Optimal Sampling from Distributed Streams

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Joint work with S. Muthukrishnan (Rutgers)
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Reservoir sampling [Waterman '??; Vitter '85]

- Maintain a (uniform) sample (w/o replacement) of size s from a stream of n items
  - Every subset of size s has equal probability to be the sample
- When the i-th item arrives
  - With probability s/i, use it to replace an item in the current sample chosen uniformly at ranfom
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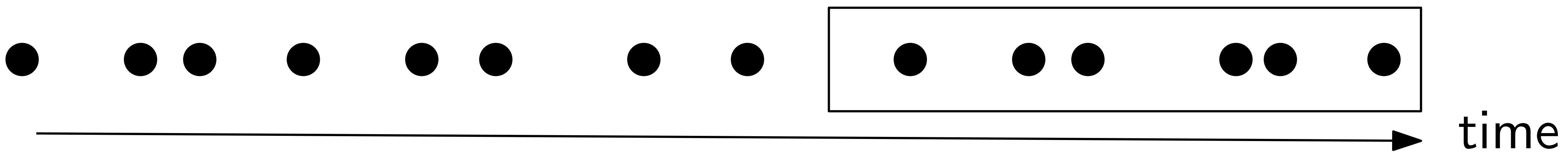
- Correctness: intuitive
- Space: $O(s)$, time $O(1)$

Sampling from a sliding window

[Babcock, Datar, Motwani, SODA'02; Gemulla, Lehner, SIGMOD'08; Braverman, Ostrovsky, Zaniolo, PODS'09]



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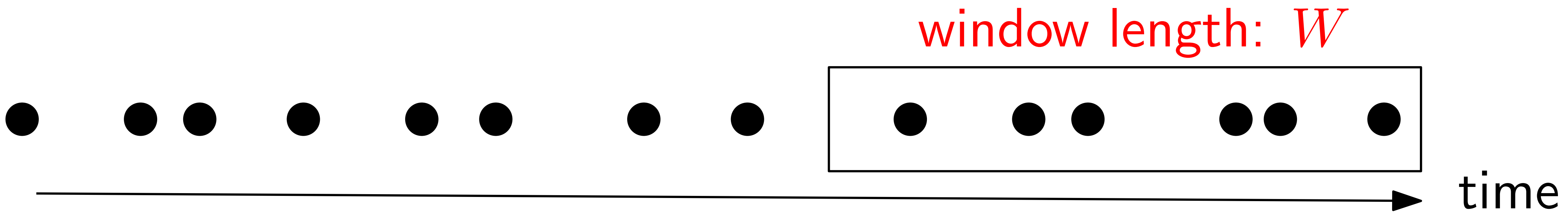

Time based window and sequence based window

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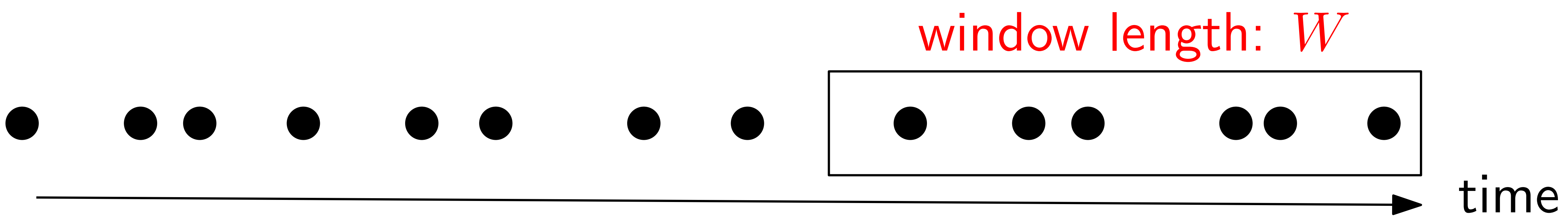
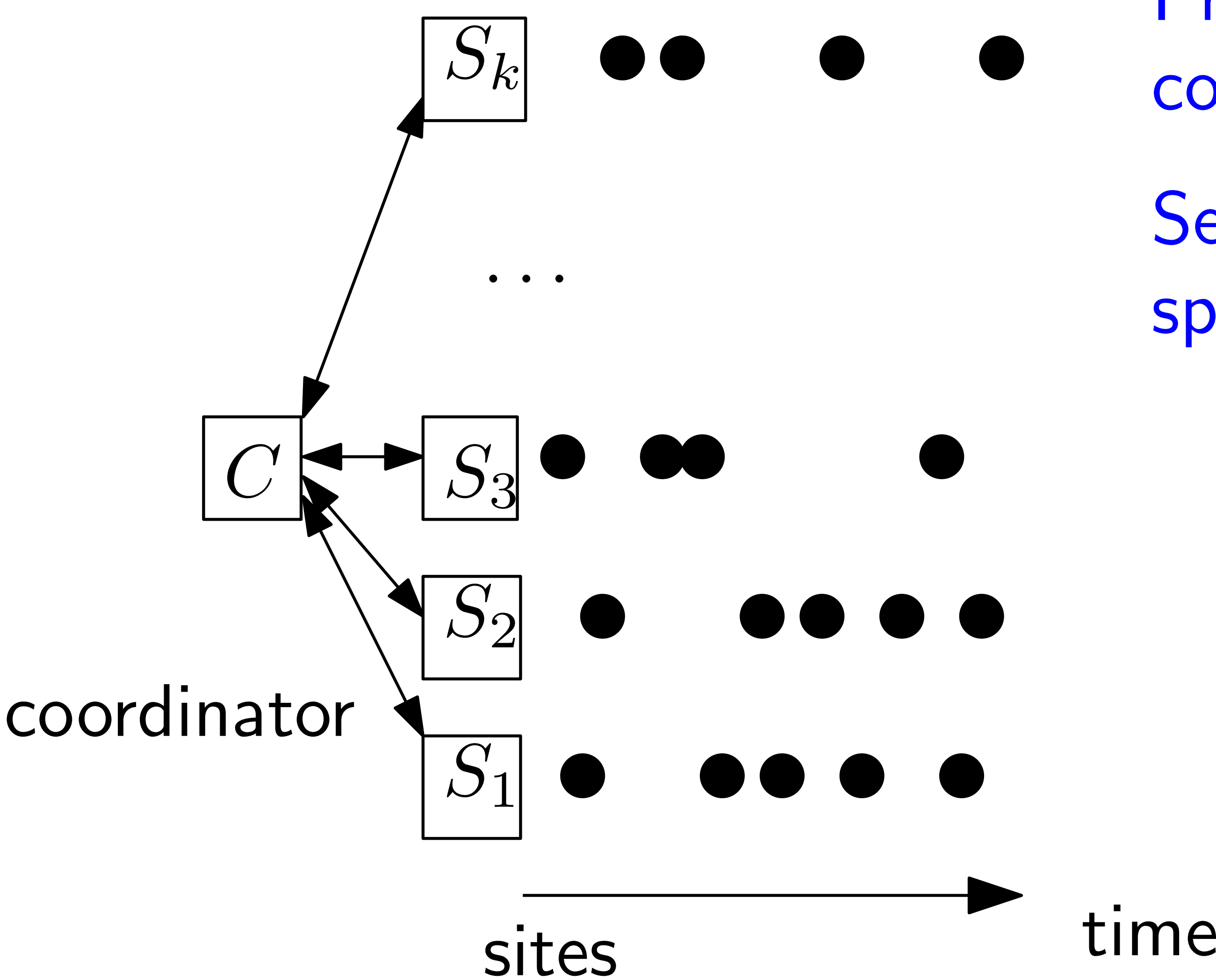


Time based window and sequence based window

- Space: $\Theta(s \log w)$
  - w: number of items in the sliding window
- Time: $\Theta(\log w)$

Sampling from distributed streams

- Maintain a (uniform) sample (w/o replacement) of size s from k streams of a total of n items



Primary goal:
communication

Secondary goal:
space/time at coordinator/site

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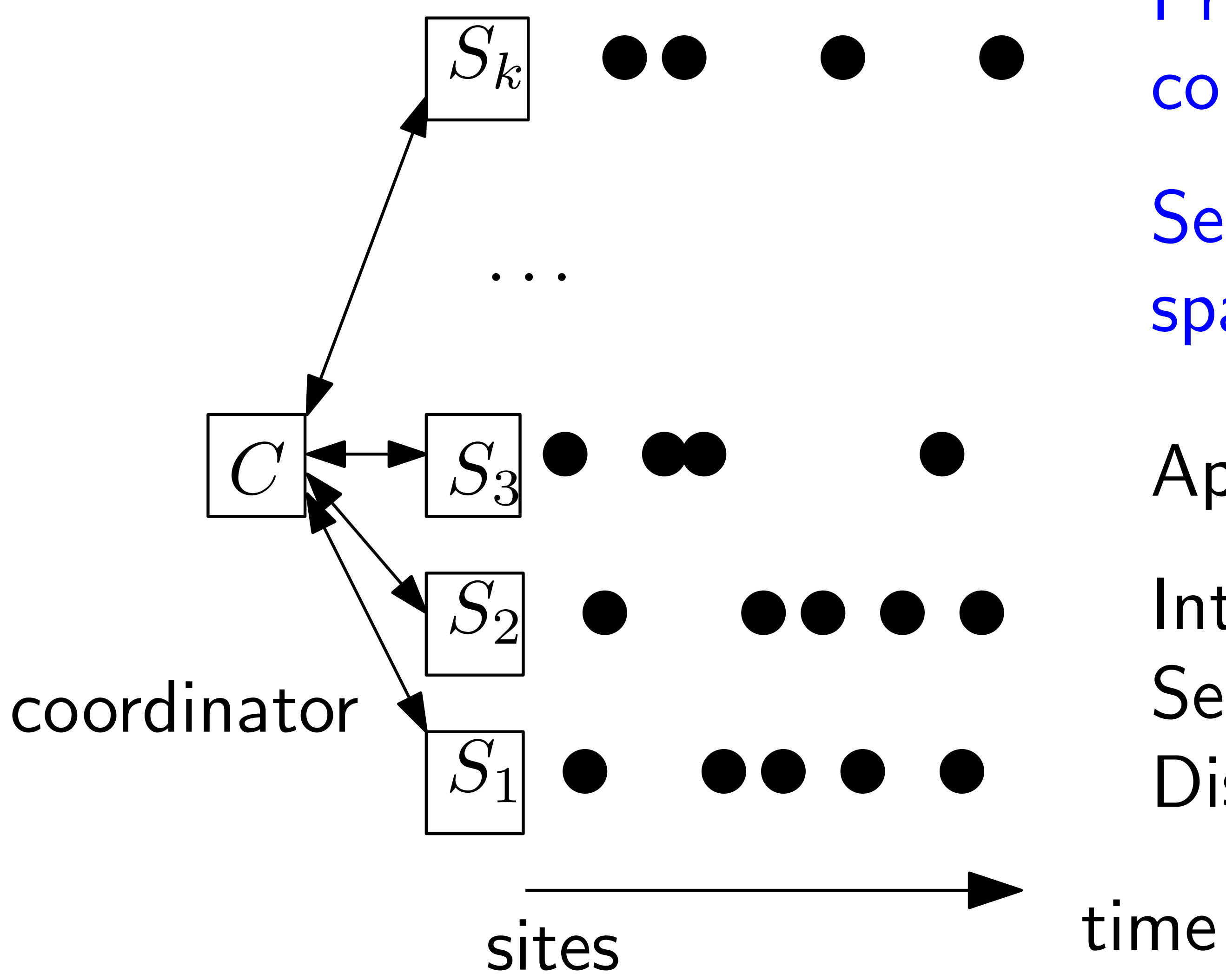

Primary goal:
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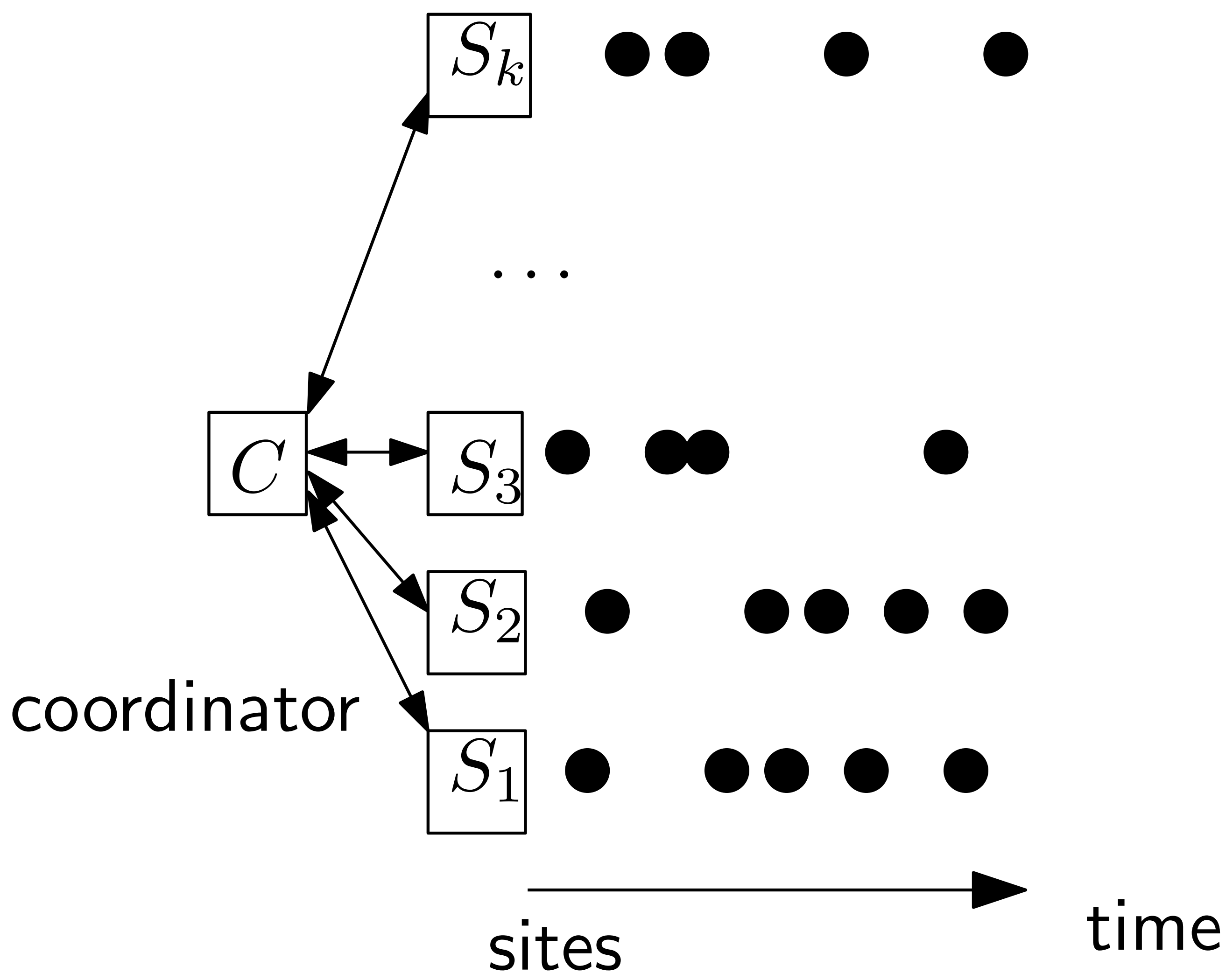
Applications:

Internet routers
Sensor networks
Distributed computing

Why existing solutions don't work

- When $k = 1$, reservoir sampling has communication $\Theta(s \log n)$



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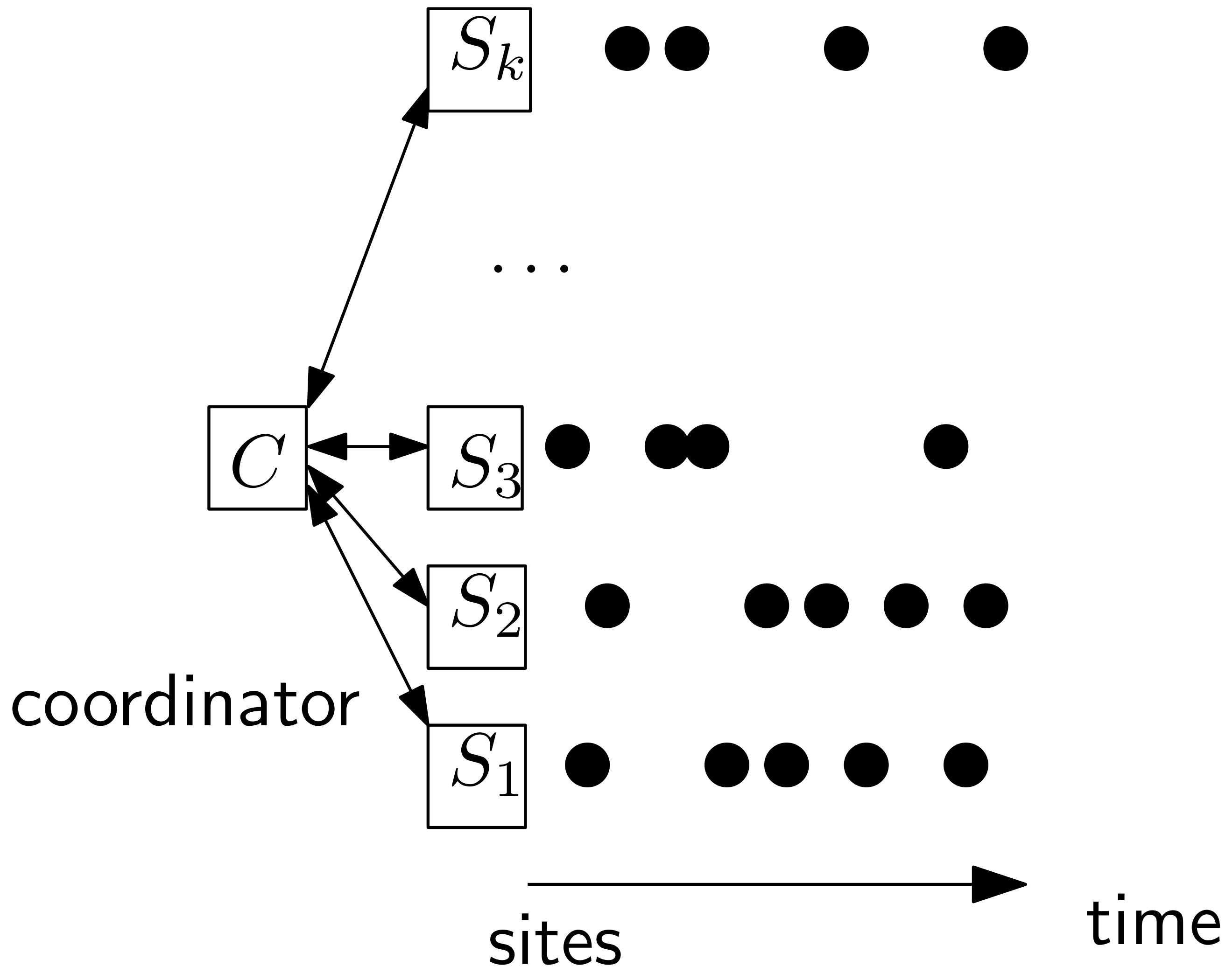
- When $k \geq 2$, reservoir sampling has cost $O(n)$ because it's costly to track i



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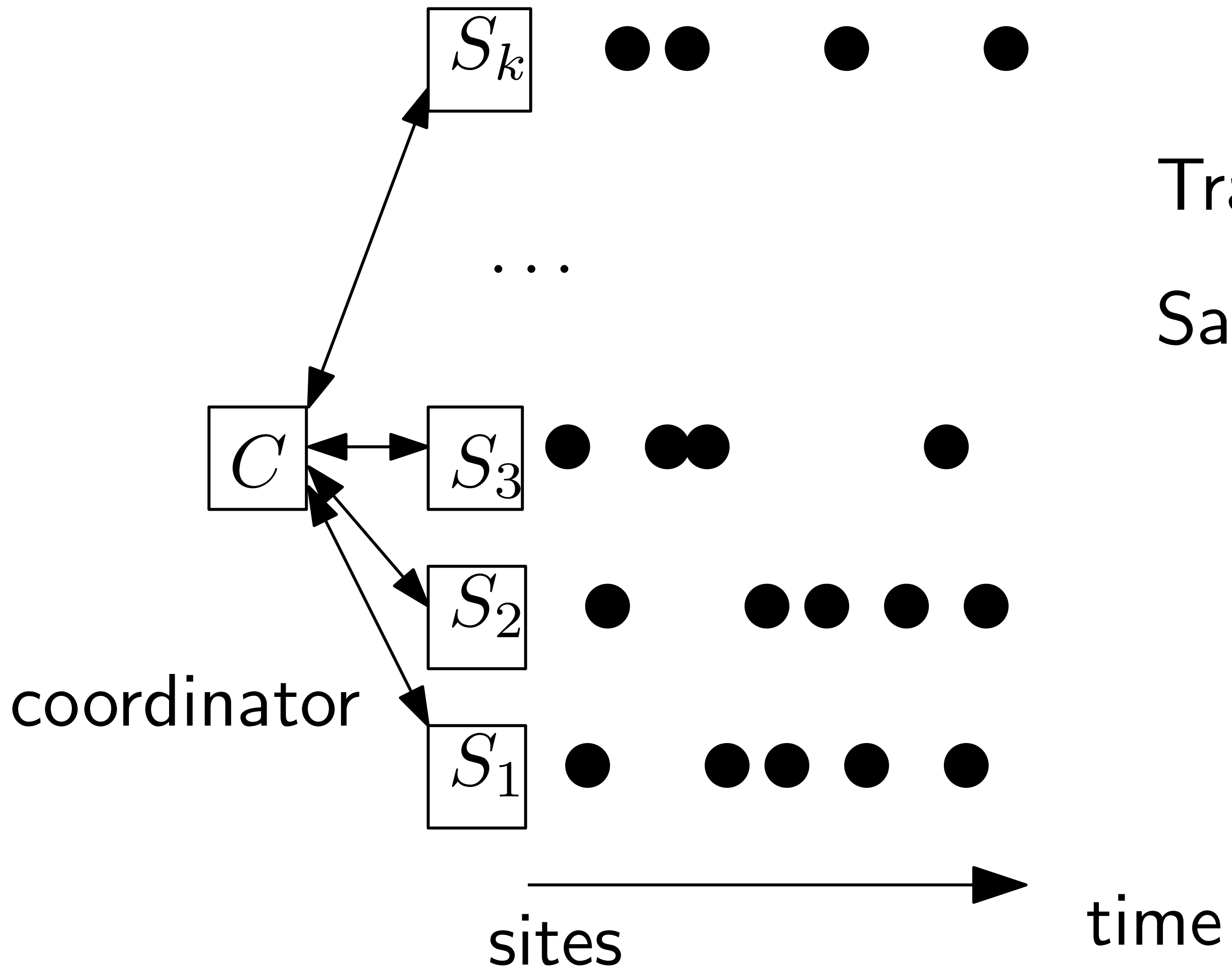

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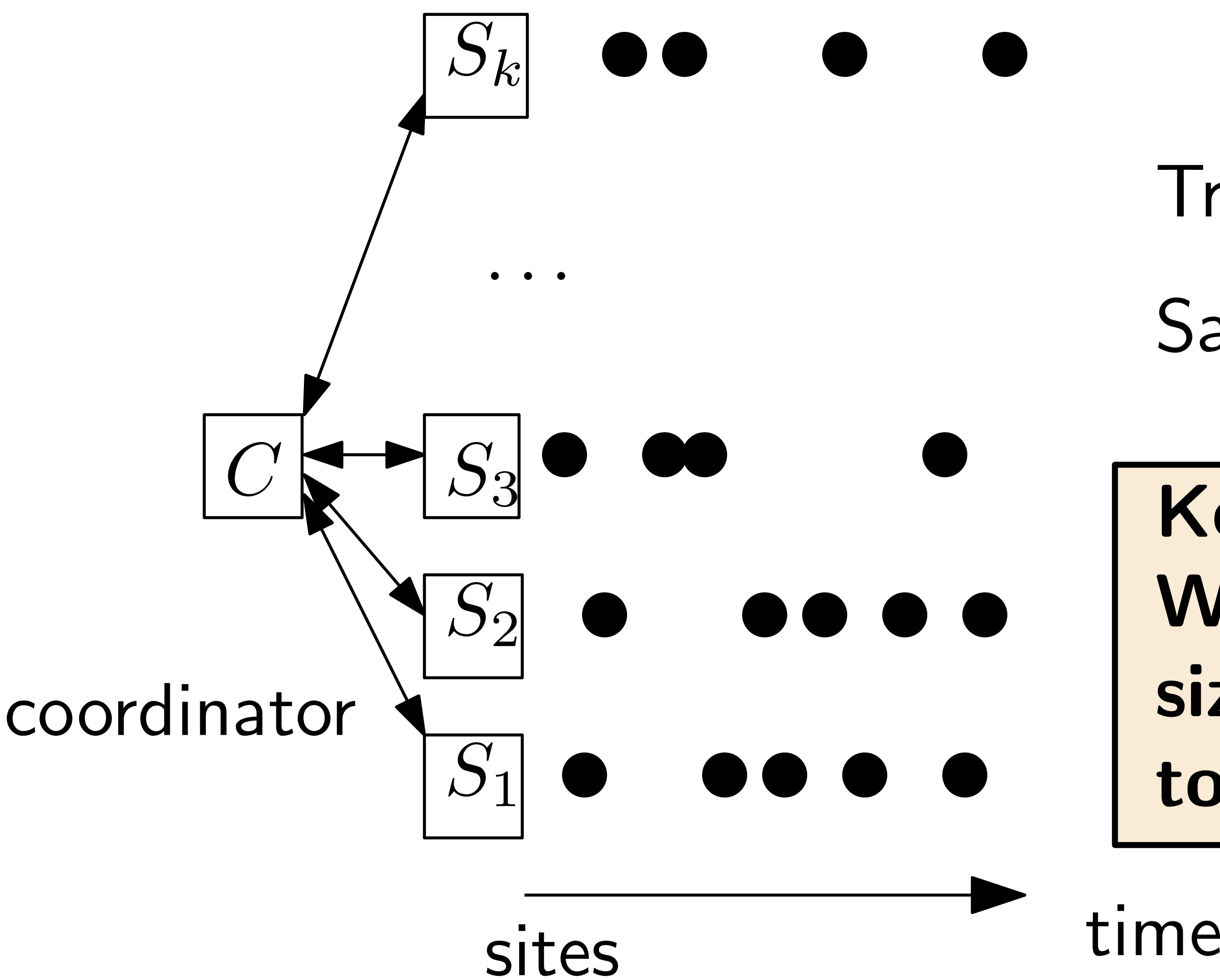

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Key observation:
We don't have to know the size of the population in order to sample!

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- Threshold monitoring, frequency moments [Cormode, Muthukrishnan, Yi, SODA'08]
- Entropy [Arackaparambil, Brody, Chakrabarti, ICALP'08]
- Heavy hitters and quantiles [Yi, Zhang, PODS'09]
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- All of them are deterministic algorithms, or use randomized sketches as black boxes

Our results on random sampling

window	upper bounds	lower bounds
infinite	$O((k+s)\log n)$	$\Omega(k+s\log n)$
sequence-based	$O(ks\log(w/s))$	$\Omega(ks\log(w/ks))$
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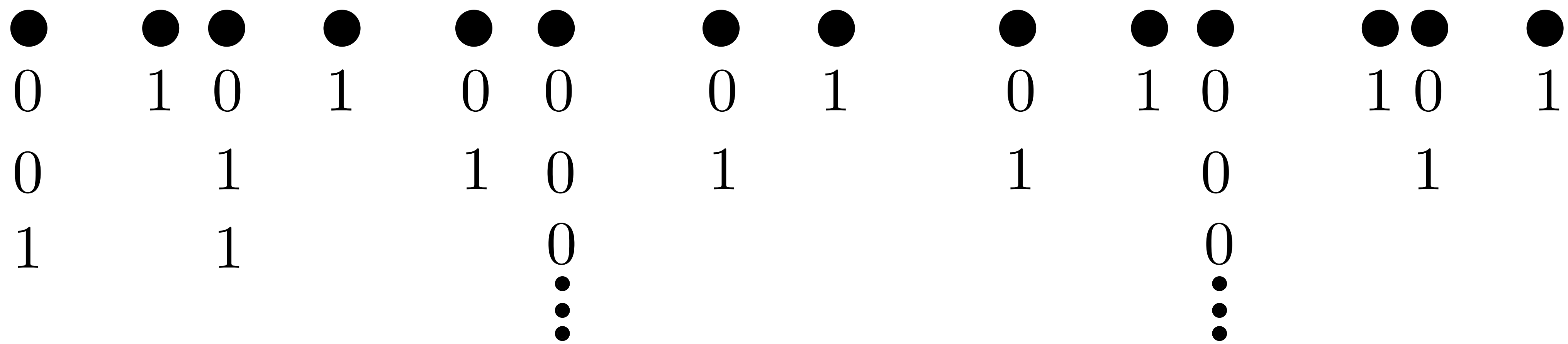

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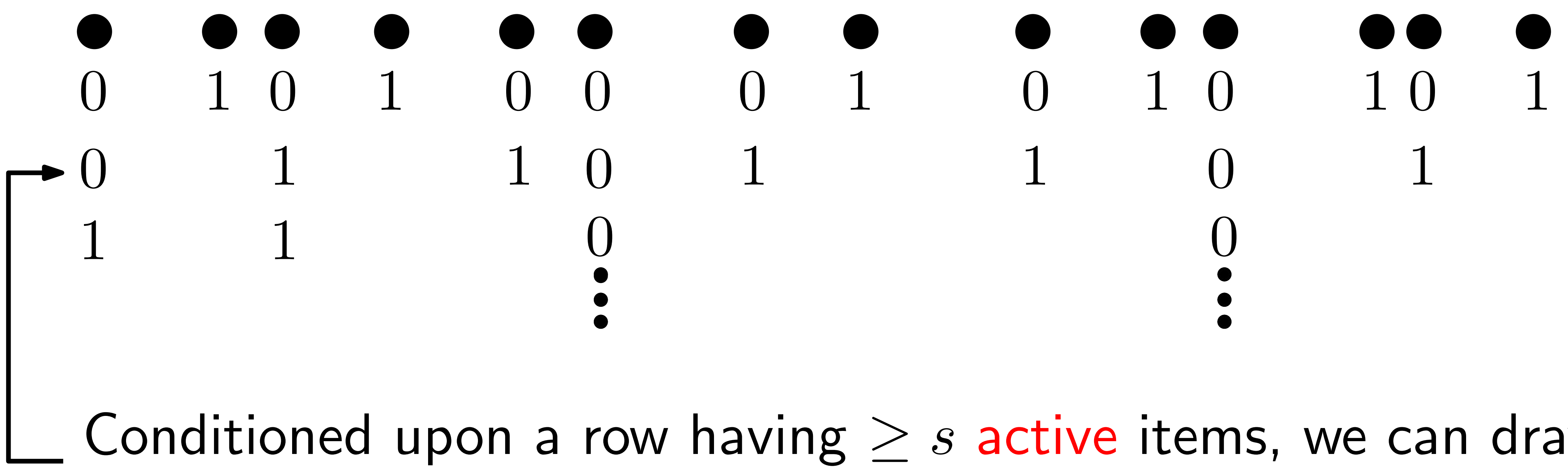

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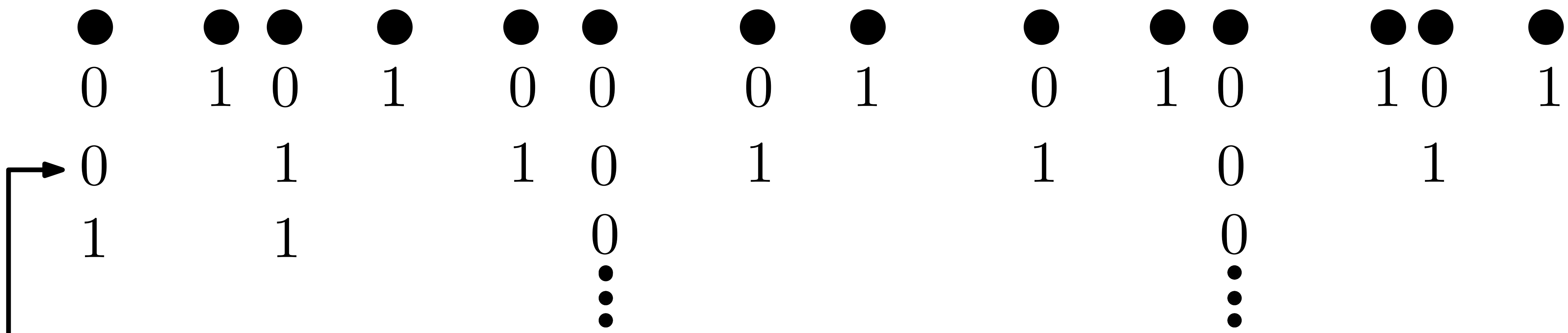

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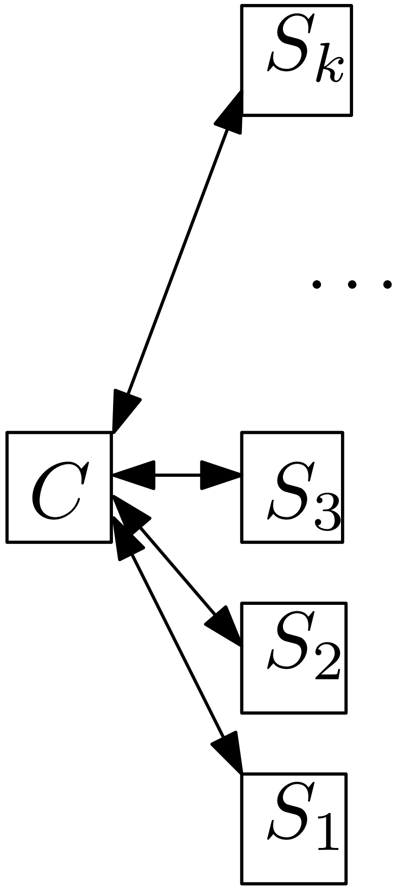
The coordinator could maintain a Bernoulli sample of size between s and $O(s)$

Sampling from an infinite window



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    (This is a Bernoulli sample with prob. 2^{-i})

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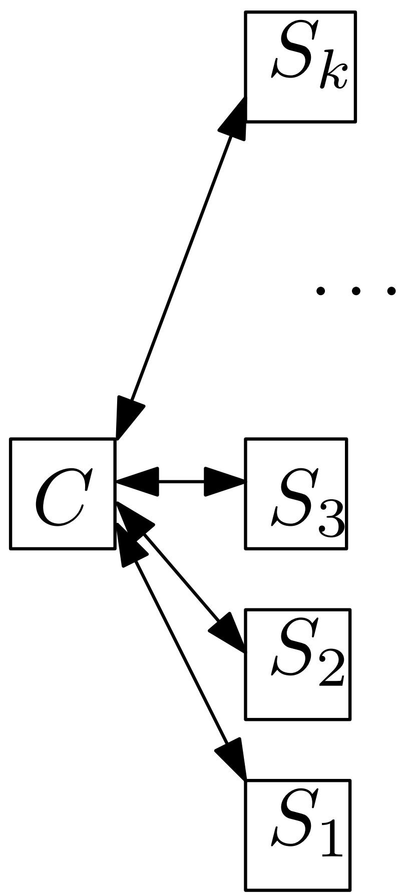

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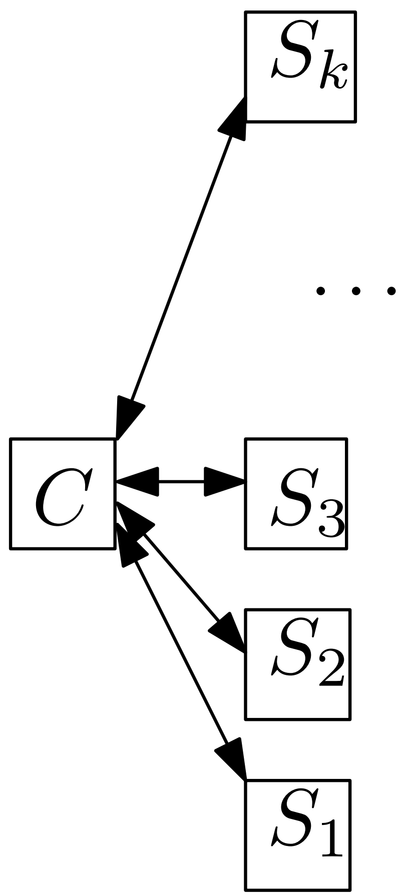

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    (The lower sample is a Bernoulli sample with prob. 2^{-i-1})
  - When the lower sample reaches size s, the coordinator broadcasts to advance to round $i \leftarrow i + 1$

    Discard the upper sample

    Split the lower sample into a new lower sample and a higher sample

Sampling from an infinite window: Analysis

- Communication cost of round i: $O(k + s)$
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  - In round i, need $\Theta(s)$ items being sampled to end round
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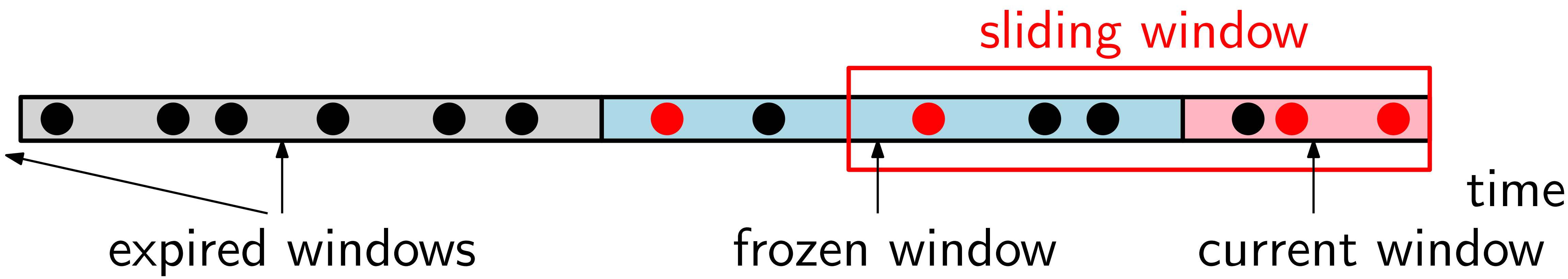
  Coordinator space: $O(s)$, total time: $O((k+s)\log n)$

Sampling from a sliding window: Idea



Sample for sliding window =
a subsample of the (unexpired) sample of frozen window +
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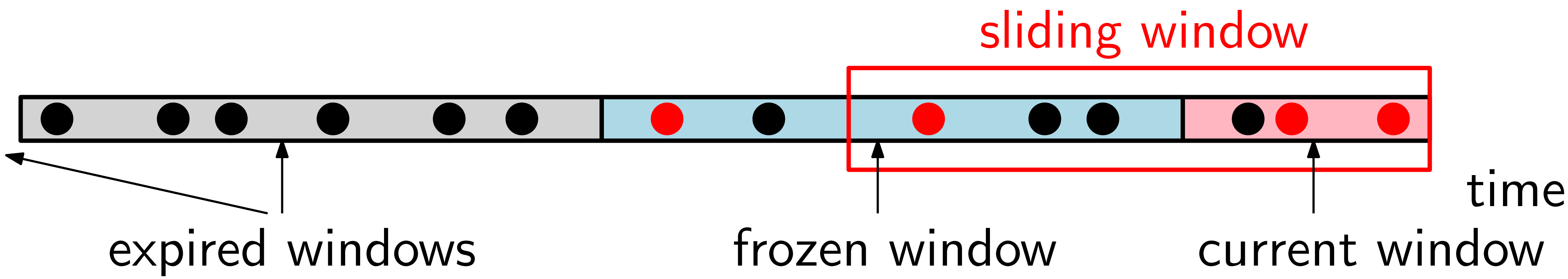

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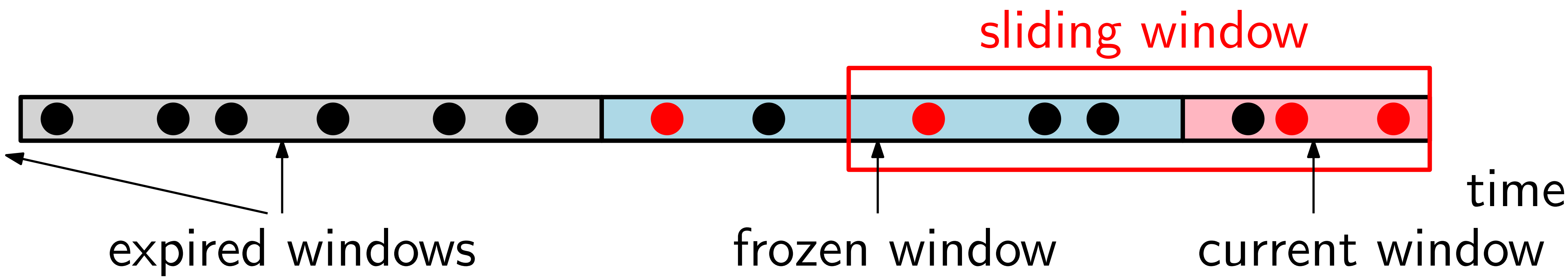

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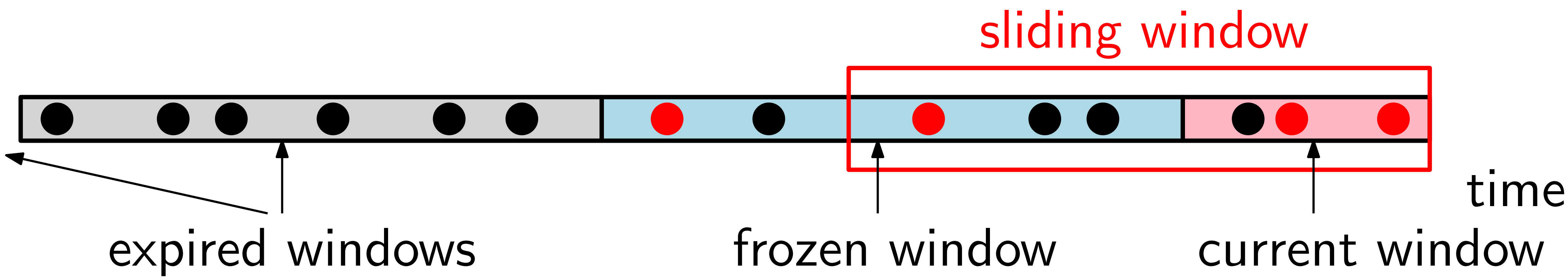

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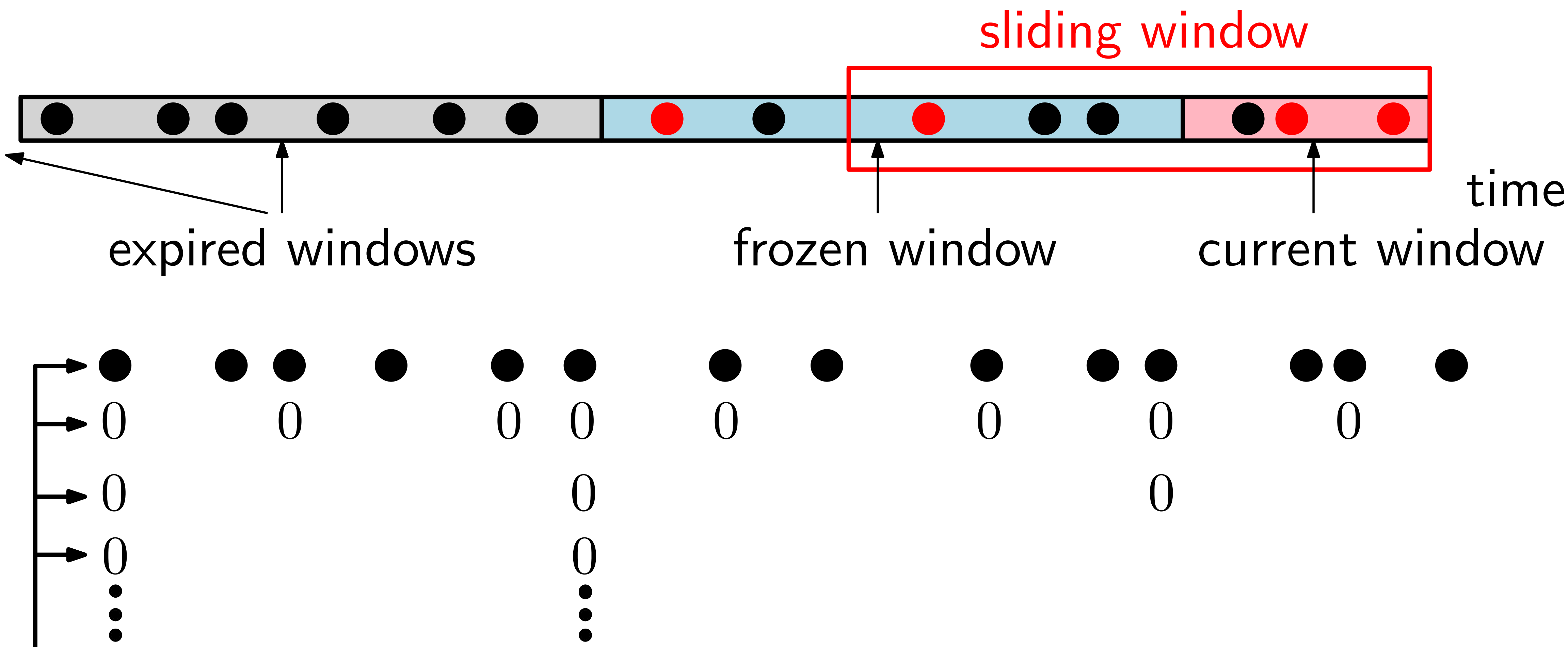
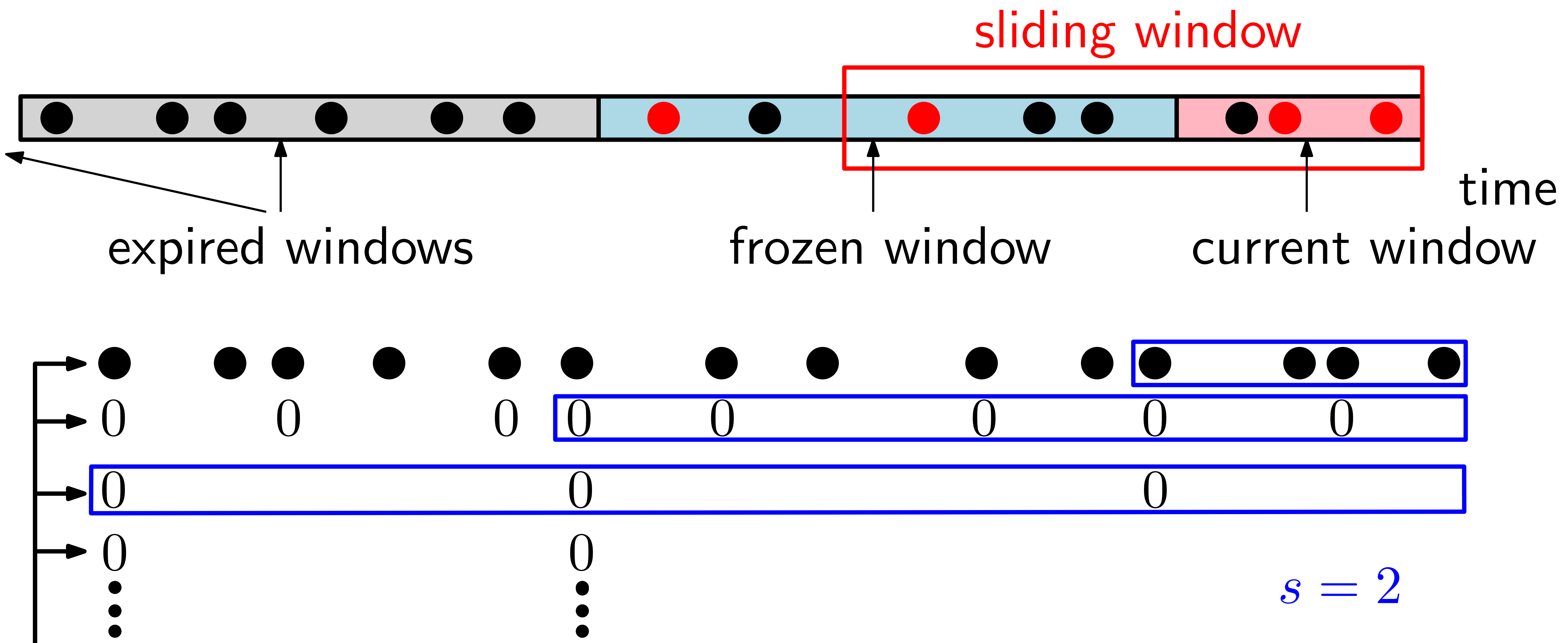
- Frozen window: Need to have the same

Dealing with the frozen window



Keep all the levels? Need $O(w)$ communication

Dealing with the frozen window



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Keep most recent sampled items in a level until s of them are also sampled at the next level. Total size: $O(s \log w)$

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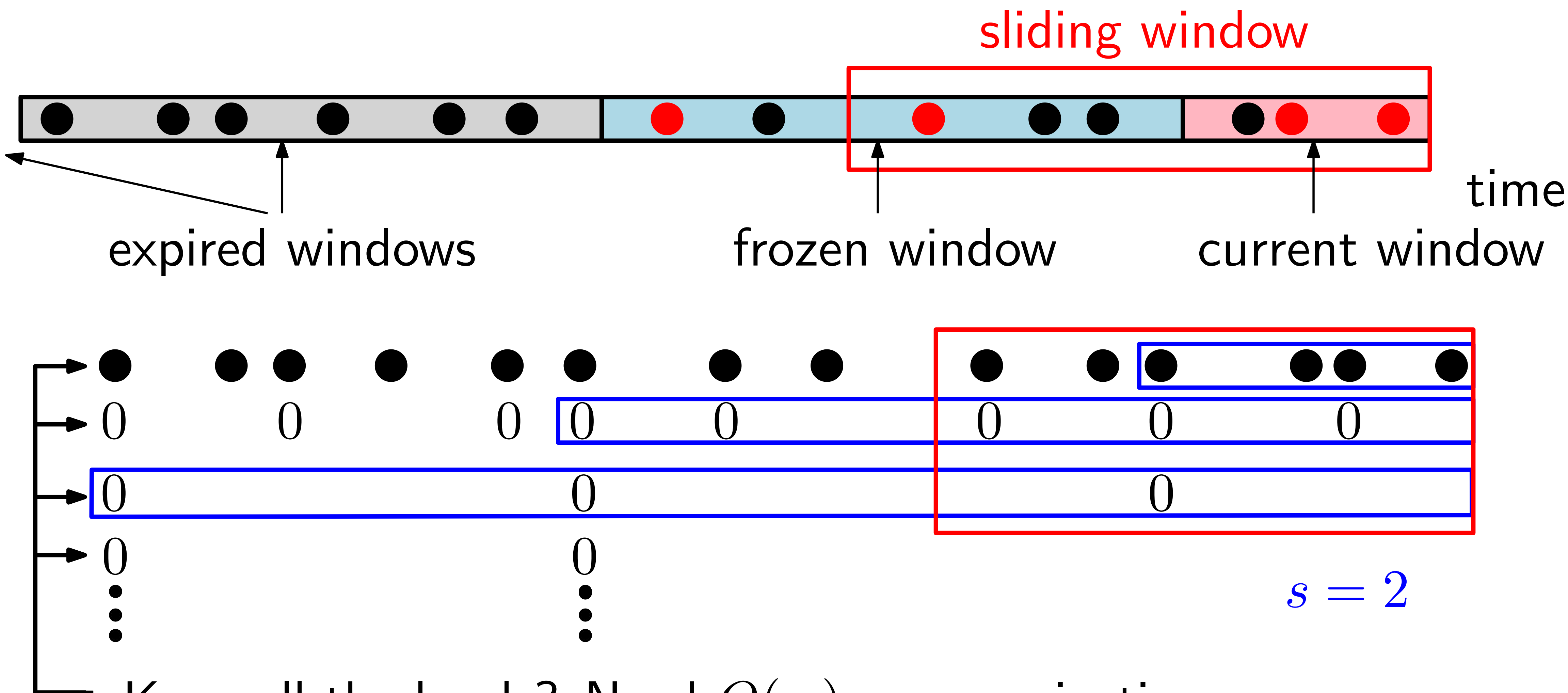

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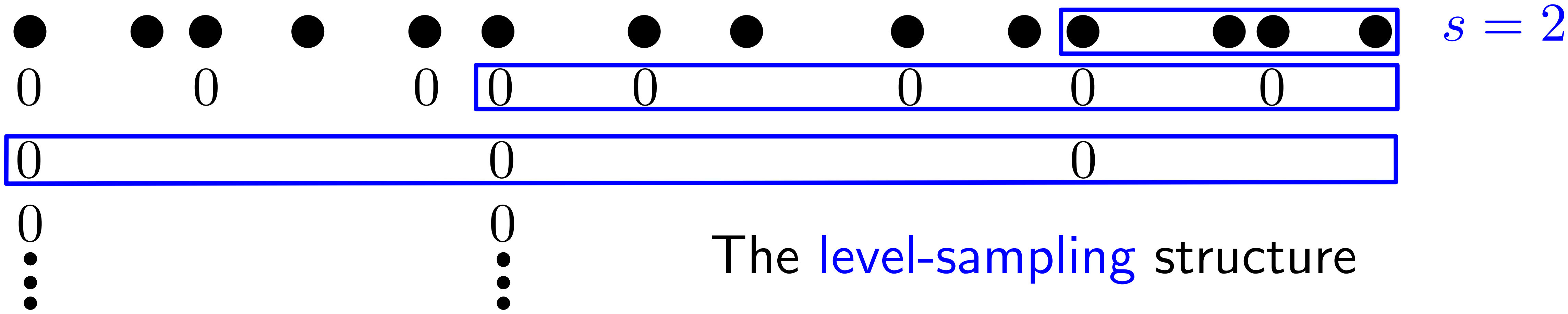
Guaranteed: There is a blue window with $\geq s$ sampled items that covers the unexpired portion of the frozen window

Dealing with the frozen window: The algorithm



The level-sampling structure

- Each site builds its own level-sampling structure for the current window until it freezes
  - Needs $O(s \log w)$ space and $O(1)$ time per item
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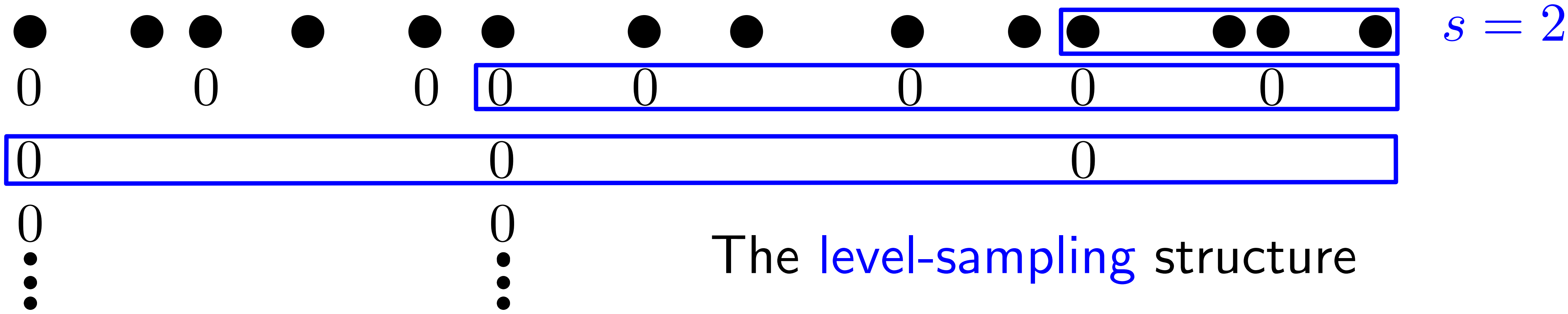

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  - Needs $O(s \log w)$ space and $O(1)$ time per item
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- When the current window freezes
  - For each level, do a k-way merge to build the level of the global structure at the coordinator
    Total communication $O((k + s) \log w)$

Future directions

- Applications
  - Heavy hitters and quantiles can be tracked in $\tilde{O}(k + 1/\epsilon^2)$
    Beats deterministic bound $\tilde{\Theta}(k/\epsilon)$ for $k \gg 1/\epsilon$
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- Is random sampling the best way to solve these problems?
  - New result: Heavy hitters and quantiles can be tracked in $\tilde{O}(k + \sqrt{k}/\epsilon)$, using a different sampling method

The End

THANK YOU

Q and A