n \geq 3$. This was proved by Andrew Wiles in 1993.

In this problem, we consider the Diophantine equation $x^2 + y^2 + z^2 = 3xyz$. It follows easily that if any of the numbers x, y, or z is 0, then $xyz = 0$ and $x = y = z = 0$.
(a) Verify that if $(x, y, z) = (a, b, c)$ is a solution, so also are $(x, y, z) = (-a, -b, c), (-a, b, -c)$, and $(a, -b, -c)$. Explain why this implies that we may assume that $a > 0$, $b > 0$, $c > 0$.
(b) The Fibonacci sequence $f_1, f_2, f_3, f_4, \ldots, f_n, \ldots$ is defined by $f_1 = 1$, $f_2 = 1$, and if $n \geq 1$, by the recursion relation $f_{n+2} = f_{n+1} + f_n$. In other words, after the first two terms, each term of the sequence is the sum of the two previous terms. The resulting sequence is:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 \ldots.$$

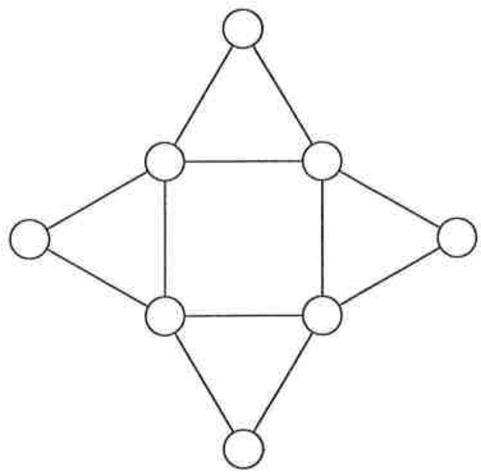
A new sequence $g_1, g_2, g_3, g_4, \ldots, g_m, \ldots$ is found by deleting the 2^{nd}, 4^{th}, 6^{th}, 8^{th}, 10^{th} terms of the Fibonaccii sequence to obtain $1, 2, 5, 13, 34, 89, 233, 610 \ldots$.

Since these are the terms: $f_1, f_3, f_5, f_7, f_9, \ldots$, it follows that $g_m = f_{2m-1}$ for $m \geq 1$.

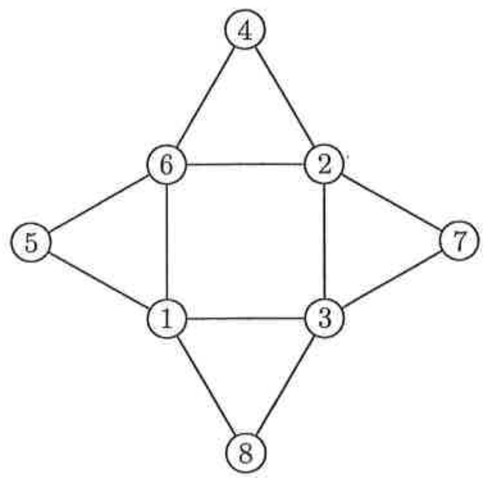
Find constants A, B so that, if $m \geq 1$, the terms g_m can be calculated from the recursion relation $g_{m+2} = Ag_{m+1} + Bg_m$.
(c) Verify that $(x, y, z) = (1, 1, 2), (1, 2, 5), (1, 5, 13), (1, 13, 34)$ are all solutions of $x^2+y^2+z^2 = 3xyz$.
(d) Verify that if $g_1, g_2, g_3, g_4, \ldots, g_m, \ldots$ is the sequence of (b), then $(1, g_m, g_{m+1})$ is a solution of $x^2 + y^2 + z^2 = 3xyz$.
(e) There is a sequence $h_1, h_2, h_3, h_4, \ldots, h_m, \ldots$ with $h_1 = 5$, $h_2 = 29$ such that for all $m \geq 1$, $h_m < h_{m+1}$ and $(2, h_m, h_{m+1})$ is a solution of $x^2 + y^2 + z^2 = 3xyz$. Find h_3, h_4, h_5, h_6, h_7.
(f) As in (c) above, there are numbers U, V so that $h_{m+2} = Uh_{m+1} + Vh_m$ for all $m \geq 1$. Find U and V.

PROBLEM 15: NUMBERING A FOUR-POINTED STAR

The diagram represents a planar graph. The small circles are called the vertices of the graph and the lines are called the edges. The connected regions into which the graph divides the plane are called the faces. For this graph there are 8 vertices, 12 edges, and 6 faces (including the outside region, called the outer face).



The objective of the problem is to number the vertices 1,2,3,4,5,6,7,8 so that the sum of the numbers in each triangular face is the same number T. In the numbering below the number T is 12.



Find all of the possible values of T so that the graph can be numbered with $1, \ldots, 8$ with the sum of the numbers in each triangular face the same number T. For each value of T give an expicit numbering to show that it is possible.