

# Data Science and Privacy Preservation

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# Schedule

- ◆ **Part 1 (today):** Centralized privacy models
  - The Privacy Problem
  - Syntactic Approaches to Privacy (1998 onwards)
  - (Centralized) Differential Privacy (2006 onwards)
- ◆ **Part 2 (tomorrow):** Local privacy models (2014 onwards)
  - Local Differential Privacy technical foundations
  - Current directions and open problems
- ◆ **Note:** This material can be quite technical and mathematical!
- ◆ Slides available from <http://cormode.org/ghent>

# Why Privacy?

- ◆ Data subjects have inherent right and expectation of privacy
  - A lot of new data gives detailed descriptions of people’s behaviour
- ◆ “Privacy” is a complex concept
  - What exactly does “privacy” mean? When does it apply?
  - Could there exist societies without a concept of privacy?
- ◆ Concretely: at collection “small print” outlines privacy rules
  - Most companies have adopted a **privacy policy**
  - E.g. Facebook privacy policy [facebook.com/policy.php](https://www.facebook.com/policy.php)
- ◆ Significant legal framework relating to privacy
  - UN Declaration of Human Rights
  - EU General Data Protection Regulation (GDPR)
  - US: HIPAA, Video Privacy Protection, Data Protection Acts

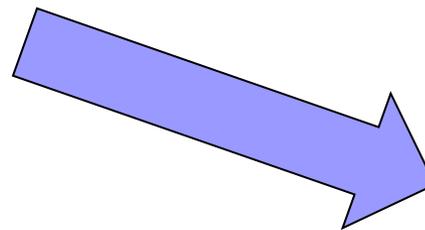
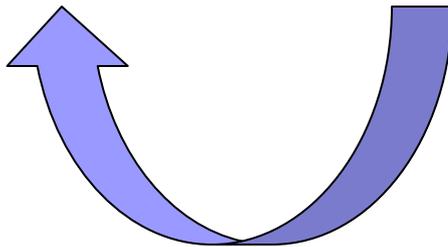
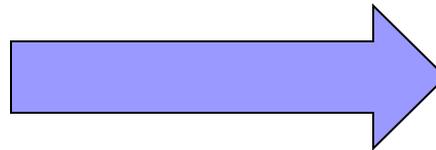
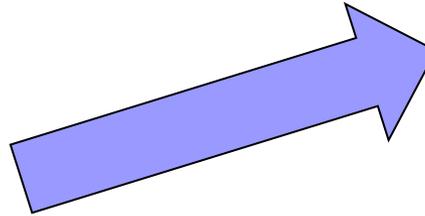


# The Privacy Problem



- ◆ Goals for privacy in companies and cities:
  - Enable appropriate use of data while protecting customers
  - Keep CTO/minister off front page of the newspapers!
- ◆ **Security is binary\***: allow access to data **iff** you have the key
  - Encryption is robust, reliable and widely deployed
- ◆ **Privacy comes in many shades**:  
reveal some information, disallow unintended uses
  - Hard to control what may be inferred
  - Possible to combine with other data sources to breach privacy
  - Privacy technology is still maturing

# The data release scenario



# Why Anonymize?

## ◆ For **Data Sharing**

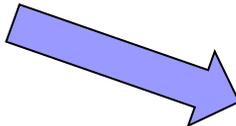
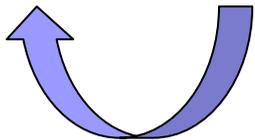
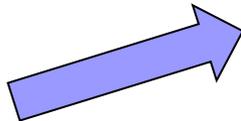
- Give real(istic) data to others to study without compromising privacy of individuals in the data
- Allows third-parties to try new analysis and mining techniques not thought of by the data owner

## ◆ For **Data Retention and Usage**

- Various requirements prevent companies from retaining customer information indefinitely
- E.g. Google progressively anonymizes IP addresses in search logs
- Internal sharing across departments (e.g. billing → marketing)

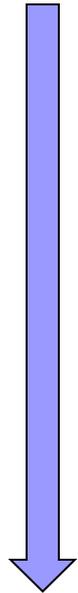
# Dimensions to consider

- ◆ How much **privacy** do we need?
- ◆ How much **utility** do we want from the anonymized data?
- ◆ How will data be accessed: as data feed, as data set, via API?



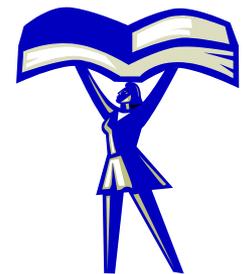
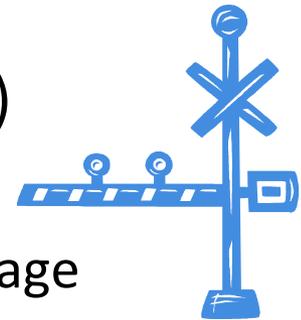
Who will use the data?

1. Permanent employees  
Temporary employees  
(students, contractors)
2. External organizations  
Data purchasers
3. General Public



# Models of Anonymization

- ◆ **Interactive Model** (akin to statistical databases)
  - Data owner acts as “gatekeeper” to data
  - Researchers pose queries in some agreed language
  - Gatekeeper gives an (anonymized) answer, or refuses to answer
- ◆ **“Send me your code”** model
  - Data owner executes code on their system and reports result
  - Cannot be sure that code is not malicious or steganographic
- ◆ **Offline**, aka “publish and be damned” model
  - Data owner somehow anonymizes data set
  - Publishes the results to the world, and retires
  - The model used in most real data releases

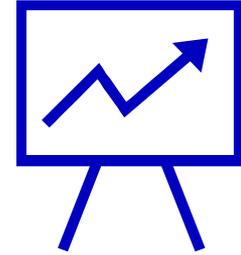


# Objectives for Anonymization



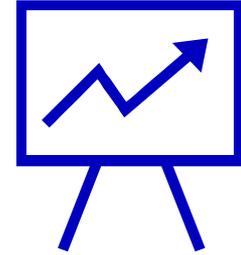
- ◆ Prevent (high confidence) inference of **associations**
  - Prevent inference of salary for an individual in “census”
  - Prevent inference of individual’s viewing history in “video”
  - Prevent inference of individual’s search history in “search”
  - All aim to prevent **linking** sensitive information to an individual
- ◆ Prevent inference of **presence** of an individual in the data set
  - Satisfying “presence” also satisfies “association” (not vice-versa)
  - Presence in a data set can violate privacy (eg STD clinic patients)
- ◆ Have to consider what knowledge might be known to attacker
  - **Background knowledge**: facts about the data set (X has salary Y)
  - **Domain knowledge**: broad properties of data (illness Z rare in men)

# Utility



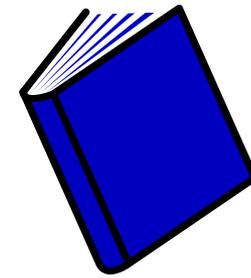
- ◆ Anonymization is meaningless if **utility** of data not considered
  - The empty data set has perfect privacy, but no utility
  - The original data has full utility, but no privacy
- ◆ What is “**utility**”? Depends what the application is...
  - For fixed query set, can look at maximum or average error
  - Problem for publishing: want to support unknown applications!
  - Need some way to **quantify** utility of alternate anonymizations

# Measures of Utility

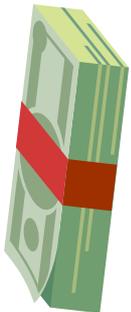


- ◆ Define a **surrogate measure** and try to optimize
  - Often based on the “**information loss**” of the anonymization
  - Simple example: number of examples deleted from a data set
- ◆ Give a guarantee for all queries in some **fixed class**
  - Hope the class is representative, so other uses have low distortion
  - Costly: some methods enumerate all queries, or all anonymizations
- ◆ **Empirical Evaluation**
  - Perform experiments with a reasonable workload on the result
  - Compare to results on original data (e.g. Netflix prize problems)
- ◆ Combinations of multiple methods
  - Optimize for some surrogate, but also evaluate on real queries

# Definitions of Technical Terms



- ◆ **Identifiers**—uniquely identify, e.g. Social Security Number (SSN)
  - Step 0: remove all identifiers
  - Was not enough for AOL search data
- ◆ **Quasi-Identifiers (QI)**—such as DOB, Sex, ZIP Code
  - Enough to partially identify an individual in a dataset
  - DOB+Sex+ZIP unique for 87% of US Residents [Sweeney 02]
- ◆ **Sensitive attributes (SA)**—the associations we want to hide
  - Salary in the “census” example is considered sensitive
  - Not always well-defined: only some “search” queries sensitive
  - In “video”, association between user and video is sensitive
  - One SA can reveal others: bonus may identify salary...



# Summary of Anonymization Motivation

- ◆ Anonymization needed for safe data sharing and retention
  - Many legal **requirements** apply
- ◆ Various privacy **definitions** possible
  - Primarily, prevent inference of sensitive information
  - Under some assumptions of background knowledge
- ◆ **Utility** of the anonymized data needs to be carefully studied
  - Different data types imply different classes of query
- ◆ **Main focus**: the publishing model with consideration of utility

# Case Study: US Census



- ◆ **Raw data:** information about every US household
  - Who, where; age, gender, racial, income and educational data
- ◆ **Why released:** determine representation, planning
- ◆ **How anonymized:** aggregated to geographic areas (Zip code)
  - Broken down by various combinations of dimensions
  - Released in full after 72 years
  - Census 2020 will use differential privacy techniques
- ◆ **Attacks:** no reports of successful deanonymization so far
  - Attempts by FBI to access raw data have been rebuffed
- ◆ **Consequences:** greater understanding of US population
  - Affects representation, funding of civil projects
  - Rich source of data for future historians and genealogists

# Case Study: Netflix Prize

The Netflix logo, consisting of the word "NETFLIX" in a bold, white, sans-serif font with a black outline, set against a red rectangular background.

- ◆ **Raw data:** 100M dated ratings from 480K users to 18K movies
- ◆ **Why released:** improve predicting ratings of unlabeled examples
- ◆ **How anonymized:** exact details not described by Netflix
  - All direct customer information removed
  - Only subset of full data; dates modified; some ratings deleted,
  - Movie title and year published in full
- ◆ **Attacks:** dataset was claimed vulnerable [[Narayanan Shmatikov 08](#)]
  - Attack links data to IMDB where same users also rated movies
  - Find matches based on similar ratings or dates in both
- ◆ **Consequences:** rich source of user data for researchers
  - Unclear how serious the attacks are in practice

# Case Study: AOL Search Data



- ◆ **Raw data:** 20M search queries for 650K users from 2006
- ◆ **Why released:** allow researchers to understand search patterns
- ◆ **How anonymized:** user identifiers removed
  - All searches from same user linked by an arbitrary identifier
- ◆ **Attacks:** many successful attacks identified individual users
  - Ego-surfers: people typed in their own names
  - Zip codes and town names identify an area
  - NY Times identified user 4417749 as 62yr old GA widow
- ◆ **Consequences:** CTO resigned, two researchers fired
  - Well-intentioned effort failed due to inadequate anonymization

# Exercises

- ◆ Think of a data set or data source that you are familiar with
- ◆ Is some of the data (potentially) private? Has the data already been anonymized in some way to protect privacy?
- ◆ What are the privacy implications of the raw original data being revealed? What could be discovered?
- ◆ In the data, which are the identifying attributes? Which are the quasi-identifiers? Which are the sensitive attributes?
- ◆ If all sensitive information was erased, what analyses would no longer be possible?

# Working Examples

- ◆ Will study an example data set with few attributes
- ◆ “Census” data recording incomes and demographics
  - Format: **(SSN, DOB, Sex, Zip, Salary)**
    - “Zip” = postal code, reveals approximate region
  - Similar to UCI adult.data set (can have other attributes)
- ◆ Many other kinds of data are relevant to privacy
  - “Video” data recording movies viewed
    - Graph data—graph properties should be retained
  - “Search” data recording web searches
    - Set data—each user has different set of keywords



# Tabular Data Example

- ◆ Census data recording incomes and demographics

SSN	DOB	Sex	ZIP	Salary
11-1-111	1/21/76	M	53715	50,000
22-2-222	4/13/86	F	53715	55,000
33-3-333	2/28/76	M	53703	60,000
44-4-444	1/21/76	M	53703	65,000
55-5-555	4/13/86	F	53706	70,000
66-6-666	2/28/76	F	53706	75,000

- ◆ Releasing SSN → Salary association **violates** individual's privacy
  - SSN is an identifier, Salary is a sensitive attribute (SA)

# Tabular Data Example: De-Identification

- ◆ **Census data:** remove SSN to create a de-identified table

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

- ◆ Does the de-identified table preserve an individual's privacy?
  - Depends on what other information an attacker knows

# Tabular Data Example: Linking Attack

- ◆ De-identified private data + publicly available data

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

SSN	DOB
11-1-111	1/21/76
33-3-333	2/28/76

- ◆ Cannot uniquely identify either individual's salary
  - DOB is a **quasi-identifier** (QI)

# Tabular Data Example: Linking Attack

- ◆ De-identified private data + publicly available data

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

SSN	DOB	Sex
11-1-111	1/21/76	M
33-3-333	2/28/76	M

- ◆ Uniquely identified one individual's salary, but not the other's
  - DOB, Sex are **quasi-identifiers** (QI)

# Tabular Data Example: Linking Attack

- ◆ De-identified private data + publicly available data

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

SSN	DOB	Sex	ZIP
11-1-111	1/21/76	M	53715
33-3-333	2/28/76	M	53703

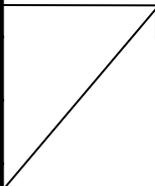
- ◆ Uniquely identified both individuals' salaries
  - [DOB, Sex, ZIP] is unique for lots of US residents [Sweeney 02]

# Tabular Data Example: Anonymization

- ◆ Anonymization through **tuple suppression**

DOB	Sex	ZIP	Salary
*	*	*	*
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
*	*	*	*
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

SSN	DOB	Sex	ZIP
11-1-111	1/21/76	M	53715



- ◆ Cannot link to private table even with knowledge of QI values
  - Missing tuples could take any value from the space of all tuples
  - Introduces a lot of uncertainty

# Tabular Data Example: Anonymization

- ◆ Anonymization through QI **attribute generalization**

The diagram illustrates the process of attribute generalization for anonymization. It shows two tables. The left table has columns for DOB, Sex, ZIP, and Salary. The right table has columns for SSN, DOB, Sex, and ZIP. Lines connect the rows of the left table to the rows of the right table, showing how specific values are replaced by generalizable ones.

DOB	Sex	ZIP	Salary
1/21/76	M	537**	50,000
4/13/86	F	537**	55,000
2/28/76	*	537**	60,000
1/21/76	M	537**	65,000
4/13/86	F	537**	70,000
2/28/76	*	537**	75,000

SSN	DOB	Sex	ZIP
11-1-111	1/21/76	M	53715
33-3-333	2/28/76	M	53703

- ◆ Cannot uniquely identify tuple with knowledge of QI values
  - More precise form of uncertainty than tuple suppression
  - E.g., ZIP = 537\*\*  $\rightarrow$  ZIP  $\in$  {53700, ..., 53799}

# Tabular Data Example: Anonymization

- ◆ Anonymization through sensitive attribute (SA) **permutation**

DOB	Sex	ZIP	Salary
1/21/76	M	53715	55,000
4/13/86	F	53715	50,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	75,000
2/28/76	F	53706	70,000

SSN	DOB	Sex	ZIP
11-1-111	1/21/76	M	53715
33-3-333	2/28/76	M	53703

- ◆ Can uniquely identify tuple, but uncertainty about SA value
  - Much more precise form of uncertainty than generalization

# Tabular Data Example: Anonymization

- ◆ Anonymization through sensitive attribute (SA) **perturbation**

DOB	Sex	ZIP	Salary
1/21/76	M	53715	60,000
4/13/86	F	53715	45,000
2/28/76	M	53703	60,000
1/21/76	M	53703	55,000
4/13/86	F	53706	80,000
2/28/76	F	53706	75,000

SSN	DOB	Sex	ZIP
11-1-111	1/21/76	M	53715
33-3-333	2/28/76	M	53703

- ◆ Can uniquely identify tuple, but get “noisy” SA value

# k-Anonymization [Samarati, Sweeney 98]

- ◆ **k-anonymity**: Table  $T$  satisfies k-anonymity wrt quasi-identifier  $QI$  iff each tuple in (the multiset)  $T[QI]$  appears at least  $k$  times
  - Protects against “linking attack”
- ◆ **k-anonymization**: Table  $T'$  is a k-anonymization of  $T$  if  $T'$  is a generalization/suppression of  $T$ , and  $T'$  satisfies k-anonymity

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000



DOB	Sex	ZIP	Salary
1/21/76	M	537**	50,000
4/13/86	F	537**	55,000
2/28/76	*	537**	60,000
1/21/76	M	537**	65,000
4/13/86	F	537**	70,000
2/28/76	*	537**	75,000

# k-Anonymization and Uncertainty

- ◆ **Intuition:** A k-anonymized table  $T'$  represents the set of all “possible world” tables  $T_i$  s.t.  $T'$  is a k-anonymization of  $T_i$ 
  - With no background knowledge, all possible worlds are equally plausible
- ◆ **Query Answering**
  - Queries should (implicitly) range over all possible worlds
  - **Example query:** what is the salary of individual (1/21/76, M, 53715)?  
Best guess is 57,500 (weighted average of 50,000 and 65,000)
  - **Example query:** what is the maximum salary of males in 53706?  
Could be as small as 50,000, or as big as 75,000

# Computing k-Anonymizations

- ◆ **Huge literature**: variations depend on search space and algorithm
  - Generalization vs (tuple) suppression
  - Global (e.g., full-domain) vs local (e.g., multidimensional) recoding
  - Hierarchy-based vs partition-based (e.g., numerical attributes)

Algorithm	Model	Properties	Complexity
Samarati 01	G+TS, FD, HB	One exact, binary search	$O(2^{ Q })$
Sweeney 02	G+TS, FD, HB	Exact, exhaustive	$O(2^{ Q })$
Bayardo+ 05	G+TS, FD, PB	Exact, top-down	$O(2^{ Q })$
LeFevre+ 05	G+TS, FD, HB	All exact, bottom-up cube	$O(2^{ Q })$

Algorithm	Model	Properties	Complexity
Meyerson+ 04	S, L	NP-hard, $O(k \log k)$ approximation	$O(n^{2k})$
Aggarwal+ 05a	S, L	$O(k)$ approximation	$O(kn^2)$
Aggarwal+ 05b	G, L, HB	$O(k)$ approximation	$O(kn^2)$
LeFevre+ 06	G, MD, PB	Constant-factor approximation	$O(n \log n)$

# Incognito [LeFevre+ 05]

- ◆ Every full-domain generalization described by a “domain vector”
  - $B_0 = \{1/21/76, 2/28/76, 4/13/86\} \rightarrow B_1 = \{76-86\}$
  - $S_0 = \{M, F\} \rightarrow S_1 = \{*\}$
  - $Z_0 = \{53715, 53710, 53706, 53703\} \rightarrow Z_1 = \{5371^*, 5370^*\} \rightarrow Z_2 = \{537^{**}\}$

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

$B_0, S_1, Z_2$



DOB	Sex	ZIP	Salary
1/21/76	*	537**	50,000
4/13/86	*	537**	55,000
2/28/76	*	537**	60,000
1/21/76	*	537**	65,000
4/13/86	*	537**	70,000
2/28/76	*	537**	75,000

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DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

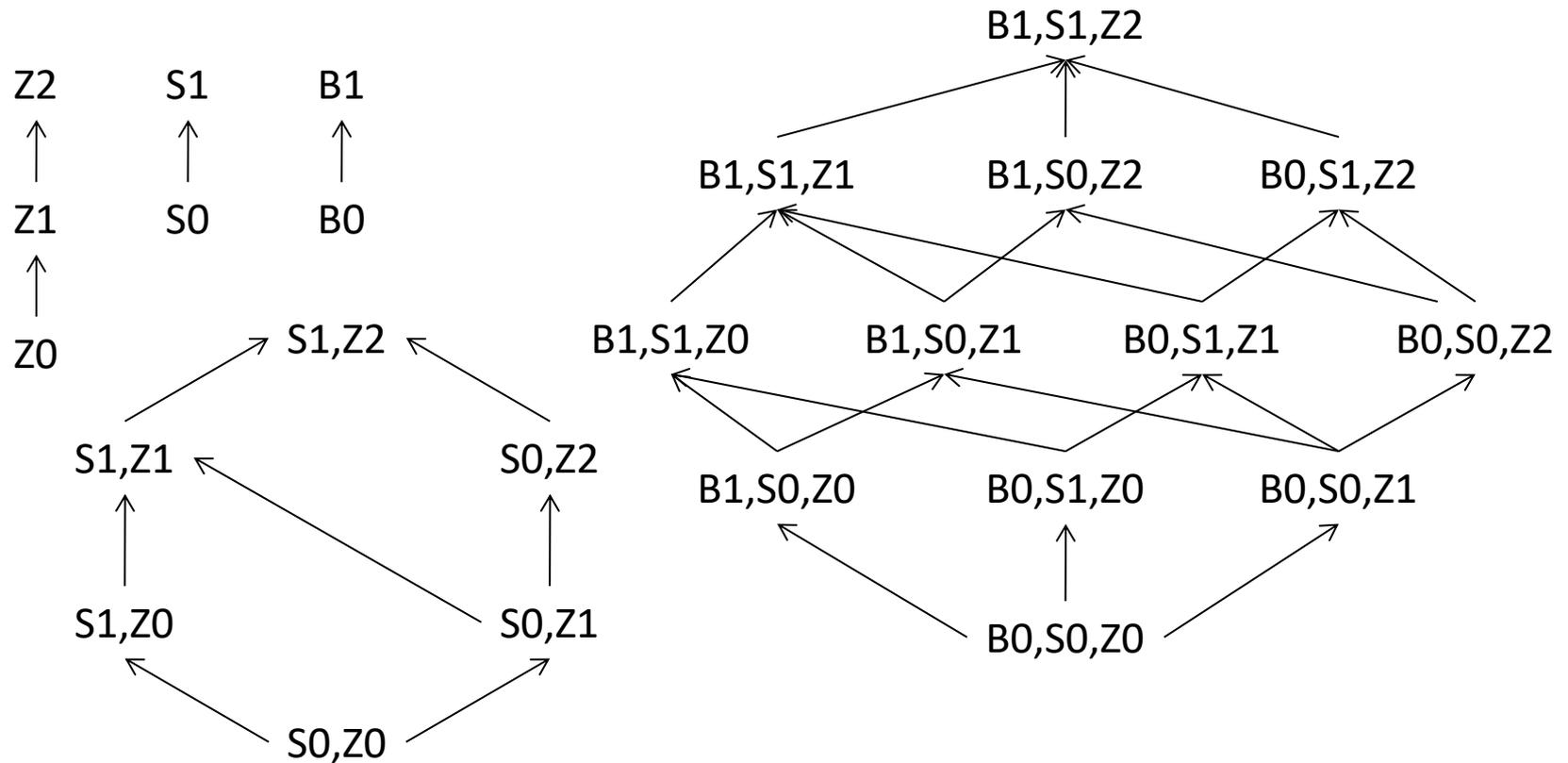
$B_1, S_0, Z_2$



DOB	Sex	ZIP	Salary
76-86	M	537**	50,000
76-86	F	537**	55,000
76-86	M	537**	60,000
76-86	M	537**	65,000
76-86	F	537**	70,000
76-86	F	537**	75,000

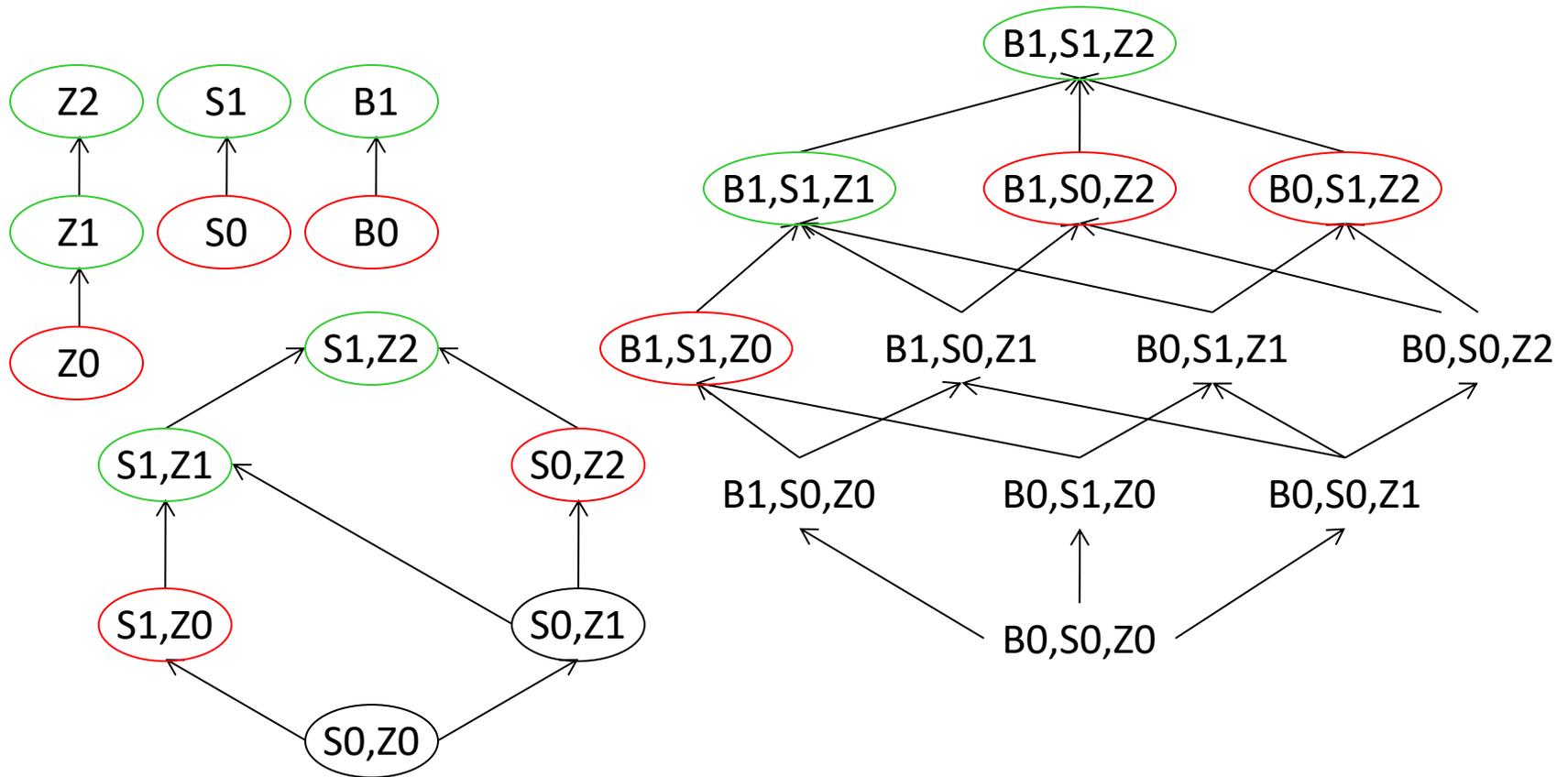
# Incognito [LeFevre+ 05]

## ◆ Lattice of domain vectors



# Incognito [LeFevre+ 05]

## ◆ Lattice of domain vectors



# Incognito [LeFevre+ 05]

- ◆ **Subset Property**: If table  $T$  is  $k$ -anonymous wrt attributes  $Q$ , then  $T$  is  $k$ -anonymous wrt any set of attributes that is a subset of  $Q$
- ◆ **Generalization Property**: If table  $T_2$  is a generalization of table  $T_1$ , and  $T_1$  is  $k$ -anonymous, then  $T_2$  is  $k$ -anonymous
- ◆ Computes all “**minimal**” full-domain generalizations
  - Set of minimal full-domain generalizations forms an anti-chain
  - Can use any reasonable utility metric to choose “**optimal**” solution

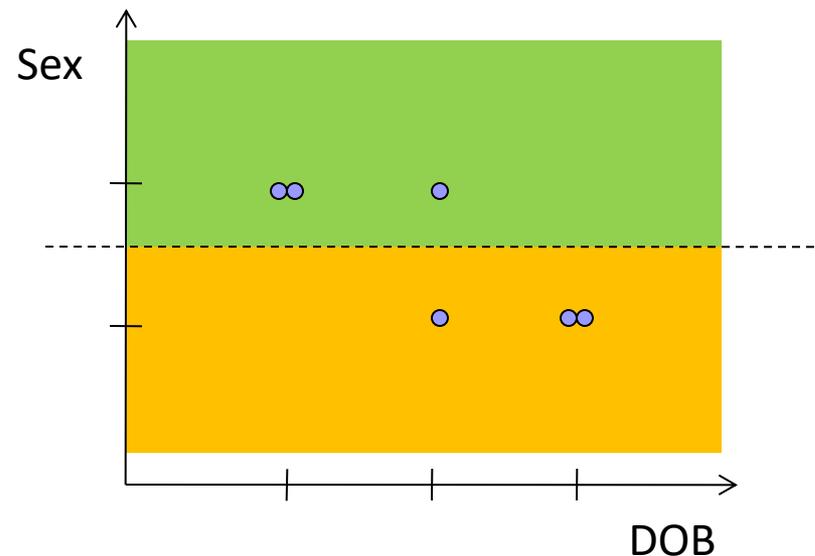
# Mondrian [LeFevre+ 06]

- ◆ Computes one “good” multi-dimensional generalization
  - Uses local recoding to explore a larger search space
  - Treats all attributes as ordered, chooses partition boundaries
- ◆ Utility metrics considered in the paper
  - **Discernability**: sum of squares of group sizes
  - **Normalized average group size** = (total tuples / total groups) /  $k$
- ◆ **Efficient**: greedy  $O(n \log n)$  heuristic for NP-hard problem
- ◆ **Quality guarantee**: solution is a constant-factor approximation

# Mondrian [LeFevre+ 06]

- ◆ Uses ideas from spatial kd-tree construction
  - QI tuples = points in a multi-dimensional space
  - Hyper-rectangles with  $\geq k$  points =  $k$ -anonymous groups
  - Choose axis-parallel line to partition point-multiset at median

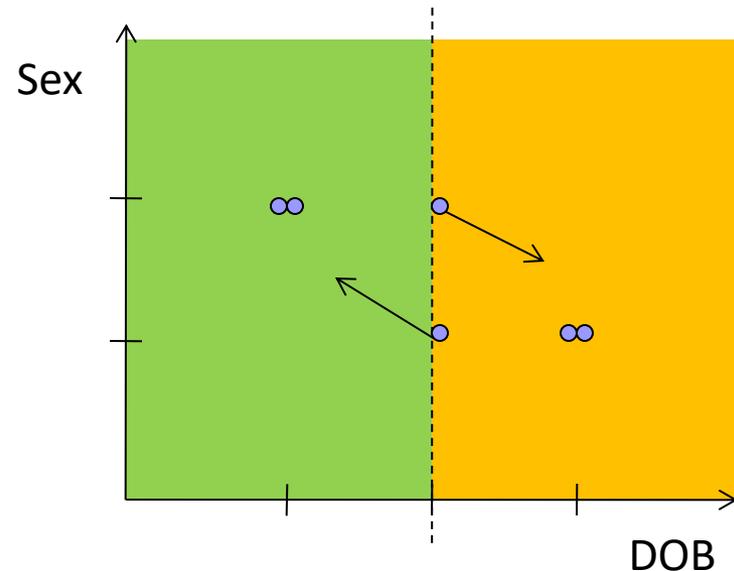
DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000



# Mondrian [LeFevre+ 06]

- ◆ Uses ideas from spatial kd-tree construction
  - QI tuples = points in a multi-dimensional space
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4/13/86	F	53706	70,000
2/28/76	F	53706	75,000



# Homogeneity Attack [Machanavajjhala+ 06]

- ◆ **Issue:** k-anonymity requires each tuple in (the multiset)  $T[QI]$  to appear  $\geq k$  times, but does not say anything about the SA values
  - If (almost) all SA values in a QI group are equal, loss of privacy!
  - The problem is with the choice of grouping, not the data

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
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1/21/76	M	53703	50,000
4/13/86	F	53706	55,000
2/28/76	F	53706	60,000

Not Ok!  
→

DOB	Sex	ZIP	Salary
1/21/76	*	537**	50,000
4/13/86	*	537**	55,000
2/28/76	*	537**	60,000
1/21/76	*	537**	50,000
4/13/86	*	537**	55,000
2/28/76	*	537**	60,000

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  - If (almost) all SA values in a QI group are equal, loss of privacy!
  - The problem is with the choice of grouping, not the data
  - For some groupings, no loss of privacy

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	50,000
4/13/86	F	53706	55,000
2/28/76	F	53706	60,000

Ok!  
→

DOB	Sex	ZIP	Salary
76-86	*	53715	50,000
76-86	*	53715	55,000
76-86	*	53703	60,000
76-86	*	53703	50,000
76-86	*	53706	55,000
76-86	*	53706	60,000

# $l$ -Diversity [Machanavajjhala+ 06]

- ◆  **$l$ -Diversity Principle**: a table is  $l$ -diverse if each of its QI groups contains at least  $l$  “well-represented” values for the SA
- ◆ Different definitions of  $l$ -diversity based on formalizing the intuition of a “well-represented” value
  - **Entropy  $l$ -diversity**: for each QI group  $g$ ,  $\text{entropy}(g) \geq \log(l)$
  - **Recursive  $(c, l)$ -diversity**: for each QI group  $g$  with  $m$  SA values, and  $r_i$  the  $i$ 'th highest frequency,  $r_1 < c (r_1 + r_{l+1} + \dots + r_m)$
  - **Folk  $l$ -diversity**: for each QI group  $g$ , no SA value should occur more than  $1/l$  fraction of the time = Recursive( $1/l$ , 1)-diversity
- ◆ **Intuition**: Most frequent value does not appear too often compared to the less frequent values in a QI group

# Computing $l$ -Diversity [Machanavajjhala+ 06]

- ◆ **Key Observation:** entropy  $l$ -diversity and recursive( $c, l$ )-diversity possess the Subset Property and the Generalization Property
- ◆ **Algorithm Template:**
  - Take an algorithm for  $k$ -anonymity and replace the  $k$ -anonymity test for a generalized table by the  $l$ -diversity test
  - Easy to check based on counts of SA values in QI groups

# t-Closeness [Li+ 07]

## ◆ Limitations of *l*-diversity

- Similarity attack: SA values are distinct, but semantically similar

DOB	Sex	ZIP	Salary
1/21/76	*	537**	50,000
4/13/86	*	537**	55,000
2/28/76	*	537**	60,000
1/21/76	*	537**	50,001
4/13/86	*	537**	55,001
2/28/76	*	537**	60,001

SSN	DOB	Sex	ZIP
11-1-111	1/21/76	M	53715

- ◆ **t-Closeness Principle**: a table has t-closeness if in each of its QI groups, the distance between the distribution of SA values in the group and in the whole table is no more than threshold  $t$

# Answering Queries on Generalized Tables

- ◆ **Observation:** Generalization loses a lot of information, resulting in inaccurate aggregate analyses
- ◆ How many people were born in 1976?
  - Bounds = [1,5], selectivity estimate = 1, actual value = 4

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000



DOB	Sex	ZIP	Salary
76-86	M	537**	50,000
76-86	F	537**	55,000
76-86	M	537**	60,000
76-86	M	537**	65,000
76-86	F	537**	70,000
76-86	F	537**	75,000

# Answering Queries on Generalized Tables

- ◆ **Observation:** Generalization loses a lot of information, resulting in inaccurate aggregate analyses
- ◆ What is the average salary of people born in 1976?
  - Bounds = [50K,75K], actual value = 62.5K

DOB	Sex	ZIP	Salary
1/21/76	M	53715	50,000
4/13/86	F	53715	55,000
2/28/76	M	53703	60,000
1/21/76	M	53703	65,000
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DOB	Sex	ZIP	Salary
76-86	M	537**	50,000
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76-86	M	537**	60,000
76-86	M	537**	65,000
76-86	F	537**	70,000
76-86	F	537**	75,000

# Subsequent Attacks and Developments

- ◆ **Minimality Attack [Wong+ 07]:**
  - Uses knowledge of anonymization algorithm to argue some possible worlds are not consistent with output
- ◆ **deFinetti Attack [Kifer 09]:**
  - Uses knowledge from anonymized data to argue some associations are more likely than others
- ◆ **Further development:**
  - Due to such attacks, work on “syntactic methods” has slowed
  - Few if any significant deployments have been reported
  - Continued interest in areas such as graph data anonymization

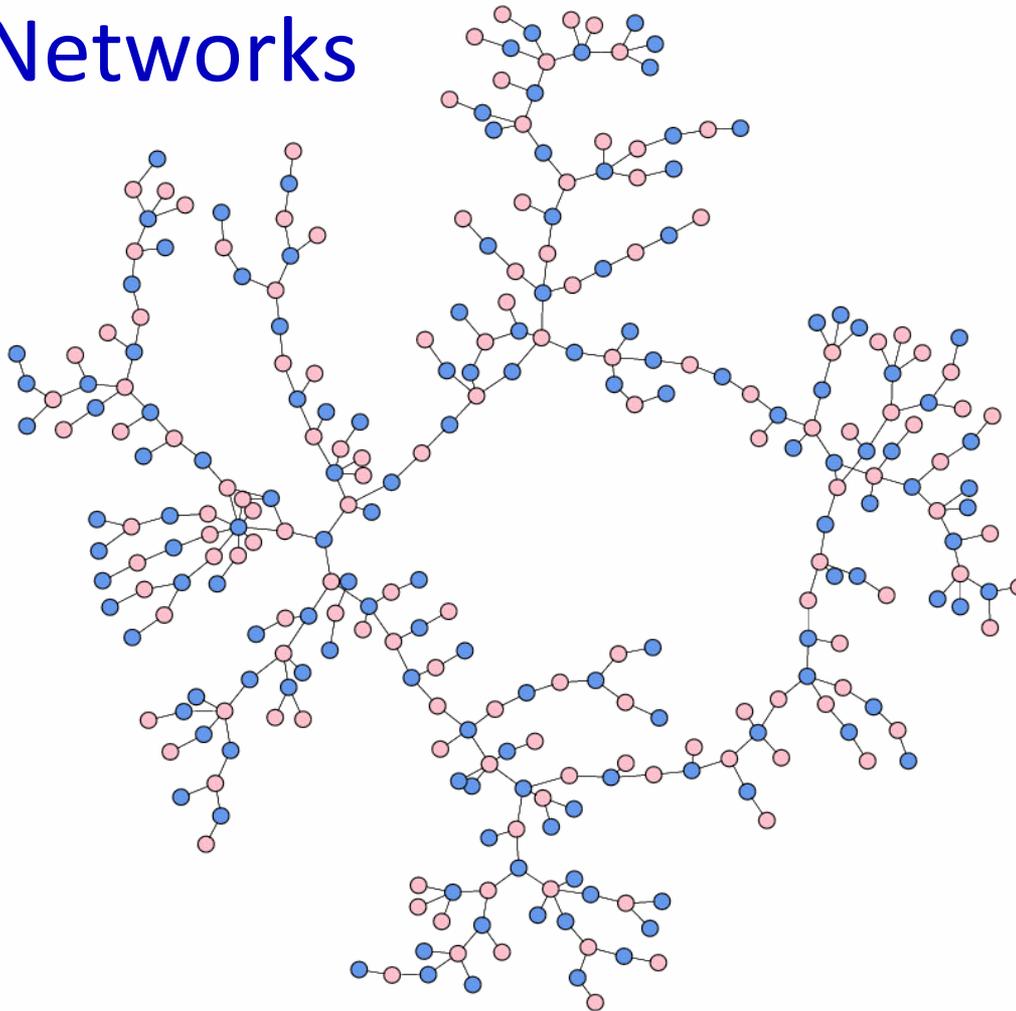
# More to life than tables...



# Recommendation Data



# Social Networks



Plot from Mark Newman, based on data in  
*"The structure of adolescent romantic and sexual networks"*, *American Journal of Sociology* 110, 44-91 (2004) .  
Males are red, females are blue

# Location and Trajectory Data



# Web Search Logs

AOL

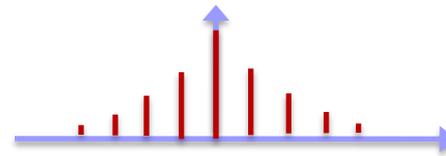
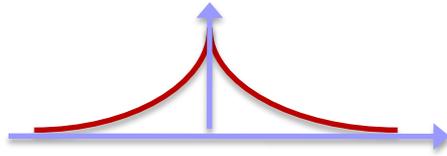


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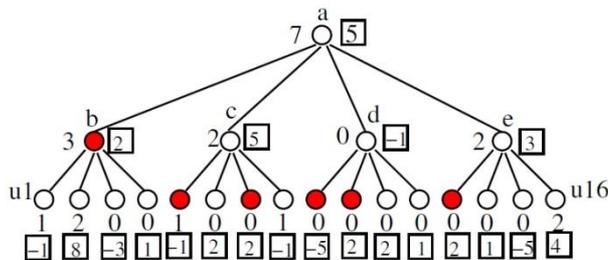
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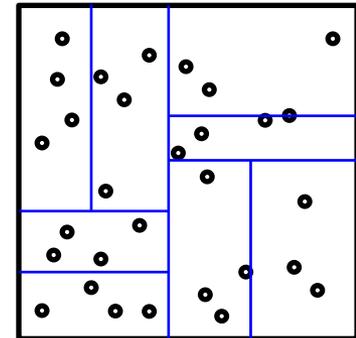


# Building Blocks of Privacy: Differentially Private Mechanisms



**Graham Cormode**

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# Differential Privacy: a new hope

- ◆ **Principle:** released info reveals little about any individual
  - Even if adversary knows (almost) everything about everyone else!
- ◆ Thus, individuals should be secure about contributing their data
  - What is learnt about them is about the same either way
- ◆ Much work on providing differential privacy (DP)
  - Simple recipe for some data types e.g. numeric answers
  - Simple rules allow us to reason about composition of results
  - More complex algorithms for arbitrary data (many DP mechanisms)
- ◆ Adopted and used by several organizations:
  - US Census, Common Data Project, Facebook (?), Google, Apple...



# Differential Privacy Definition

The output distribution of a differentially private algorithm changes very little whether or not any individual's data is included in the input (so it's OK to contribute your data)

A randomized algorithm  $K$  satisfies  $\epsilon$ -differential privacy if:  
Given any pair of neighboring data sets,  
 $D$  and  $D'$ , and  $S$  in  $\text{Range}(K)$ :

$$\Pr[K(D) = S] \leq e^\epsilon \Pr[K(D') = S]$$

Neighboring datasets differ in one individual: we say  $|D - D'| = 1$

# Achieving Differential Privacy

- ◆ Suppose we want to output the number of left-handed people in our data set
  - Can reduce the description of the data to just the answer,  $n$
  - Want a randomized algorithm  $K(n)$  that will output an integer
  - Consider the distribution  $\Pr[K(n) = m]$  for different  $m$
- ◆ Write  $\exp(\varepsilon) = \alpha$ , and  $\Pr[K(n) = n] = p_n$ . Then:
  - $\Pr[K(n) = n-1] \leq \alpha \Pr[K(n-1) = n-1] = \alpha p_{n-1}$
  - $\Pr[K(n) = n-2] \leq \alpha \Pr[K(n-1) = n-2] \leq \alpha^2 \Pr[K(n-2) = n-2] = \alpha^2 p_{n-2}$
  - $\Pr[K(n) = n-i] \leq \alpha^i p_{n-i}$
  - Similarly,  $\Pr[K(n) = n+i] \leq \alpha^i p_{n+i}$

# Achieving Differential Privacy

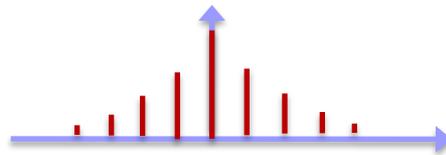
- ◆ We have  $\Pr[K(n) = n-i] \leq \alpha^i p_{n-i}$  and  $\Pr[K(n) = n+i] \leq \alpha^i p_{n+i}$
- ◆ Within these constraints, we want to maximize  $p_n$ 
  - This maximizes the probability of returning “correct” answer
  - Means we turn the inequalities into equalities
- ◆ For simplicity, set  $p_n = p$  for all  $n$ 
  - Means the distribution of “shifts” is the same whatever  $n$  is
- ◆ Yields:  $\Pr[K(n) = n-i] = \alpha^i p$  and  $\Pr[K(n) = n+i] = \alpha^i p$ 
  - Sum over all shifts  $i$ :
$$p + \sum_{i=1}^{\infty} 2\alpha^i p = 1$$
$$p + 2p \alpha/(1-\alpha) = 1$$
$$p(1 - \alpha + 2\alpha)/(1-\alpha) = 1$$
$$p = (1-\alpha)/(1+\alpha)$$

# Geometric Mechanism

- ◆ What does this mean?

- For input  $n$ , output distribution is  $\Pr[K(n) = m] = \alpha^{|m-n|} \cdot (1-\alpha)/(1+\alpha)$

- ◆ What does this look like?



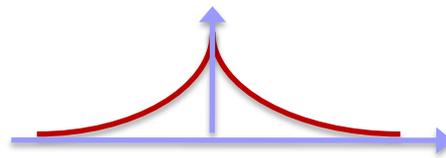
- **Symmetric geometric distribution**, centered around  $n$
- We draw from this distribution centered around zero, and add to the true answer
- We get the “true answer plus (symmetric geometric) noise”
- ◆ A first differentially private mechanism for outputting a count
  - We call this “the **geometric mechanism**”

# Truncated Geometric Mechanism

- ◆ Some practical concerns:
  - This mechanism could output any value, from  $-\infty$  to  $+\infty$
- ◆ **Solution:** we can “truncate” the output of the mechanism
  - E.g. decide we will never output any value below zero, or above  $N$
  - Any value drawn below zero is “rounded up” to zero
  - Any value drawn above  $N$  is “rounded down” to  $N$
  - This does not affect the differential privacy properties
  - Can directly compute the closed-form probability of these outcomes
- ◆ (Truncated) geometric mechanism is unique, optimal mechanism
  - Properties proved in [Ghosh Roughgarden Sundarajan 08]

# Laplace Mechanism

- ◆ Sometimes we want to output **real values** instead of integers
- ◆ The **Laplace Mechanism** naturally generalizes **Geometric**



- Add noise from a symmetric continuous distribution to true answer
- **Laplace distribution** is a **symmetric exponential distribution**
- Is DP for same reason as geometric: shifting the distribution changes the probability by at most a constant factor
- PDF:  $\Pr[X = x] = 1/2\lambda \exp(-|x|/\lambda)$   
Variance =  $2\lambda^2$

# Sensitivity of Numeric Functions

- ◆ For more complex functions, we need to calibrate the noise to the influence an individual can have on the output
  - The (global) sensitivity of a function  $F$  is the maximum (absolute) change over all possible adjacent inputs
  - $S(F) = \max_{D, D' : |D-D'|=1} \|F(D) - F(D')\|_1$
  - **Intuition:**  $S(F)$  characterizes the scale of the influence of one individual, and hence how much noise we must add
- ◆  $S(F)$  is small for many common functions
  - $S(F) = 1$  for COUNT
  - $S(F) = 2$  for HISTOGRAM
  - Bounded for other functions (MEAN, covariance matrix...)

# Laplace Mechanism with Sensitivity

- ◆ Release  $F(x) + \text{Lap}(S(F)/\epsilon)$  to obtain  $\epsilon$ -DP guarantee
  - $F(x)$  = true answer on input  $x$
  - $\text{Lap}(\lambda)$  = noise sampled from Laplace dbn with parameter  $\lambda$
  - **Exercise**: show this meets  $\epsilon$ -differential privacy requirement
- ◆ Intuition on impact of parameters of differential privacy (DP):
  - Larger  $S(F)$ , more noise (need more noise to mask an individual)
  - Smaller  $\epsilon$ , more noise (more noise increases privacy)
  - Expected magnitude of  $|\text{Lap}(\lambda)|$  is (approx)  $\lambda$

# Sequential Composition

- ◆ What happens if we ask multiple questions about same data?
  - We reveal more, so the bound on  $\epsilon$  differential privacy weakens
- ◆ Suppose we output via  $K_1$  and  $K_2$  with  $\epsilon_1, \epsilon_2$  differential privacy:  
For any neighbouring  $D, D'$ , we have
$$\Pr[ K_1(D) = S_1 ] \leq \exp(\epsilon_1) \Pr[K_1(D') = S_1], \text{ and}$$
$$\Pr[ K_2(D) = S_2 ] \leq \exp(\epsilon_2) \Pr[K_2(D') = S_2]$$
$$\Pr[ (K_1(D) = S_1), (K_2(D) = S_2)] = \Pr[K_1(D)=S_1] \Pr[K_2(D) = S_2]$$
$$\leq \exp(\epsilon_1) \Pr[K_1(D') = S_1] \exp(\epsilon_2) \Pr[K_2(D') = S_2]$$
$$= \exp(\epsilon_1 + \epsilon_2) \Pr[(K_1(D') = S_1), (K_2(D') = S_2)]$$
  - Use the fact that the noise distributions are independent
- ◆ **Bottom line:** result is  $\epsilon_1 + \epsilon_2$  differentially private
  - Can reason about **sequential composition** by just “adding the  $\epsilon$ ’s”

# Parallel Composition

- ◆ **Sequential composition** is pessimistic
  - Assumes outputs are correlated, so privacy budget is diminished
- ◆ If the inputs are disjoint, then result is  $\max(\epsilon_1, \epsilon_2)$  private
- ◆ **Example:**
  - Ask for count of people broken down by handedness, hair color

	Redhead	Blond	Brunette
Left-handed	23	35	56
Right-handed	215	360	493

- Each cell is a disjoint set of individuals
- So can release each cell with  $\epsilon$ -differential privacy (**parallel composition**) instead of  $6\epsilon$  DP (**sequential composition**)

# Exponential Mechanism

- ◆ What happens when we want to output non-numeric values?
- ◆ **Exponential mechanism** is most general approach
  - Captures all possible DP mechanisms
  - But ranges over all possible outputs, may not be efficient
- ◆ **Requirements:**
  - Input value  $x$
  - Set of possible outputs  $O$
  - Quality function,  $q$ , assigns “score” to possible outputs  $o \in O$ 
    - $q(x, o)$  is bigger the “better”  $o$  is for  $x$
  - Sensitivity of  $q = S(q) = \max_{x, x', o} |q(x, o) - q(x', o)|$

# Exponential Mechanism

- ◆ Sample output  $o \in O$  with probability
$$\Pr[K(x) = o] = \exp(\varepsilon q(x,o)) / (\sum_{o' \in O} \exp(\varepsilon q(x,o')))$$
- ◆ Result is  $(2\varepsilon S(q))$ -DP
  - Shown by considering change in numerator and denominator under change of  $x$  is at most a factor of  $\exp(\varepsilon S(q))$
- ◆ **Scalability**: need to be able to draw from this distribution
- ◆ **Generalizations**:
  - $O$  can be continuous,  $\sum$  becomes an integral
  - Can apply a prior distribution over outputs as  $P(o)$ 
    - We assume a uniform prior for simplicity

# Exponential Mechanism Example 1: Count

- ◆ Suppose input is a count  $n$ , we want to output (noisy)  $n$ 
  - Outputs  $O$  = all integers
  - $q(n,o) = \alpha^{-|o-n|}$
  - $S(q) = 1$
  - Then  $\Pr[K(n) = o] = \exp(-\varepsilon |o-n|) / (\sum_o \exp(-\varepsilon |o-n|)) = \alpha^{-|o-n|} \cdot (1-\alpha) / (1-\alpha)$
  - Simplifies to the **Geometric mechanism!**
- ◆ Similarly, if  $O$  = all reals, applying exponential mechanism results in the **Laplace Mechanism**
- ◆ Illustrates the claim that **Exponential Mechanism** captures all possible DP mechanisms

# Exponential Mechanism, Example 2: Median

- ◆ Let  $M(X)$  = median of set of values in range  $[0, T]$  (e.g. median age)
- ◆ Try **Laplace Mechanism**:  $S(M) = T$ 
  - There can be datasets  $X, X'$  where  $M(X) = 0, M(X') = T, |X - X'| = 1$
  - Consider  $X = [0^n, 0, T^n], X' = [0^n, T, T^n]$
  - Noise from Laplace mechanism outweighs the true answer!
- ◆ **Exponential Mechanism**: set  $q(X, o) = -| \text{rank}_X(o) - |X|/2 |$ 
  - Define  $\text{rank}_X(o)$  as the number of elements in  $X$  dominated by  $o$
  - Note,  $\text{rank}_X(M(X)) = |X|/2$  : median has rank half
  - $S(q) = 1$ : adding or removing an individual changes  $q$  by at most 1
  - Then  $\Pr[ K(X) = o ] = \exp(\varepsilon q(X, o)) / (\sum_{o' \in O} \exp(\varepsilon q(X, o')))$
  - **Problem**: Output set  $O$  could be very large, how to make efficient?

# Exponential Mechanism, Example 2: Median

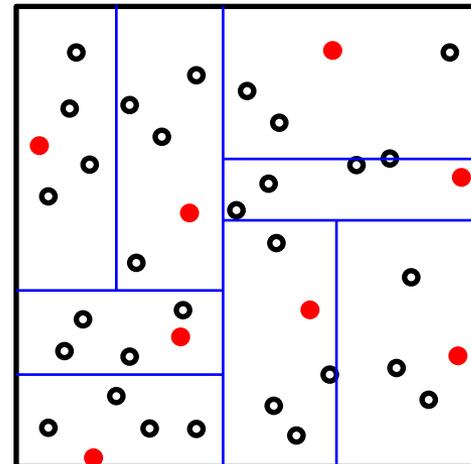
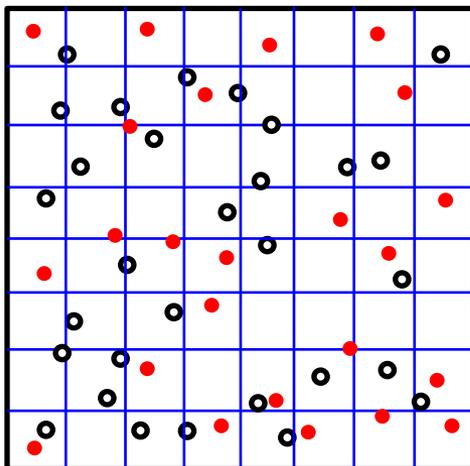
- ◆ **Observation**: for many values of  $o$ ,  $q(X, o)$  is the same:
  - Index  $X$  in sorted order so  $x_1 \leq x_2 \leq x_3 \leq \dots \leq x_n$
  - Then for any  $x_i \leq o < o' \leq x_{i+1}$ ,  $\text{rank}_X(o) = \text{rank}_X(o')$
  - Hence  $q(X, o) = q(X, o')$
- ◆ Break possible outputs into ranges:
  - $O_0 = [0, x_1]$        $O_1 = [x_1, x_2]$       ...       $O_n = [x_n, T]$
  - Pick range  $O_j$  with probability proportional to  $|O_j| \exp(\epsilon q(X, O_j))$
  - Pick output  $o \in O_j$  uniformly from the range
  - Time cost is proportional to number of ranges  $n$  (after sorting  $X$ )
- ◆ Similar tricks make **exponential mechanism** practical elsewhere

# Recap

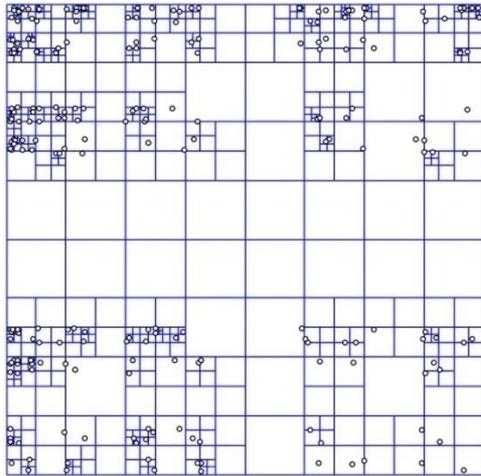
- ◆ Have developed a number of **building blocks** for DP:
  - **Geometric** and **Laplace mechanism** for numeric functions
  - **Exponential mechanism** for sampling from arbitrary sets
- ◆ And “**cement**” to glue things together:
  - **Parallel** and **sequential composition** theorems
- ◆ With these blocks and cement, can build a lot
  - Many papers arrive from careful combination of these tools!
- ◆ **Useful fact**: any post-processing of DP output remains DP
  - (so long as you don't access the original data again)
  - Helps reason about privacy of data release processes

# Case Study: Sparse Spatial Data

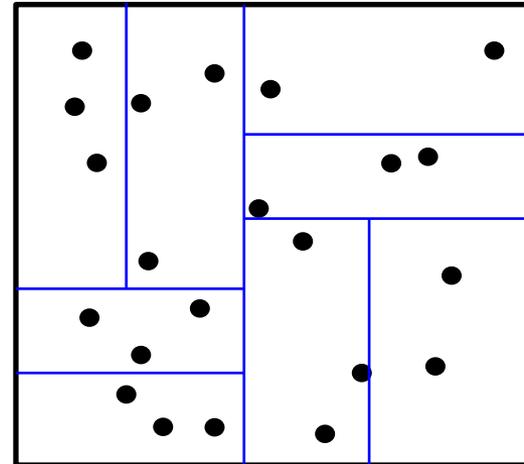
- ◆ Consider location data of many individuals
  - Some dense areas (towns and cities), some sparse (rural)
- ◆ Applying DP naively simply generates noise
  - lay down a fine grid, signal overwhelmed by noise
- ◆ **Instead:** compact regions with sufficient number of points



# Private Spatial decompositions



quadtree



kd-tree

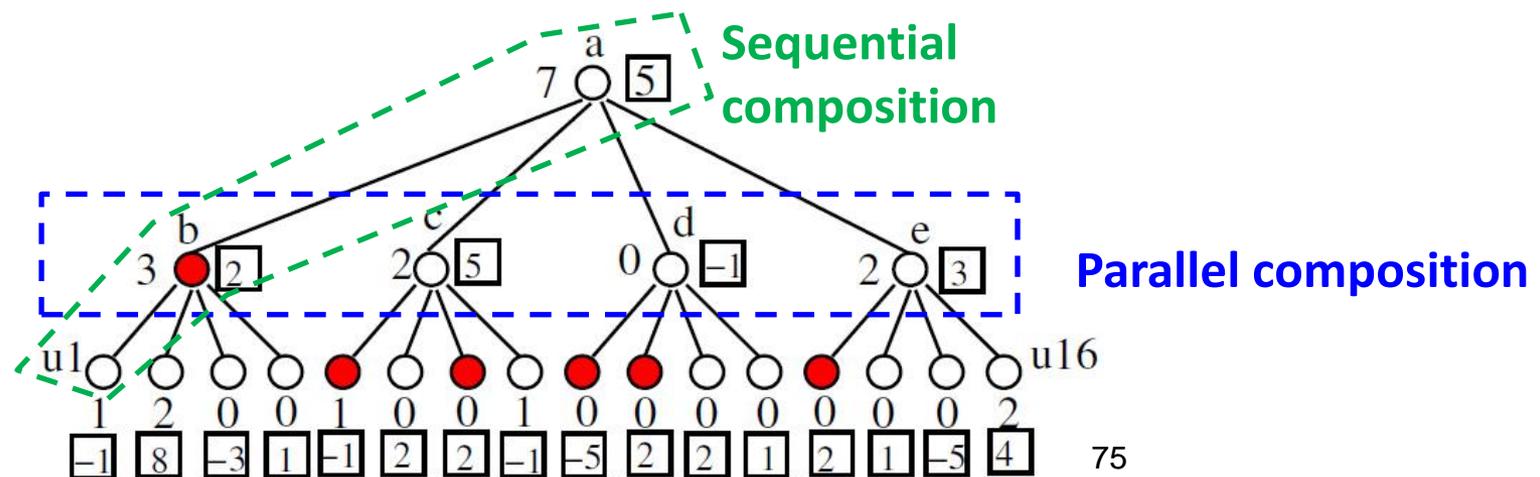
- ◆ **Build**: adapt existing methods to have differential privacy
- ◆ **Release**: a private description of data distribution (in the form of bounding boxes and noisy counts)

# Building a Private kd-tree

- ◆ Process to build a private kd-tree
  - **Input**: maximum height  $h$ , minimum leaf size  $L$ , data set
  - Choose dimension to split
  - Get (private) median in this dimension
  - Create child nodes and add noise to the counts
  - Recurse until we hit some stopping condition, e.g.:
    - Max height is reached
    - (Noisy) count of this node less than  $L$
    - Budget along the root-leaf path has used up
- ◆ The entire PSD satisfies DP by the composition property

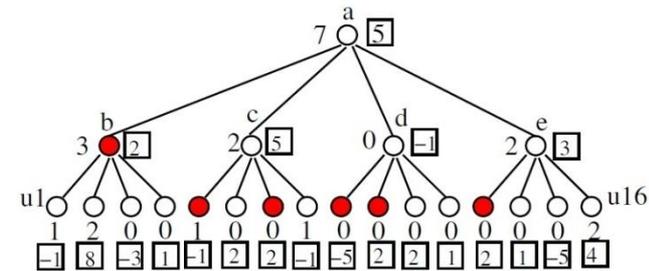
# Building PSDs – privacy budget allocation

- ◆ Data owner specifies a total budget  $\epsilon$  reflecting the level of anonymization desired
- ◆ Budget is split between medians and counts
  - Tradeoff accuracy of division with accuracy of counts
- ◆ Budget is split across levels of the tree
  - Privacy budget used along any root-leaf path should total  $\epsilon$



# Privacy budget allocation

- ◆ How to set an  $\epsilon_i$  for each level?
  - Compute the number of nodes touched by a ‘typical’ query
  - Minimize variance of such queries
  - **Optimization:**  $\min \sum_i 2^{h-i} / \epsilon_i^2$  s.t.  $\sum_i \epsilon_i = \epsilon$
  - Solved by  $\epsilon_i \propto (2^{(h-i)})^{1/3} \epsilon$  : more to leaves
  - Total error (variance) goes as  $2^h / \epsilon^2$



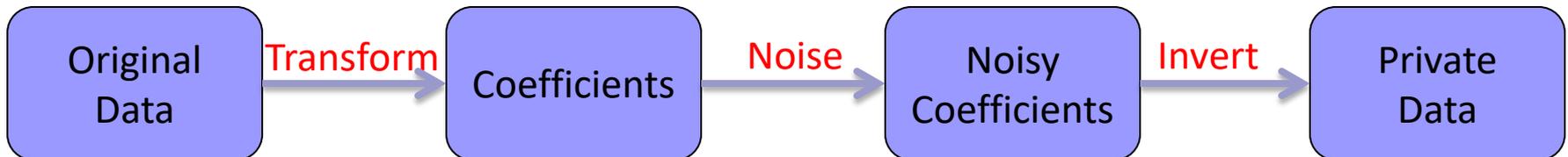
- ◆ Tradeoff between noise error and spatial uncertainty
  - Reducing  $h$  drops the noise error
  - But lower  $h$  increases the size of leaves, more uncertainty

# Post-processing of noisy counts

- ◆ Can do additional **post-processing** of the noisy counts
  - To improve query accuracy and achieve consistency
- ◆ **Intuition**: we have count estimate for a node and for its children
  - Combine these independent estimates to get better accuracy
  - Make consistent with some true set of leaf counts
- ◆ Formulate as a linear system in  $n$  unknowns [Hay et al 10]
  - Avoid explicitly solving the system
  - Expresses optimal estimate for node  $v$  in terms of estimates of ancestors and noisy counts in subtree of  $v$
  - Use the tree-structure to solve in three passes over the tree
  - Linear time to find optimal, consistent estimates

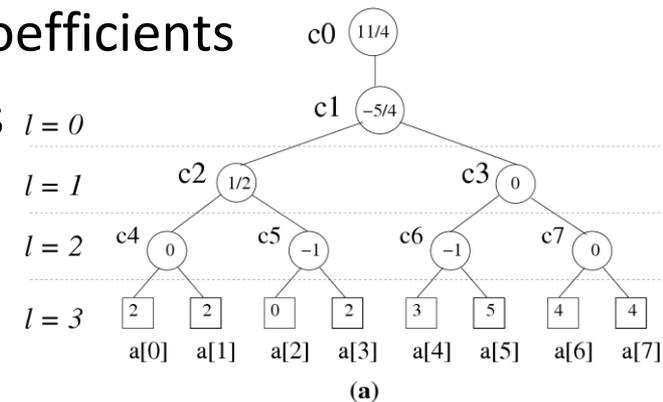
# Data Transformations

- ◆ Can think of trees as a 'data-dependent' transform of input
- ◆ Can apply other data transformations
- ◆ **General idea:**
  - Apply transform of data
  - Add noise in the transformed space (based on sensitivity)
  - Publish noisy coefficients, or invert transform (post-processing)
- ◆ **Goal:** pick a transform that preserves good properties of data
  - And which has low sensitivity, so noise does not corrupt



# Wavelet Transform

- ◆ Haar wavelet transform commonly used to approximate data
  - Any 1D range is expressed using  $2\log n$  coefficients
  - Each input point affects  $\log n$  coefficients
  - Is a linear, orthonormal transform
- ◆ Can add noise to wavelet coefficients
  - Treat input as a 1D histogram of counts
  - **Bounded sensitivity**: each individual affects coefficients by  $O(1)$
  - Can transform noisy coefficients back to get noisy histogram
- ◆ Range queries are answered well in this model
  - Each range query picks up noise (variance)  $O(\log^3 n / \epsilon^2)$
  - Directly adding noise to input would give noise  $O(n / \epsilon^2)$



# Other Transforms

Many other transforms can be applied within DP

- ◆ (Discrete) **Fourier Transform**: also bounded sensitivity
  - Often need only a fixed set of coefficients: further reduces  $S(F)$
  - Used for representing data cube counts, time series
- ◆ **Hierarchical Transforms**: binary trees and quadtrees
- ◆ **Randomized Transforms**: sketches and compressed sensing

$$A_8 = \sqrt{\frac{1}{8}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

# Local Sensitivity

- ◆ **A common fallacy**: using local sensitivity instead of global
  - **Global sensitivity**  $S(F) = \max_{x, x' : |x-x'|=1} \|F(x)-F(x')\|_1$
  - **Local sensitivity**  $S(F, x) = \max_{x' : |x-x'|=1} \|F(x)-F(x')\|_1$
  - These can be very different: local can be much smaller than global
  - It is tempting (but incorrect) to calibrate noise to local sensitivity
- ◆ **Bad case for local sensitivity: Median**
  - Consider  $X = [0^n, 0, 0, T^{n-1}]$ ,  $X' = [0^n, 0, T^n]$ ,  $X'' = [0^n, T, T^n]$
  - $S(F, X) = 0$  while  $S(F, X') = T$
  - Scale of the noise will reveal exactly which case we are in
- ◆ Still, there **has** to be something better than always using global?
  - Such bad cases seem artificial, rare

# Smooth Sensitivity

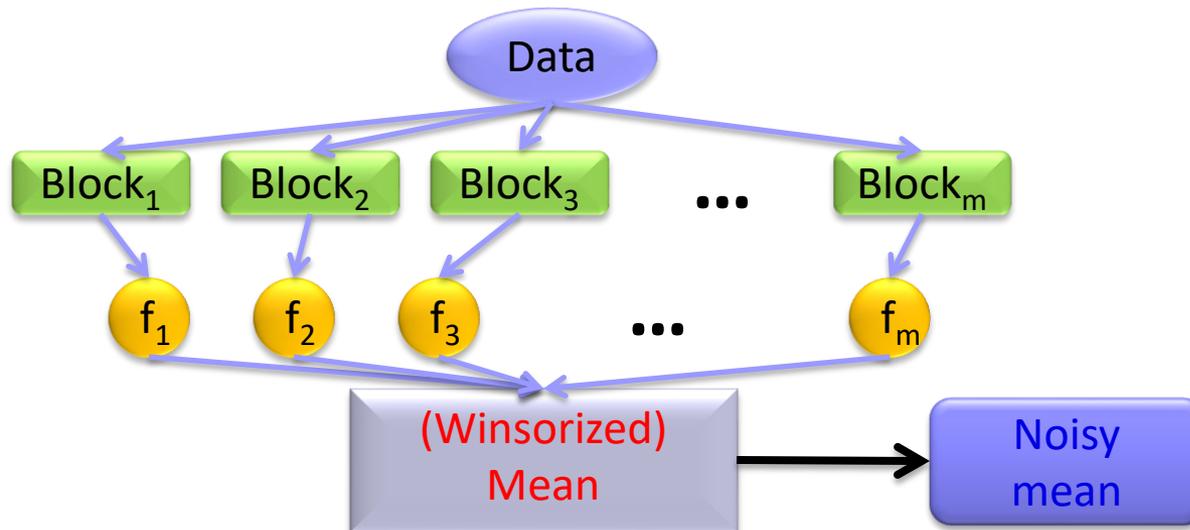
- ◆ Previous case was bad because local sensitivity was low, but “close” to a case where local sensitivity was high
- ◆ “Smooth sensitivity” combines sensitivity from all neighborhoods (based on parameter  $\beta$ )
  - $SS(F,x) = \max_{o \in \mathcal{O}} LS(F,o) \exp(-\beta |o - x|)$
  - Contribution of output  $o$  is decayed exponentially based on distance of  $o$  from  $x$ ,  $|o - x|$
  - Can add Laplace noise scaled by  $SS(F,x)$  to obtain (variant of) DP

# Smooth Sensitivity: Example

- ◆ Consider the median function  $M$  over  $n$  items again
  - Compute the maximum change in the median for each distance  $d$
  - $LS$  measures when median changes from  $x_i$  to  $x_{i+1}$
- ◆ So  $LS$  at distance  $d$  is at most  $\max_{0 \leq j \leq d} (x_{n/2+j} - x_{n/2+j-d-1})$ 
  - Largest gap that can be created by inserting/deleting at most  $d$  items
- ◆ Gives  $SS(M, x) = \max_{0 \leq d \leq n} \exp(-d\beta) \max_{0 \leq j \leq d} (x_{n/2+j} - x_{n/2+j-d-1})$ 
  - Can compute in time  $O(n^2)$
  - Empirically, exponential mechanism seems preferable
  - No generic process for computing smooth sensitivity

# Sample-and-aggregate

- ◆ **Sample-and-aggregate** gives a useful template
  - **Intuition**: sampling is almost DP - can't be sure who is included
  - Break input into moderate number of blocks,  $m$
  - Compute desired function on each block
  - Snap to some range  $[\min, \max]$  and aggregate (e.g. mean)
  - Add **Laplace noise** scaled by sensitivity ( $\max-\min$ )



# Sparse Data

- ◆ Suppose we have many (overlapping) queries, most of which have a small answer, but we don't know which
  - We are only interested in large answers (e.g. frequent itemsets)
  - **Two problems**: time efficiency, and “privacy efficiency”
- ◆ **Time efficiency**:
  - Don't want to add noise to every single zero-valued query
  - Assume we can materialize all non-zero query answers
  - Count how many are zero
  - Compute probability of noise pushing a zero-query past threshold
  - Sample from **Binomial distribution** how many to “upgrade”
  - Sample noisy value conditioned on passing threshold

# Sparse Data – Privacy Efficiency

- ◆ Only want to pay for  $c$  queries with that exceed threshold  $T$ 
  - Assume all queries have sensitivity  $S$
- ◆ Compute noisy threshold  $T' = T + \text{Lap}(2S/\epsilon)$
- ◆ For each query, add noise  $\text{Lap}(2Sc/\epsilon)$ , only output if above  $T'$
- ◆ Result is  $\epsilon$ -DP
  - For “suppressed” answers, probability of seeing same output is about the same as if  $T'$  was a little higher on neighboring input
  - For released answers, DP follows from Laplace mechanism
- ◆ Result is reasonably accurate: with high probability,
  - All suppressed answers are smaller than  $T + \alpha$
  - All released answers have error at most  $\alpha$for parameter  $\alpha(c, 1/\epsilon, S)$ , and at most  $c$  query answers  $> T - \alpha$

# Sparse Vector Technique

- ◆ **Sparse Vector Technique** allows us to save on privacy budget
  - When asking multiple questions, most of which are negative
- ◆ **Setting:** private input vector  $D$ , threshold  $T$ , budget  $\epsilon$ , limit  $c$ 
  - List of queries  $Q_i$  whether  $Q_i(D) > T$ ? Sensitivity of all queries  $< \Delta$
- ◆ Initialize: count = 0,  $\rho = \text{Lap}(2 \Delta/\epsilon)$
- ◆ For each query  $i$ 
  - Local noise  $v_i = \text{Lap}(4c \Delta / \epsilon)$
  - If  $Q_i(D) + v_i \geq T + \rho$  then
    - output “over threshold”, increment count, abort if count  $\geq c$
  - Else, output “under threshold”

# Sparse Vector Technique

- ◆ **Optimization:** can choose how to split budget between local noise  $v_i$  and global noise  $\rho$ 
  - Give more to  $v_i$ , because of the factor of  $c$
- ◆ Can easily have a different threshold for each query
- ◆ **Caution needed:**  
multiple incorrect versions of SVT have been published!
  - They neglected to use cutoff limit  $c$ , or applied noise incorrectly
- ◆ If we know all  $Q_i$  in advance, can use EM to sample from them
  - Empirically, more accurate than SVT in practice!

# Multiplicative weights [Hardt et al 12]

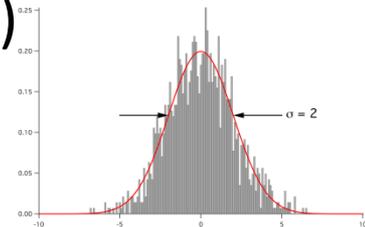
- ◆ The idea of “multiplicative weights” widely used in optimization
  - Up-weight ‘good’ answers, down-weight ‘poor’ answers
  - Applied to output of DP mechanism
- ◆ **Set-up:**
  - (Private) input, represented as vector  $D$  with  $n$  entries
  - $Q$ , set of queries over  $x$  (matrix)
  - $T$ , bound on number of iterations
  - **Output:**  $\epsilon$ -DP vector  $A$  so that  $Q(A) \approx Q(D)$

# Multiplicative Weights Algorithm

- ◆ Initialize vector  $A_0$  to assign uniform weight for each value
- ◆ For  $i=1$  to  $T$ :
  - Exponential Mechanism ( $\epsilon/2T$ ) to sample  $j$  prop. to  $|Q_j(A_i) - Q_j(D)|$ 
    - Try to find query with large error
  - Laplace Mechanism to estimate  $\Delta = (Q_j(A) - Q_j(D)) + \text{Lap}(2T/\epsilon)$ 
    - Error in the selected query
  - Set  $A_i = A_{i-1} \cdot \exp(\Delta Q_j(D)/2n)$ , normalize so that  $A_i$  is a distribution
    - (Noisily) reward good answers, penalize poor answers
- ◆ Output  $A = \text{average}_i nA_i$  — or just output  $A_n$ 
  - Privacy follows via sequential composition of EM and LM steps
  - Accuracy (should) improve in each iteration, up to  $\log$  iterations

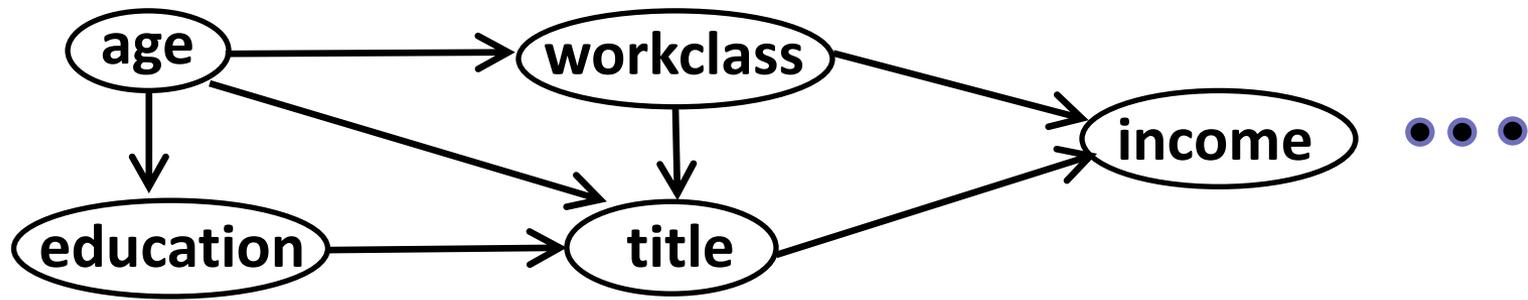
# Differential privacy for data release

- ◆ Differential privacy is an attractive model for data release
  - Achieve a fairly robust statistical guarantee over outputs
- ◆ **Problem:** how to apply to data release where  $f(x) = x$ ?
  - Trying to use global sensitivity does not work well
- ◆ **General recipe:** find a model for the data (e.g. PSDs)
  - Choose and release the model parameters under DP
- ◆ A new tradeoff in picking suitable models
  - Must be robust to privacy noise, as well as fit the data
  - Each parameter should depend only weakly on any input item
  - Need different models for different types of data
- ◆ Next 3 (biased) examples of recent work following this outline



# Example 1: PrivBayes [Zhang et al. 14]

- ◆ Directly materializing tabular data: low signal, high noise
- ◆ Use a **Bayesian network** to approximate the full-dimensional distribution by lower-dimensional ones:



$$\begin{aligned} \Pr[\mathbf{H}] \approx & \Pr[\text{age}] \cdot \Pr[\text{education}|\text{age}] \cdot \Pr[\text{workclass}|\text{age}] \cdot \\ & \Pr[\text{title}|\text{age,education,workclass}] \cdot \Pr[\text{income}|\text{workclass,title}] \cdot \\ & \Pr[\text{marital status}|\text{age,income}] \cdots \end{aligned}$$

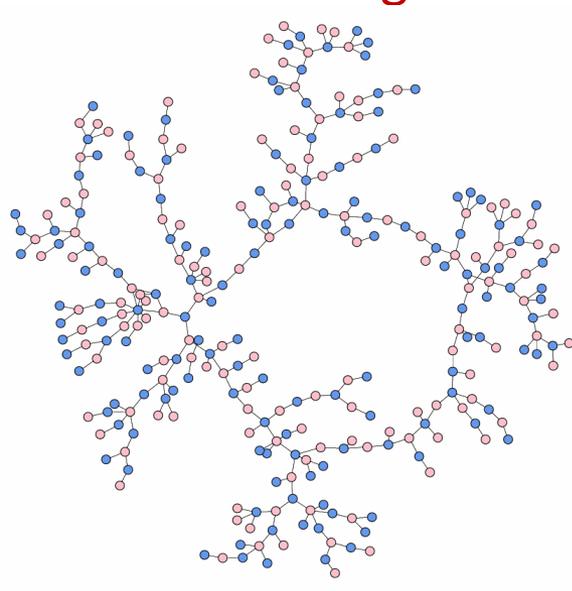
low-dimensional distributions: **high signal-to-noise**

# PrivBayes (SIGMOD14)

- ◆ **STEP 1:** Choose a suitable Bayesian Network BN
  - in a differentially private way
  - sample (via exponential mechanism) edges in the network
  - design surrogate quality function with low sensitivity
- ◆ **STEP 2:** Compute distributions implied by edges of BN
  - straightforward to do under differential privacy (Laplace)
- ◆ **STEP 3:** Generate synthetic data by sampling from the BN
  - post-processing: no privacy issues
- ◆ Evaluate utility of synthetic data for variety of different tasks
  - performs well for multiple tasks (classification, regression)

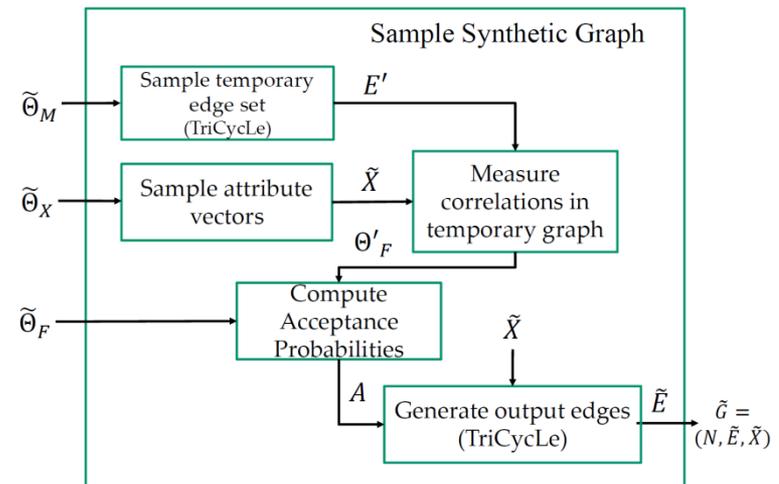
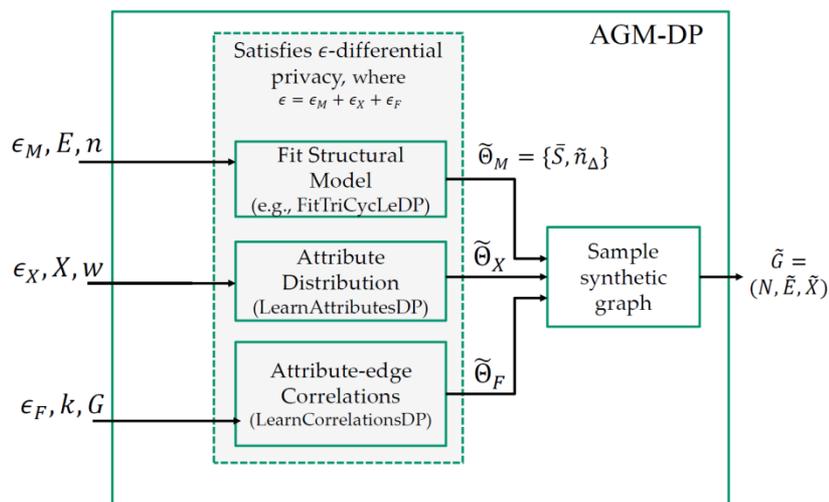
# Example 2: Graph Data

- ◆ Releasing graph structured data remains a big challenge
  - Each individual (node) can have a big impact on graph structure
- ◆ **Most current work** focuses on releasing graph statistics
  - Counts of small subgraphs like stars, triangles, cliques etc.
  - These counts are parameters for graph models
  - **Sensitivity of these counts is large**: one edge can change a lot

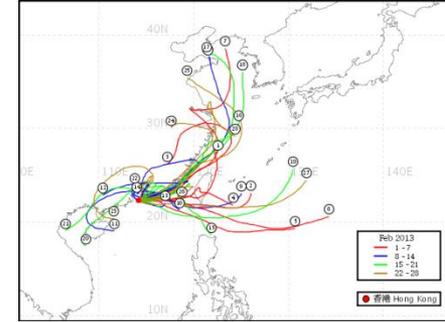


# Attributed Graph Data [Jorgensen et al. 16]

- ◆ Real graphs (e.g. social networks) have attributes
  - Different types of node, different types of edge
- ◆ Define graph models that have attribute distributions
  - Capture real graph structure e.g. number of triangles
- ◆ Learn parameters from input graphs (under differential privacy)
- ◆ Sample “realistic” graphs from the learned model

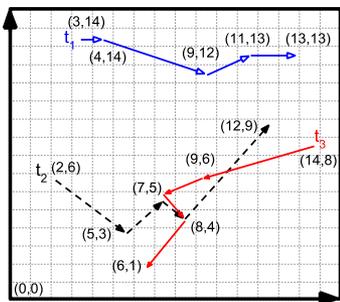


# Example 3: Trajectory Data

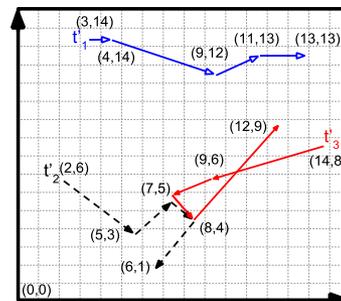


- ◆ More and more location and mobility data available
  - From GPS enabled devices, approximate location from wifi/phone
- ◆ Location and movements are **very sensitive!**
- ◆ Location and movements are **very identifying!**
  - Easy to identify ‘work’ and ‘home’ locations from traces
  - 4 random points identify 95% of individuals [Montjoye et al 2013]
- ◆ Aim for **Differentially Private Trajectories** [He et al. 15]
  - Find a model that works for trajectory data
  - Based on Markov models at multiple resolutions

# Original Trajectories



# Synthetic Trajectories



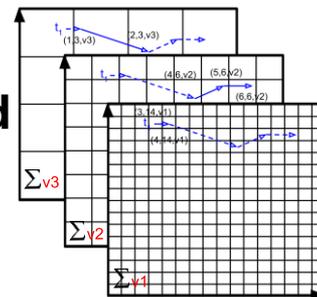
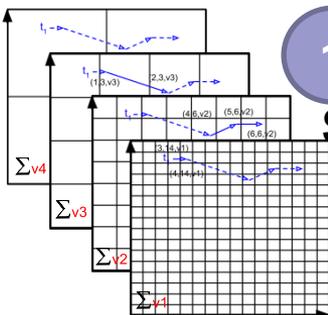
## DPT System Overview



1 Hierarchical Reference System Mapping

6

Direction-weighted Sampling



4 Noise Infusion

5

Adaptive Pruning

2 Prefix Tree Construction

3 Model Selection

5

# Other topics

- ◆ Huge amount of work in DP across theory, security, DB...
- ◆ Many topics not touched on in this tutorial:
  - Connections to game theory and auction design
  - Mining primitives: regression, clustering, frequent itemsets
  - Efforts in programming languages and systems to support DP
  - Variant definitions:  $(\epsilon, \delta)$ -DP, other privacy/adversary models
  - Lower bounds for privacy (what is not possible)
  - Applications to graph data (social networks), mobility data etc.
  - Applications to machine learning: classifiers that don't leak
  - Privacy over data streams: pan-privacy and continual observation

# State of Anonymization

- ◆ Data privacy and anonymization is a subject of ongoing research today
- ◆ **Many unresolved challenges:**
  - How can a social network release a substantial data set without revealing private connections between users?
  - How can a video website release information on viewing patterns without disclosing who watched what?
  - How can a search engine release information on search queries without revealing who searched for what?
  - How to release private information efficiently over large scale data?

# Concluding Remarks

- ◆ Differential privacy can be applied effectively for data release
- ◆ **Care is still needed** to ensure that release is allowable
  - Can't just apply DP and forget it: must analyze whether data release provides sufficient privacy for data subjects
- ◆ Many open problems remain:
  - **Transition** these techniques to tools for data release
  - Want data in same form as input: **private synthetic data?**
  - Allow **joining** anonymized data sets accurately
  - Obtain alternate (workable) **privacy definitions**

**Thank you!**

# References – Basic Building Blocks

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## ◆ Geometric Mechanism

- Universally utility-maximizing privacy mechanisms. Arpita Ghosh, Tim Roughgarden, Mukund Sundararajan. STOC 2009

## ◆ Sequential and Parallel Composition, Median Example

- Privacy integrated queries: an extensible platform for privacy-preserving data analysis. Frank McSherry. SIGMOD 2009.

## ◆ Exponential Mechanism

- Mechanism Design via Differential Privacy. Frank McSherry and Kunal Talwar. FOCS, 2007

# References – Applications & Transforms

## ◆ Spatial Data Application

- Differentially private spatial decompositions. Graham Cormode, Magda Procopiu, Entong Shen, Divesh Srivastava, and Ting Yu. In International Conference on Data Engineering (ICDE), 2012

## ◆ Data Transforms

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- Privacy, accuracy, and consistency too: a holistic solution to contingency table release. Boaz Barak, Kamalika Chaudhuri, Cynthia Dwork, Satyen Kale, Frank Mcsherry, Kunal Talwar. PODS 2007
- Differentially Private Aggregation of Distributed Time-Series with Transformation and Encryption. Vibhor Rastogi and Suman Nath, SIGMOD 2010

# References – Advanced Mechanisms

## ◆ Smooth Sensitivity, Sample and Aggregate

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## ◆ Sparse Data Processing

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# References – DP for Data Release

- PrivBayes: Private Data Release via Bayesian Networks. Jun Zhang, Graham Cormode, Cecilia M. Procopiuc, Divesh Srivastava, Xiaokui Xiao. ACM Trans. Database Syst. 42(4): 25:1-25:41, 2017
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- Xi He, Graham Cormode, Ashwin Machanavajjhala, Cecilia M. Procopiuc, Divesh Srivastava. DPT: Differentially Private Trajectory Synthesis Using Hierarchical Reference Systems. VLDB 2015