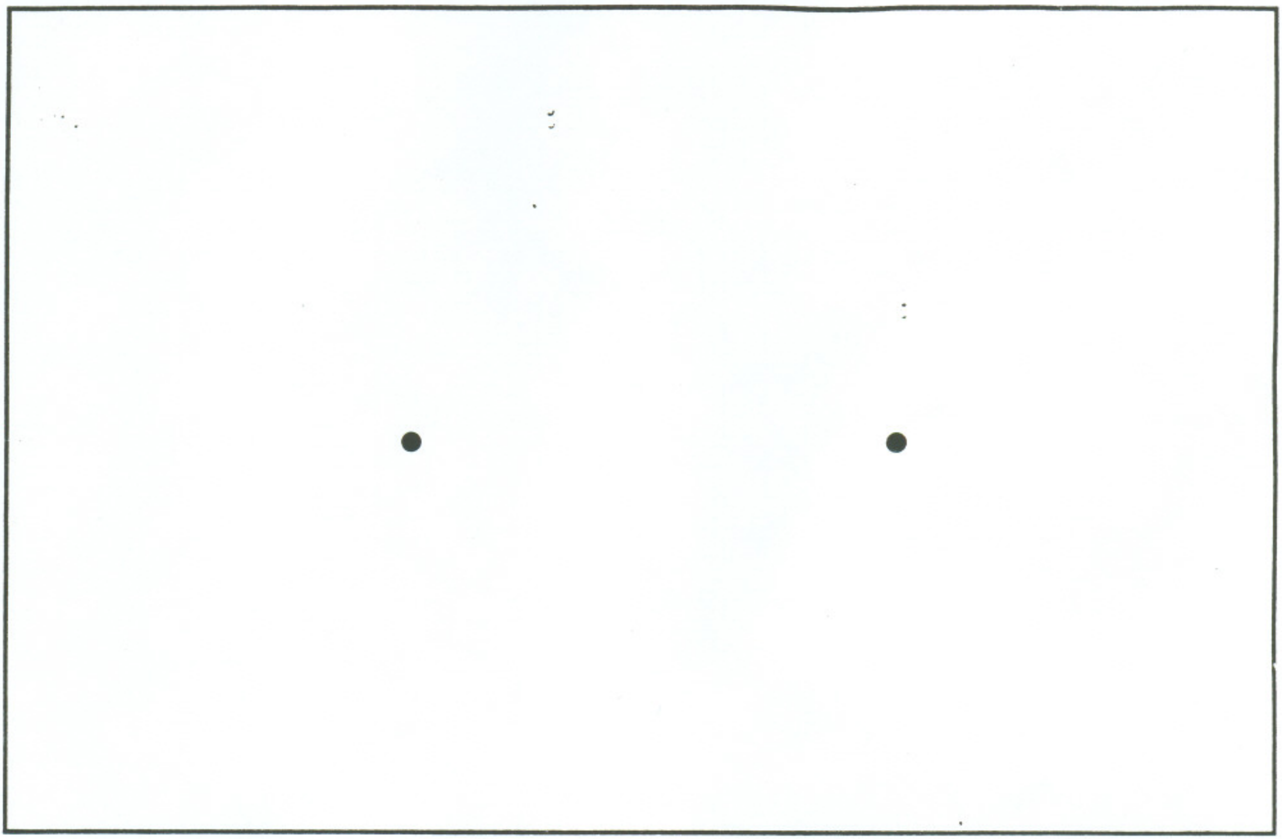
Voronoi Diagrams: an Introduction

L. Charles Biehl, The Charter School of Wilmington, DE

Activity 1

Suppose the two points on the "map" below represent the only two MacDonnelly's fast-food restaurants in the entire state (the box.) Construct an accurate boundary between them such that anyone living in the state can tell which MacDonnelly's is closer to them. The map will then define the "service region" for each of the two restaurants.



Writing assignment: Explain how you constructed this boundary and describe what geometric properties this boundary has.

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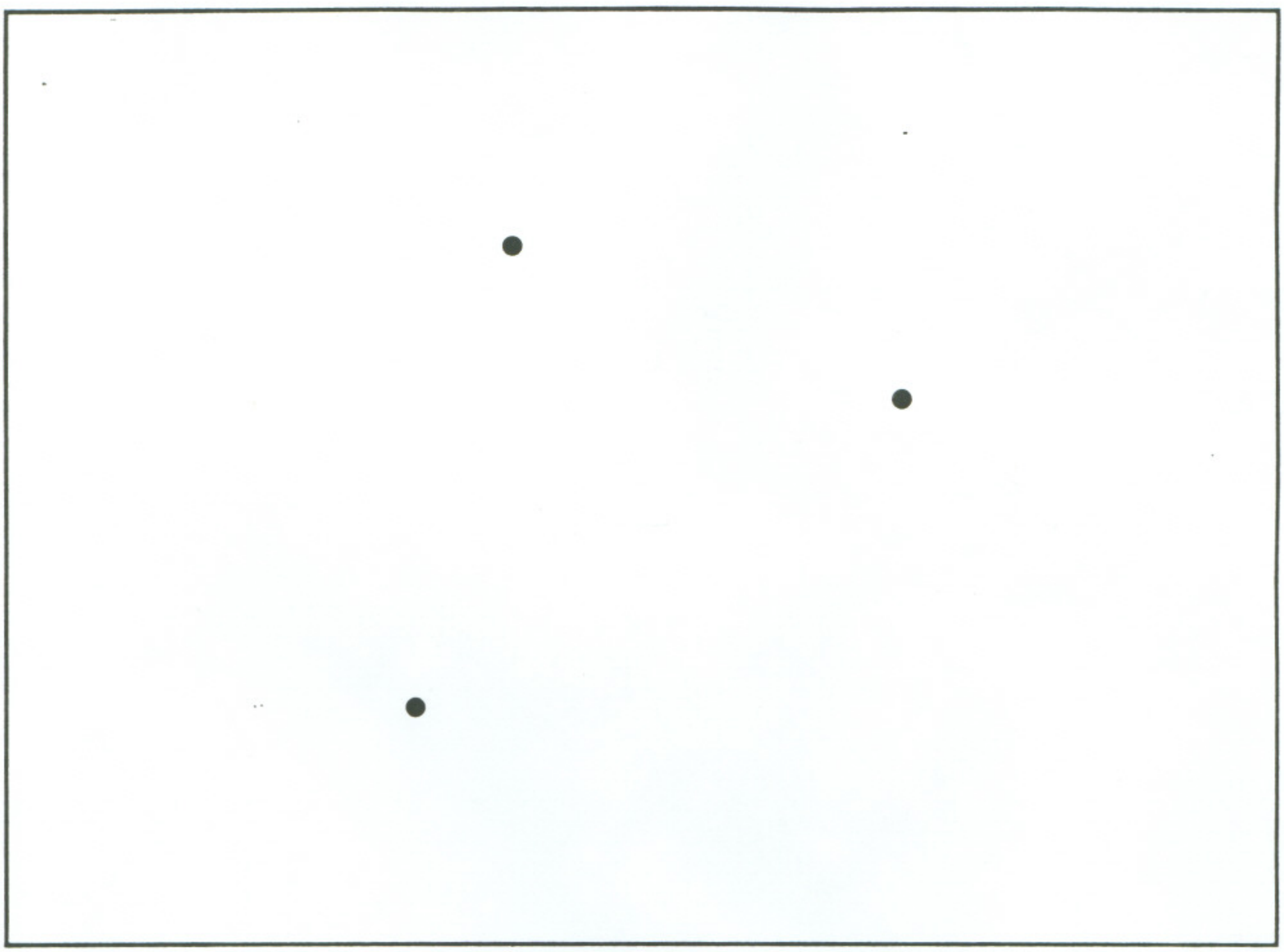
Activity 2

In Activity 1, you saw that you could define the service region for the restaurants by constructing the perpendicular bisector of the segment which connects them. Here's a slightly different problem setting:

Suppose that the three points below represent the dens of three predatory animals in a certain region (the box again.) See if you can extend what you learned in Activity 1 in order to construct the boundaries between these dens which would define the "domains", or hunting regions, for each of the animals. Assume that all of the animals have equal strength and influence, so that the perpendicular bisector of the segment between any two of them would define a boundary.



Writing assignment: How did you have to approach this problem differently from Activity 1, and how was it the same? Notice that there is a point where all the boundaries intersect. What is the significance of this point?

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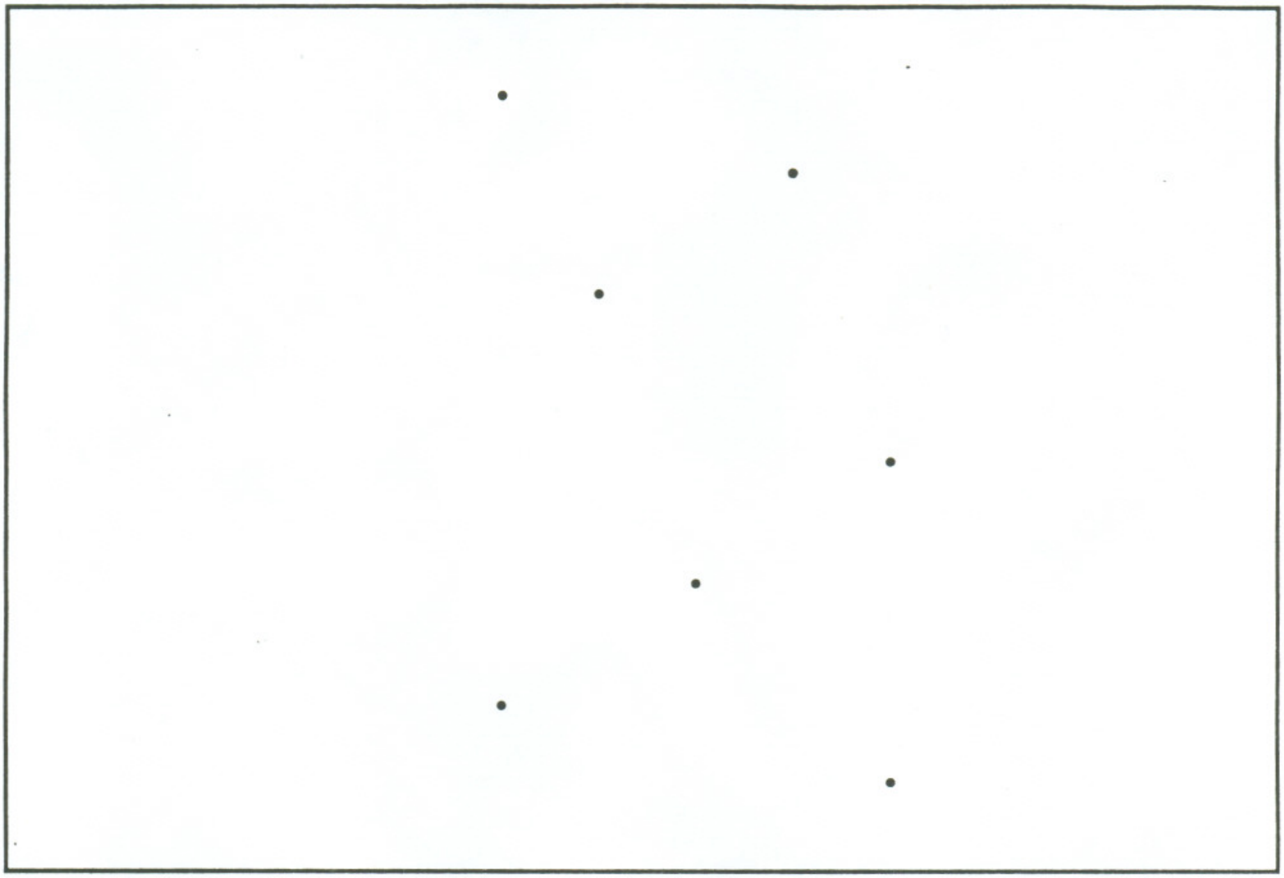
Activity 3

Constructing the domains in Activity 2 may have been a bit more challenging with three points compared to two. In this activity, use the construction skills you have learned and practiced to solve a more complicated problem.

Suppose that each point below represents a <u>nucleation site</u>, or center, for the growth of crystals on a microscope slide (the box, of course.) If each of the crystals grows in radius at the same rate, use the same process as in the previous activity to construct the boundaries as the crystals grow to touch each other.

(Be careful and take your time... accuracy is very important as the problem becomes more complicated.)

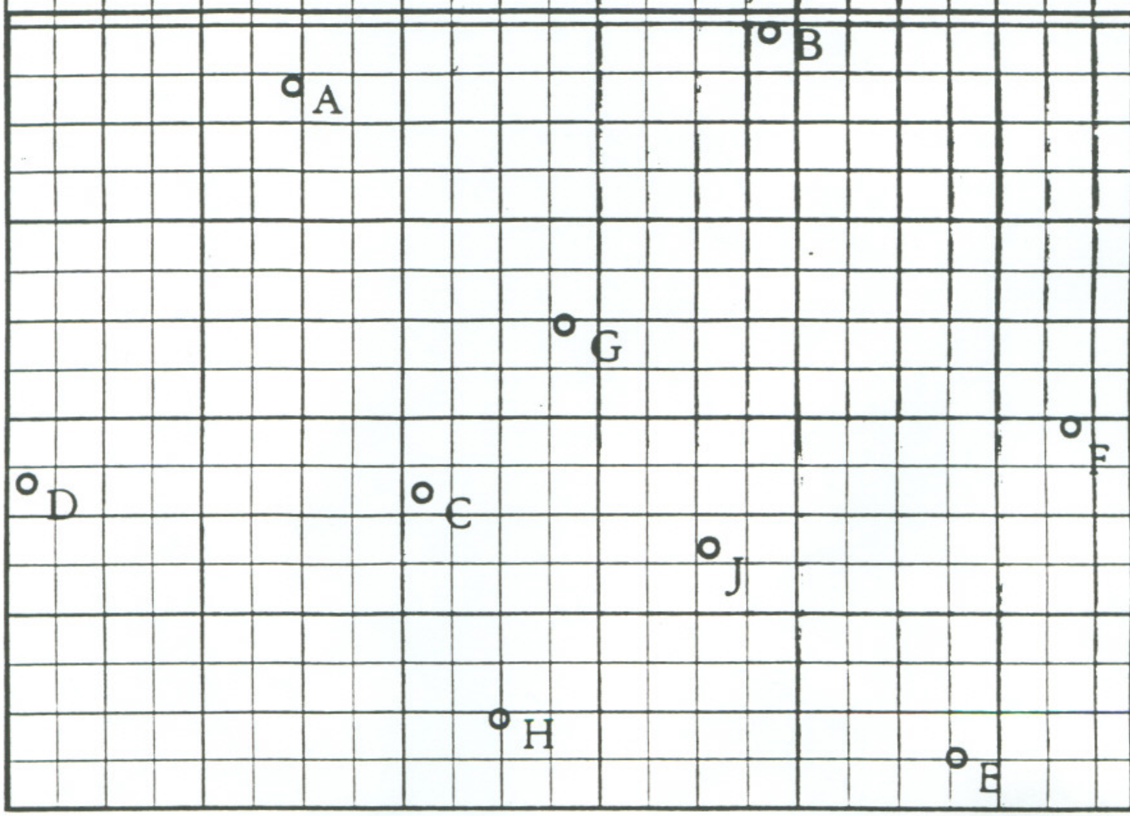


Your completed picture is called a *Voronoi Diagram.* The boundaries are called *Voronoi edges* and their intersections are called *Voronoi vertices.*

Writing assignment: Describe at least one other situation in which the Voronoi diagram would be constructed based on growth. In addition, what significant property or properties do the Voronoi vertices have?

EXERCISES

1. **Figure 13** shows a set of sites. On the worksheet provided, construct Voronoi diagram of this set, and find the largest empty circle.



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Activity 5: Extension Project

Each of the preceding activities has given you a different setting in which the Voronoi diagram shows the best way to divide a region to solve some problem. Other situations include providing public services in cities, defining areas of influence, service areas for a wide variety of businesses, and various kinds of growth. You also now have the tools by which the solution can bet determined by computation rather than construction.

An extension of these problems is called "The Greatest Empty Circle." Suppose that the 1st National Bank of Yahoo has its ATM's located as points on a map, along with their coordinates. Once the Voronoi Diagram has been computed to determine the service areas for each ATM, each Voronoi vertex is equidistant from exactly three of them, which means it is the center of a circle which contains the three ATM's. If the bank wants to determine the best place to locate a new ATM, where should it go? At the center of the greatest empty circle, of course!

First of all, determine and explain why each circle with a Voronoi vertex as its center and its three neighbors on its surface has to be empty. Then determine and explain why a new ATM should be put at the center of the greatest empty circle. Finally, take all the mathematics you have used in these activities and use it to do the following:

a. Compute the Voronoi edges for an existing set of ATM's;

b. Find the locations of the Voronoi vertices;

c. Calculate the sizes of all empty circles centered at Voronoi vertices; and

d. Determine the best location for a new ATM.