Exponentially Decayed Aggregates on Data Streams

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Data Stream Computations

- Streams of updates must be processed in single pass
  - IP traffic measurements, stock feeds, sensor readings
- Need to mine holistic aggregates
  - Medians (quantiles), frequent items
- Recent updates more important than older data
  - Weight updates based on a function of age
- Need to keep small summaries, give accurate answers
  - Much work on sketches, summaries without decay

Exponential Decay

- Give items a weight that depends exponentially on age
  - Age a has weight $\exp(-\lambda a)$, for parameter λ
- Given a stream of timestamps t_i, easy to compute decayed count
  - Track current decayed count C, last update t
  - Given t_i, $C \leftarrow C * \exp(-\lambda(t_i - t)) + 1$
    $t \leftarrow t_i$
  - Generalizes to exponentially decayed sum
- We study more complex (holistic) aggregates, show that there are fast, small solutions for these as well.

Aggregates of Interest

- Streaming model: Given stream of $\langle x_i, w_i, t_i \rangle$ tuples
  - x_i is an item, w_i its weight, t_i its timestamp
  - E.g. IP flow, weight in bytes, start time
  - Total weight at time t is $D(t) = \sum_i w_i \exp(\lambda(t - t_i))$
- ϕ-Heavy Hitters (under exponential decay)
  - Find items x so that $\sum_{x_i = x} w_i \exp(\lambda(t-t_i)) > (\phi \pm \varepsilon) D(t)$
- ϕ-Quantiles (under exponential decay
  - Find q so that $(\phi - \varepsilon)D \leq \sum_{x_i \leq q} w_i \exp(\lambda(t-t_i)) \leq (\phi + \varepsilon)D$
- $\lambda=0 \Rightarrow$ no decay
  - Same as standard approximate heavy hitters/quantiles

Decayed Heavy Hitters

Algorithm 1.1: HeavyHitterUpdate(x_i, w_i, t_i, λ)

Input: item x_i, timestamp t_i, weight w_i, decay factor λ
Output: Current estimate of item weight
if $\exists j.\,\text{item}[j] = x_i$;
  then $j \leftarrow \text{item}^{-1}(x_i)$
  else $j \leftarrow \arg\min_k(\text{count}[k])$;
$\text{item}[j] \leftarrow x_i$;
$\text{count}[j] \leftarrow \text{count}[j] + w_i \exp(\lambda t_i)$
return $(\text{count}[j] * \exp(-\lambda t_i))$

- We extend the "SpaceSaving" algorithm of [MAA05]
  - Keep $k = 1/\varepsilon$ items and counters
  - If next item is stored, update its count with $+ w_i \exp(\lambda\, t_i)$
  - Else, overwrite the stored item with smallest count
  - On output, scale stored counts by $\exp(-\lambda\, t)$

SpaceSaving Analysis

- Smallest counter value, min, is at most $\varepsilon D \cdot \exp(t)$
  - Counters sum to $N = D(t) \exp(t)$ by induction
  - $1/\varepsilon$ counters, so avg is εN: smallest cannot be bigger
- True count of an uncounted item is between 0 and min
  - Proof by induction, true initially, min increases monotonically
  - Hence, the count of any item stored is off by at most εN
- Any item x whose true count $> \varepsilon N$ is stored
  - By contradiction: x was evicted in past, with count $\leq min_t$
  - Every count is an overestimate, using above observation
  - So est. count of $x > \varepsilon N \geq min \geq min_t$, and would not be evicted

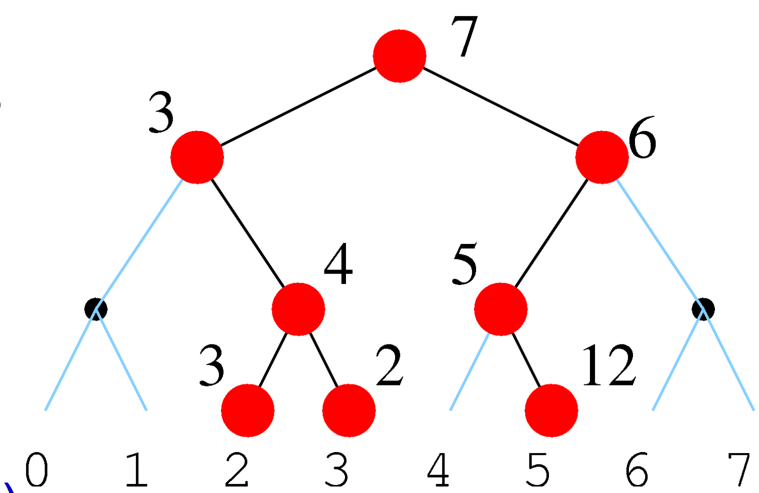
So: Find all items with count $> \varepsilon N$, error in counts $\leq \varepsilon N$

Heavy Hitters Result

- Algorithm finds ε-approximate exponentially decayed heavy hitters in space $O(1/\varepsilon)$, update time $O(\log 1/\varepsilon)$.
  - Time cost: keep a standard heap to allow find minimum
  - Index items with efficient dynamic tree structure (deterministic) or hash table (randomized)
- Space and time costs asymptotically the same as the non-decayed version.

Decayed Quantiles

- Q-digest [SBAS05] summarizes distribution over fixed domain U
- Tracks nodes and counts in binary tree over domain
- Ensures that non-leaf nodes have small counts $(\leq \varepsilon N / \log U)$
- Ensures that counts of parents + two children $> \varepsilon N/\log U$



- Guarantees any quantile can be found with εN error
- Space used is bounded by $O(\log U/\varepsilon)$

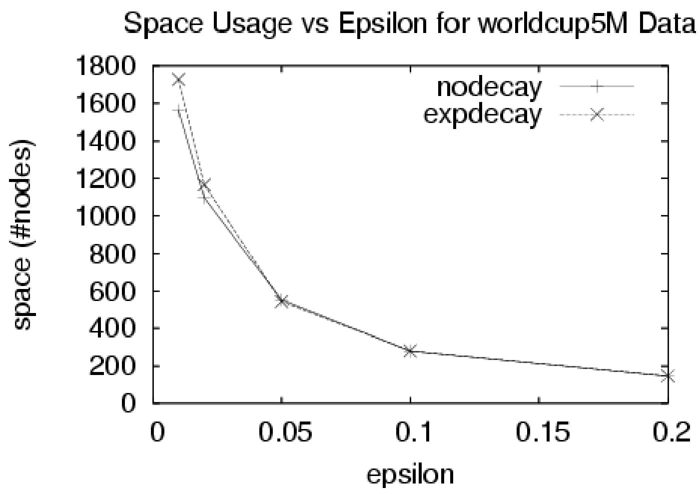
Extended Q-digest

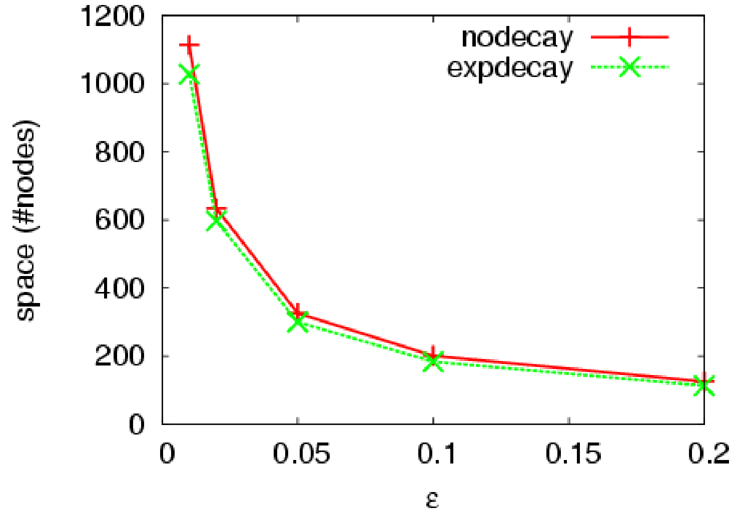
- Replace counters by exponentially decayed counts
- Observe that for the Q-digest:
  - updates can be arbitrary fractional values
  - multiplying all counts by γ gives a summary of input multiplied by γ
- Hence: sum of all counts is $D(t)$, no count $> \varepsilon D/\log U$ etc.
- Careful analysis shows that we can estimate the rank of any item using the stored counts
  - Error arises from counts of ancestor nodes from leaf to root
  - Total error is bounded by $\varepsilon\ D(t)$, as required

Quantiles Result

- Can find ε-approximate decayed quantiles in space $O(1/\varepsilon \log U)$
- Time per update $O(\log \log U)$, queries take $O(\log U/\varepsilon)$
  - Updates are fast based on a ‘lazy’ update procedure: Don’t decay all counts every update, only those affected by the update
- Space and time independent of λ.
  - Same space as non-decayed version
  - Slightly slower because of count maintenance

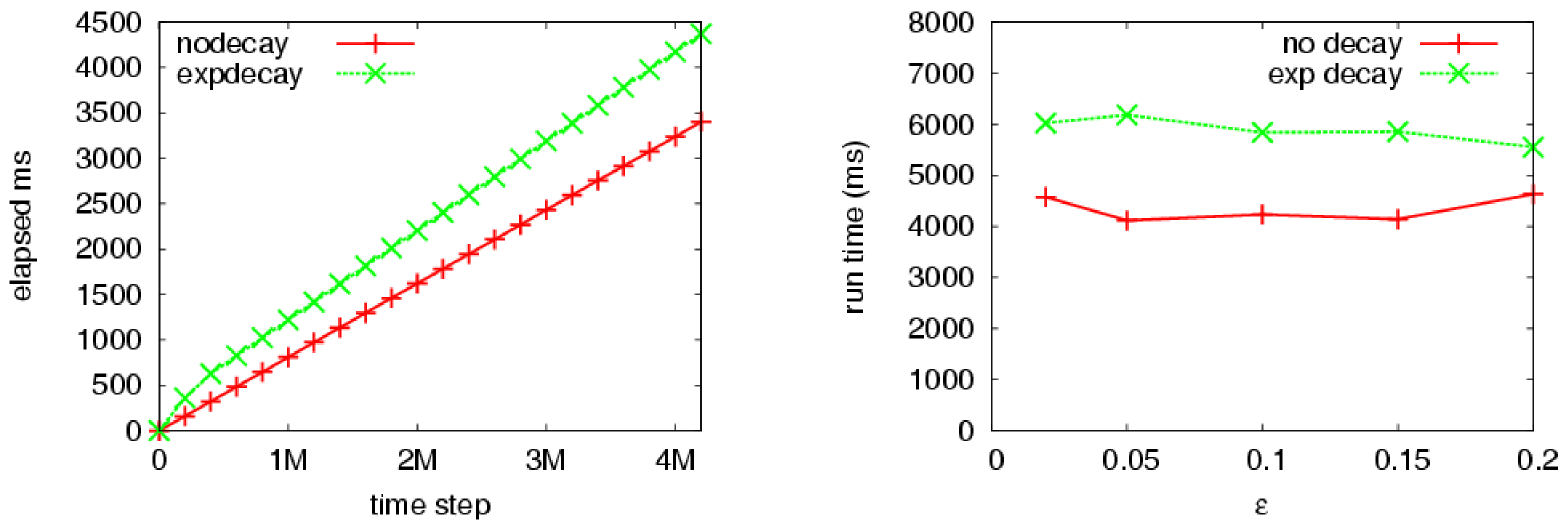
Experimental Evaluation





- Implemented q-digest in C, experimented on 5 million World Cup requests and 5 million IP flow records
- Space cost: almost identical to non-decayed space

Experimental Throughput



- Throughput: about 70-80% as fast as undecayed version
- Processing about 800K updates per second

Conclusions

- Quantiles and frequent items under exponential decay
  - Cost is very close to that for no decay
  - Adapt existing algorithms to handle decayed counts
  - Extend analysis to show correctness and throughput

- Other decay functions:
  - Polynomial decay, logarithmic decay, sliding window
  - Harder, even for simple sums and counts [CS03]
  - New algorithms for aggregates given in [CKT08], in PODS