Locally Private Release of Marginal Statistics

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Privacy with a coin toss

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- **Scenario.** Each user has a single private bit of information
  - Encoding e.g. political/sexual/religious preference, illness, etc.
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  – Can ‘unbias’ the estimate (if we know $p$) of the population fraction
  – The error in the estimate is proportional to $\frac{1}{\sqrt{N}}$
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♦ **Analysis.** Gives differential privacy with parameter $\epsilon = \ln{(p/(1-p))}$
  – Works well in theory, but would anyone ever use this?
Privacy in practice
Differential privacy based on coin tossing is widely deployed
  – In Google Chrome browser, to collect browsing statistics
  – In Apple iOS and MacOS, to collect typing statistics
  – This yields deployments of over 100 million users
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- Local Differential privacy is state of the art in 2017:
  Randomized response invented in 1965: five decade lead time!
Going beyond 1 bit of data

1 bit can tell you a lot, but can we do more?

- **Recent work**: materializing marginal distributions
  - Each user has $d$ bits of data (encoding sensitive data)
  - We are interested in the distribution of combinations of attributes
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Nail, meet hammer

- Could apply Randomized Response to each entry of each marginal
  - To give an overall guarantee of privacy, need to change $p$
  - The more bits released by a user, the closer $p$ gets to $\frac{1}{2}$ (noise)
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- **First observation**: a sampling trick
  - If we release $n$ bits of information per user, the error is $\frac{n}{\sqrt{N}}$
  - If we sample 1 out of $n$ bits, the error is $\sqrt{\frac{n}{N}}$
  - Quadratically better to sample than to share!
What to materialize?

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Still using randomized response on each entry

- Approach 1 (marginals): cost proportional to $2^{3k/2} d^{k/2} / \sqrt{N}$
- Approach 2 (full): cost proportional to $2^{(d+k)/2} / \sqrt{N}$
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Different approaches based on how information is revealed

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♦ If $k$ is small (say, 2), and $d$ is large (say 10s), Approach 1 is better
   - But there’s another approach to try...
Hadamard transform

Instead of materializing the data, we can transform it:
- Via Hadamard transform (the discrete Fourier transform for the binary hypercube)
  - Simple and fast to apply

\[
\begin{bmatrix}
H^* & H^*
\end{bmatrix}
= \begin{bmatrix}
-1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 \\
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- Yields error proportional to \(2^{k/2}d^{k/2}/\sqrt{N}\)
  - Better than both previous methods (in theory)
Empirical behaviour

- Compare three methods: Hadamard based (Inp_HT), marginal materialization (Marg_PS), Expectation maximization (Inp_EM)
- Measure sum of absolute error in materializing 2-way marginals
- $N = 0.5M$ individuals, vary privacy parameter $\varepsilon$ from 0.4 to 1.4
Applications – χ-squared test

- Anonymized, binarized NYC taxi data
- Compute χ-squared statistic to test correlation
- Want to be same side of the line as the non-private value!
Application – building a Bayesian model

- **Aim:** build the tree with highest mutual information (MI)
- **Plot shows MI on the ground truth data for evaluation purposes**