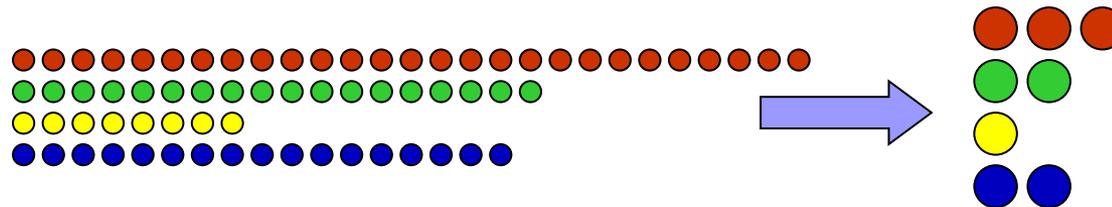


Engineering Streaming Algorithms



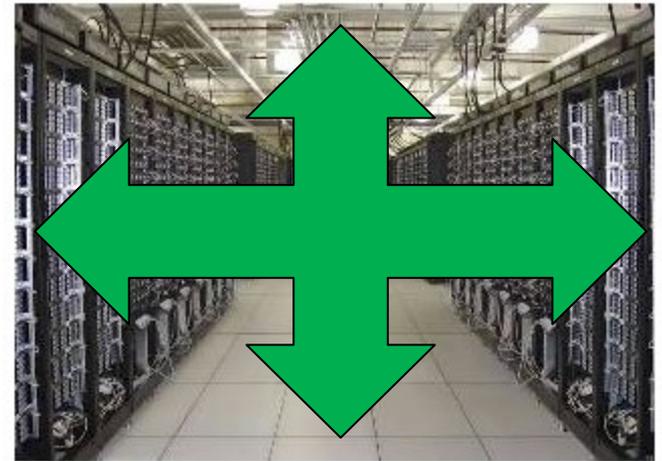
Graham Cormode

University of Warwick

G.Cormode@Warwick.ac.uk

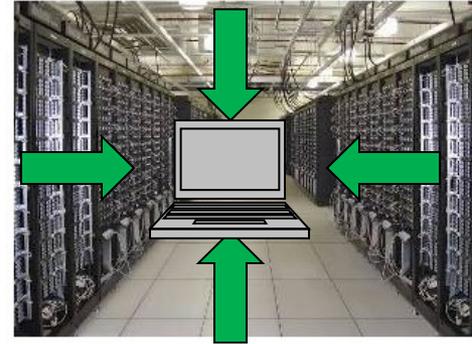
Computational scalability and “big” data

- Most work on massive data tries to **scale up the computation**
- Many great technical ideas:
 - Use many cheap commodity devices
 - Accept and tolerate failure
 - Move code to data, not vice-versa
 - MapReduce: BSP for programmers
 - Break problem into many small pieces
 - Add layers of abstraction to build massive DBMSs and warehouses
 - Decide which constraints to drop: noSQL, BASE systems
- Scaling up comes with its disadvantages:
 - Expensive (hardware, equipment, **energy**), still not always fast
- This talk is not about this approach!



Downsizing data

- A second approach to computational scalability: **scale down the data** as it is seen!
 - A compact representation of a large data set
 - Capable of being analyzed on a single machine
 - What we finally want is small: human readable analysis / decisions
 - Necessarily gives up some accuracy: **approximate answers**
 - Often **randomized** (small constant probability of error)
 - Much relevant work: samples, histograms, wavelet transforms
- Complementary to the first approach: not a case of either-or
- **Some drawbacks:**
 - Not a general purpose approach: need to fit the problem
 - Some computations don't allow any useful summary

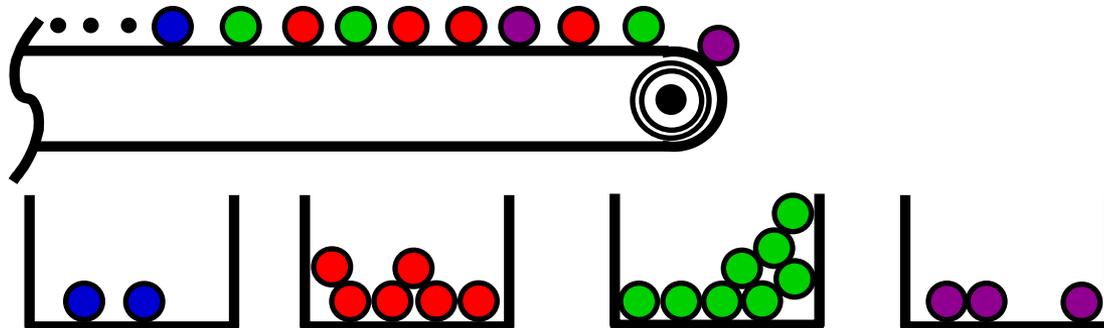


Outline for the talk

- The frequent items problem
- Engineering streaming algorithms for frequent items
 - From algorithms to prototype code
 - From prototype code to deployed code
- **Next steps:** robust code, other hardware targets
- Bulk of the talk is on two (actually, one) very simple algorithms
 - Experience and reflections on a ‘simple’ implementation task

The Frequent Items Problem

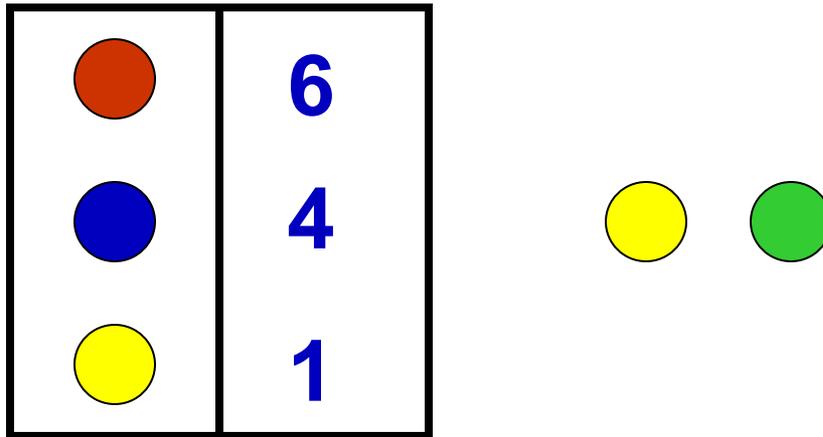
- The **Frequent Items Problem** (aka Heavy Hitters):
given stream of N items, find those that occur most frequently
 - E.g. Find all items occurring more than 1% of the time
- Formally “hard” in small space, so allow approximation
- Find all items with count $\geq \phi N$, none with count $< (\phi - \epsilon)N$
 - Error $0 < \epsilon < 1$, e.g. $\epsilon = 1/1000$
 - Related problem: estimate each frequency with error $\pm \epsilon N$



Why Frequent Items?

- A natural **question** on streaming data
 - Track bandwidth hogs, popular destinations etc.
- The subject of much streaming **research**
 - Scores of papers on the subject
- A core streaming **problem**
 - Many streaming problems connected to frequent items (itemset mining, entropy estimation, compressed sensing)
- Many practical **applications** deployed
 - In search log mining, network data analysis, DBMS optimization

Misra-Gries Summary (1982)



- **Misra-Gries (MG)** algorithm finds up to k items that occur more than $1/k$ fraction of the time in the input
- **Update:** Keep k different candidates in hand. For each item:
 - If item is monitored, increase its counter
 - Else, if $< k$ items monitored, add new item with count 1
 - Else, decrease all counts by 1

Frequent Analysis

- **Analysis:** each decrease can be charged against k arrivals of different items, so no item with frequency N/k is missed
- Moreover, $k=1/\epsilon$ counters estimate frequency with error ϵN
 - Not explicitly stated until later [Bose et al., 2003]
- **Some history:** First proposed in 1982 by Misra and Gries, rediscovered twice in 2002
 - Later papers discussed how to make fast implementations

Merging two MG Summaries [ACHPWY '12]

■ Merge algorithm:

- Merge the counter sets in the obvious way
- Take the $(k+1)$ th largest counter = C_{k+1} , and subtract from all
- Delete non-positive counters
- Sum of remaining counters is M_{12}

■ This keeps the same guarantee as Update:

- Merge subtracts at least $(k+1)C_{k+1}$ from counter sums
- So $(k+1)C_{k+1} \leq (M_1 + M_2 - M_{12})$
- By induction, error is

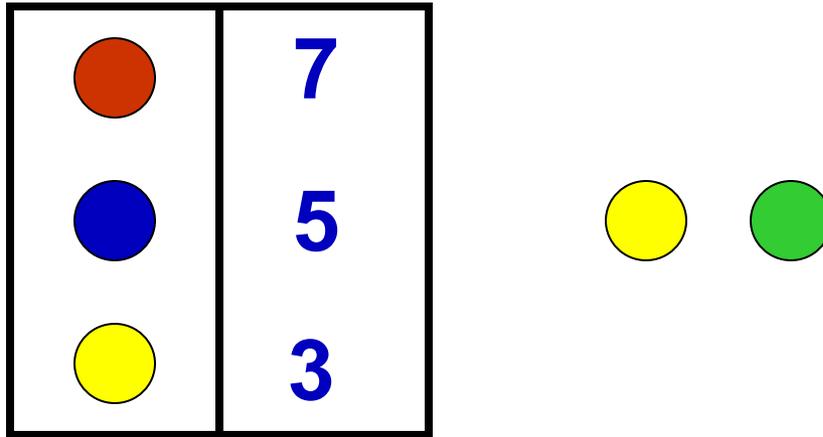
$$((N_1 - M_1) + (N_2 - M_2) + (M_1 + M_2 - M_{12})) / (k+1) = ((N_1 + N_2) - M_{12}) / (k+1)$$

(prior error)

(from merge)

(as claimed)

SpaceSaving Algorithm



- “SpaceSaving” (SS) algorithm [Metwally, Agrawal, El Abaddi 05] is similar in outline
- Keep $k = 1/\epsilon$ item names and counts, initially zero
Count first k distinct items exactly
- On seeing new item:
 - If it has a counter, increment counter
 - If not, replace item with least count, increment count

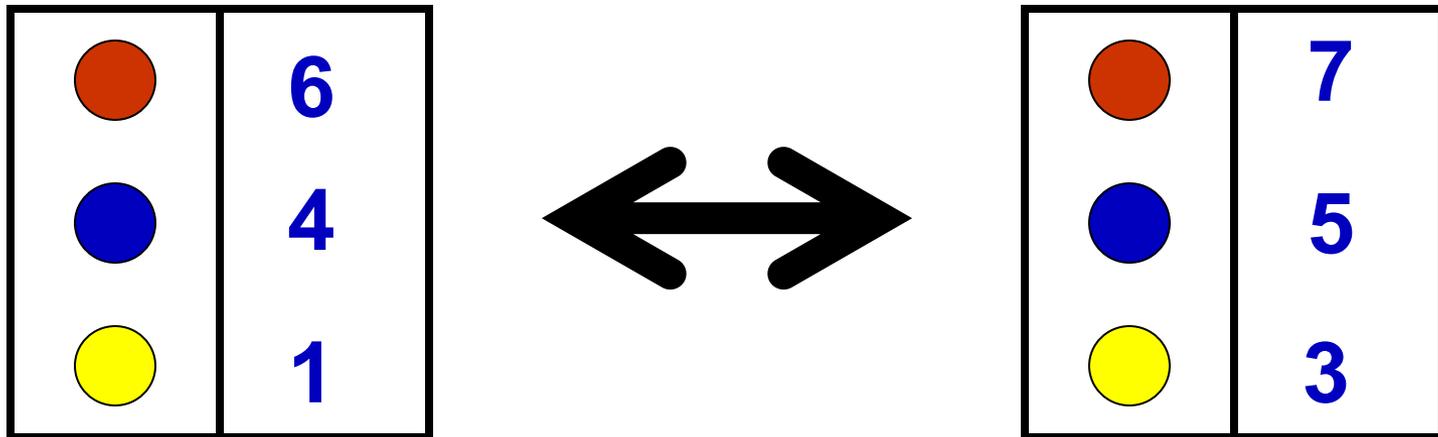
SpaceSaving Analysis

- Smallest counter value, \min , is at most ϵn
 - Counters sum to n by induction
 - $1/\epsilon$ counters, so average is ϵn : smallest cannot be bigger
- True count of an uncounted item is between 0 and \min
 - Proof by induction, true initially, \min increases monotonically
 - Hence, the count of any item stored is off by at most ϵn
- Any item x whose true count $> \epsilon n$ is stored
 - By contradiction: x was evicted in past, with count $\leq \min_t$
 - Every count is an overestimate, using above observation
 - So est. count of $x > \epsilon n \geq \min \geq \min_t$, and would not be evicted

So: Find all items with count $> \epsilon n$, error in counts $\leq \epsilon n$

Two algorithms, or one?

- **A belated realization:** SS and MG are the same algorithm!
 - Can make an isomorphism between the memory state
- **Intuition:** “overwrite the min” is conceptually equivalent to delete elements with (decremented) zero count
- The two perspectives on the same algorithm lead to different implementation choices

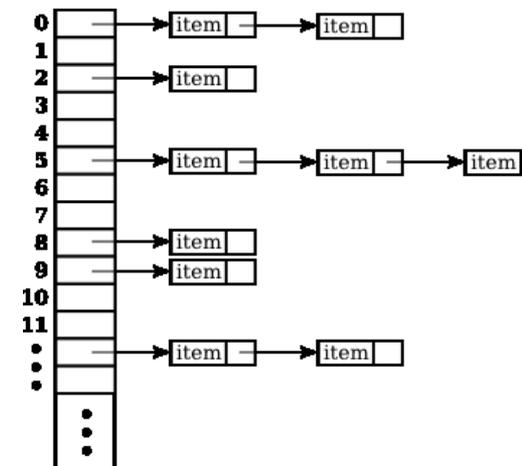
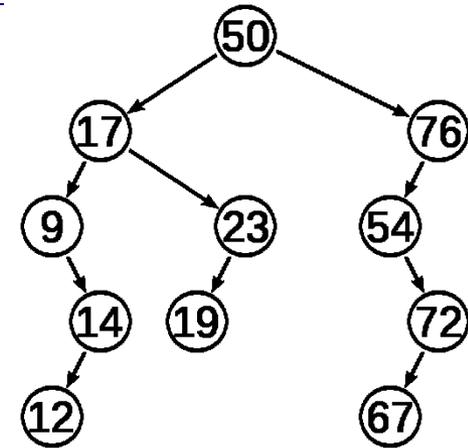


Implementation Issues

- These algorithms are really simple, so should be easy... right?
- There is surprising subtlety in implementing them
- **Basic steps:**
 - **Lookup** is current item stored? If so, update count
 - If not:
 - **Find min** weight item and overwrite it (SS)
 - **Decrement counts** and delete zero weights (MG)
- Several implementation choices for each step
 - **Optimization goals:** speed (throughput, latency) and space
 - I discuss my implementation experience and current thoughts

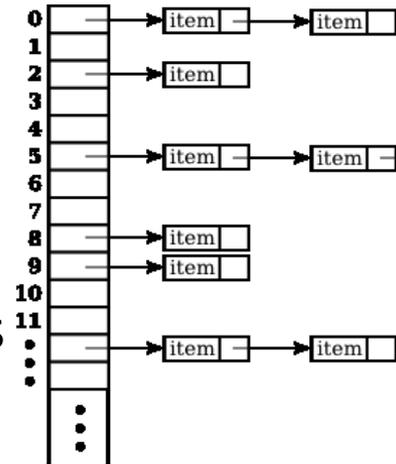
Lookup Item

- **Lookup**: is current item stored
 - The canonical dictionary data structure problem
- **Misra Gries paper**: use balanced search tree
 - $O(\log k)$ worst case time to search
- **Hash table**: hash to $O(k)$ buckets
 - $O(1)$ expected time, but now alg is randomized
 - May have bad worst case performance?
 - How to handle collisions and deletions?
 - (My implementations used chaining)
 - Could surely be further optimized...
 - Use cuckoo hashing or other options?
 - Can we use fact that table occupancy is guaranteed at most k ?



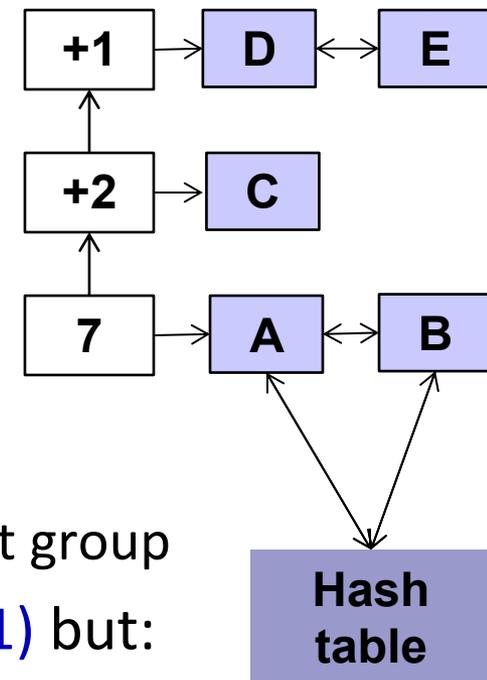
Decrement Counts

- Decrement counts could be done simply
 - Iterate through all counts, subtract by one
 - A blocking operation, $O(k)$ time
- Proof of correctness means it happens $< n/k$ times
 - So would be $O(1)$ cost amortized...
 - (considered too fiddly to deamortize when I implemented)
 - Multithreaded/double buffered approach could simplify



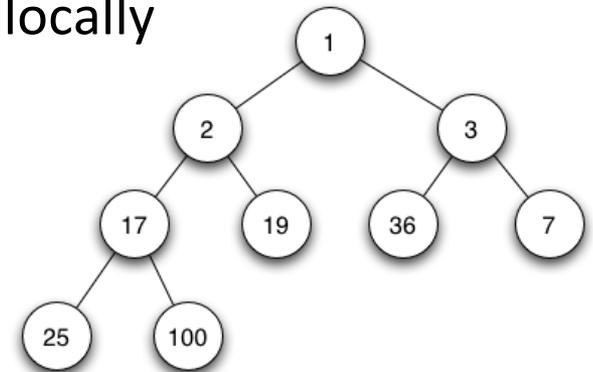
Decrement Counts: linked list approach

- **Linked list approach** (Demaine et al. 02):
 - Keep elements in a list sorted by frequency
 - Store the difference between successive items
 - Decrement now only affects the first item
- But increments are more complicated:
 - Keep elements with same frequency in a group
 - Since we only increase count by 1, move to next group
- Increments and decrements now take time $O(1)$ but:
 - Non-standard, lots of cases (housekeeping) to handle
 - Forward and backward pointers in circular linked lists
 - Significant space overhead (about 6 pointers per item)



Overwrite min

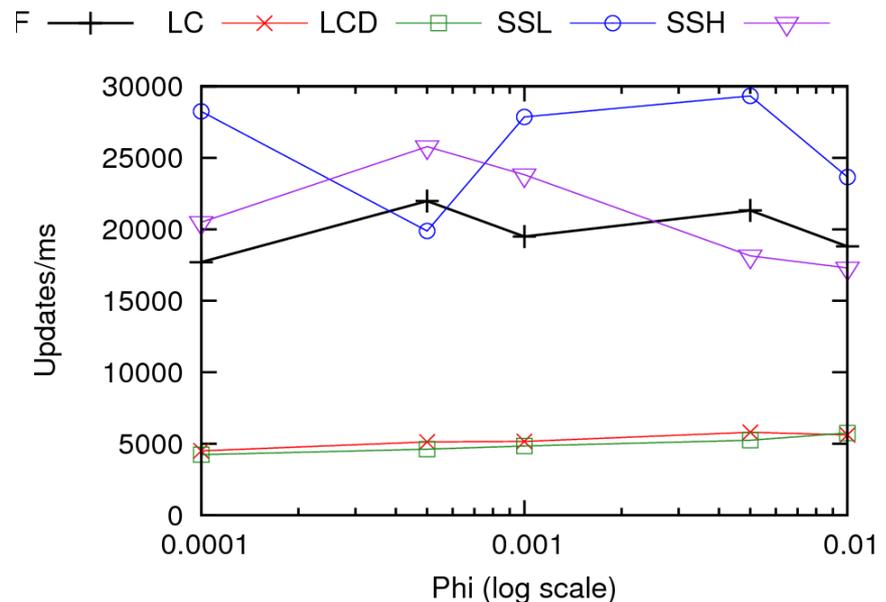
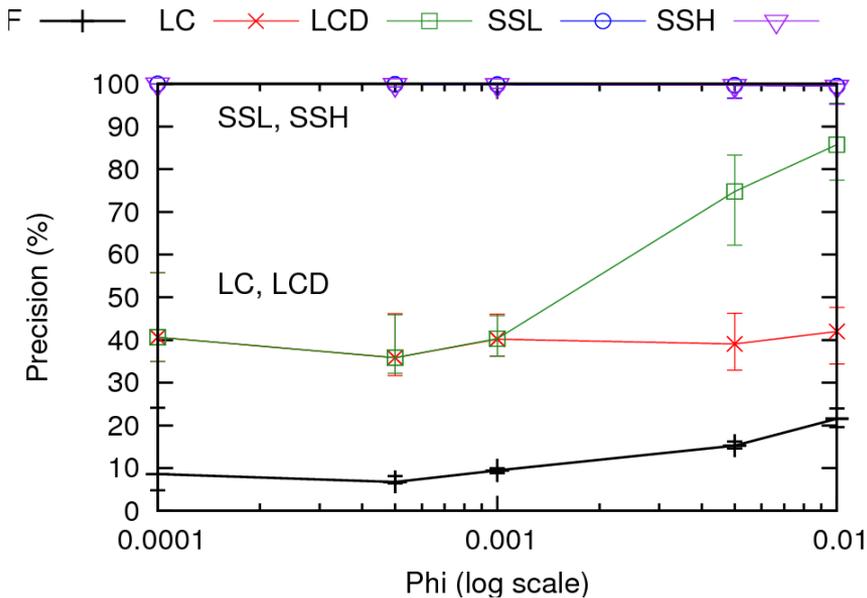
- Could also adapt the linked list approach
 - Keep items in sorted order, overwrite current min
- Findmin is a more standard data structure problem
 - Could use a minheap (binary, binomial, fibonacci...)
 - Increments easy: update and reheapify $O(\log k)$
 - Probably faster, since only adding one to the count
 - All operations $O(\log k)$ worst case, but may be faster “typically”:
 - Heap property can often be restored locally
 - Head of heap likely to be in cache
 - Access pattern non-uniform?



Experimental Comparison

- Implementation study (several years old now)
 - Best effort implementations in C (use a different language now?)
 - All low-level data structures manually implemented (using manual memory management)
 - <http://hadjieleftheriou.com/frequent-items/index.html>
- Experimental comparison highlights some differences not apparent from analytic study
 - E.g. algorithms are often more accurate than worst-case analysis
 - Perhaps because real inputs are not worst-case
- Compared on a variety of web, network and synthetic data

Frequent Algorithms Experiments

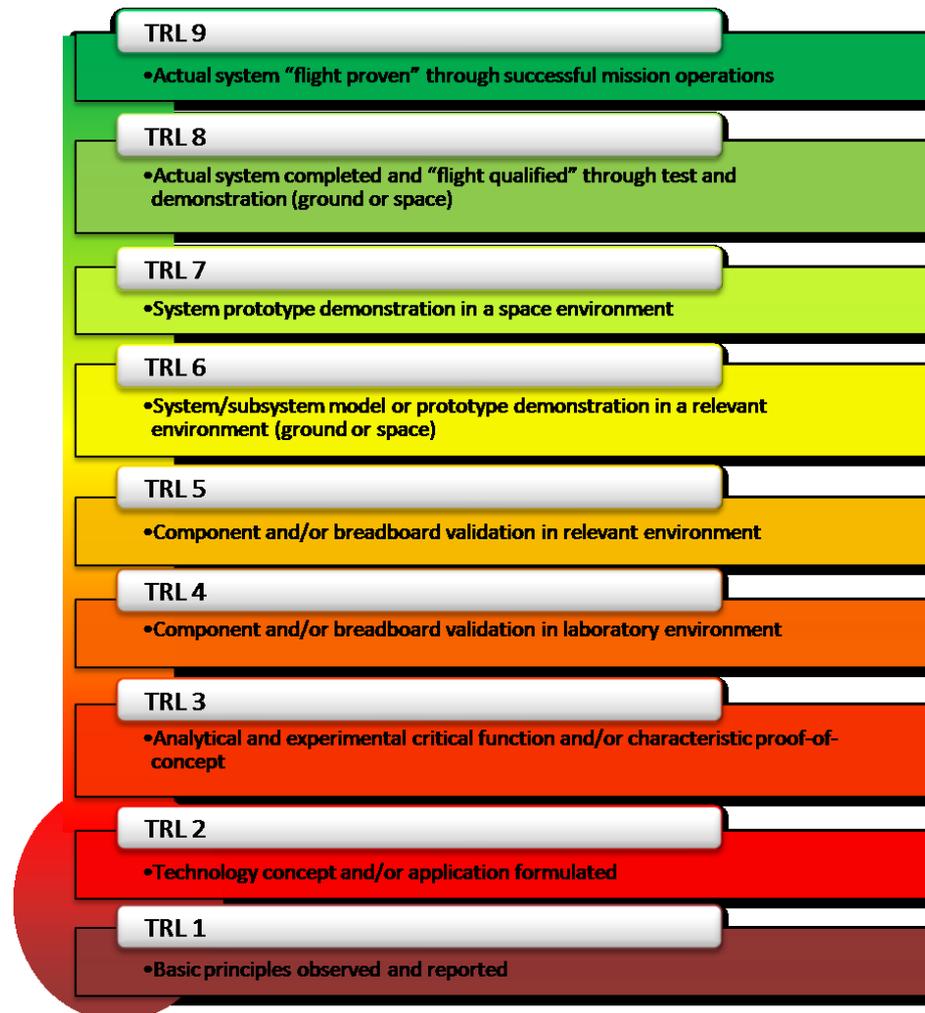


- Two implementations of **SpaceSaving** (SSL, SSH) achieve perfect accuracy in small space (10KB – 1MB)
- Misra Gries (F) has worse accuracy: different estimator used
- **Very fast**: 20M – 30M updates *per second*
 - Heap seems faster than linked list approach

Frequent Algorithms Summary

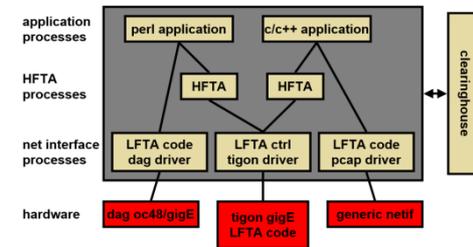
- These algorithms very efficient for arrivals-only case
 - Use $O(1/\epsilon)$ space, guarantee ϵN accuracy
 - Very fast in practice (many millions of updates per second)
- Similar algorithms, but a surprisingly clear “winner”
 - Over many data sets, parameter settings, **SpaceSaving** algorithm gives appreciably better results
- Many implementation details even for simple algorithms
 - “**Find if next item is monitored**”: search tree, hash table...?
 - “**Find item with smallest count**”: heap, linked lists...?
- Not much room left for improvement in core algorithm?
 - Maybe more explicitly model input distributions (skewed)?

Ready for prime time?



Streaming in practice: Packet stream analysis

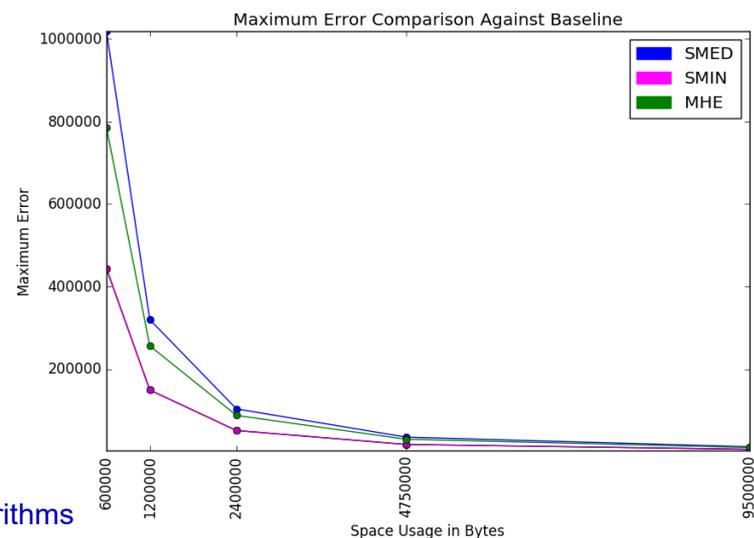
- **AT&T Gigascope / GS tool**: stream data analysis
 - Developed since early 2000s
 - Based on commodity hardware + Endace packet capture cards
- High-level (SQL like) language to express continuous queries
 - Allows “User Defined Aggregate Functions” (UDAFs) plugins
 - Sketches in gigascope since 2003 at network line speeds (Gbps)
 - Flexible use of streaming algs to summarize behaviour in groups
 - Rolled into standard query set for network monitoring
 - Software-based approach to attack, anomaly detection
- **Current status**: latest generation of GS in production use at AT&T
Also in Twitter analytics, Yahoo, other query log analysis tools



More Recent Progress

[Anderson et al '17] report their experience at Yahoo!

- **Delete min** operation can be amortized over multiple steps
- Instead of deleting based on min of k , used median of $2k$ counts
- Estimate median by sampling rather than quickselect
- May be seen as similar to a merge and prune approach
- Several times faster again than heap-based method
- Moderately increased error compared to delete min
- Java sketch library:
<https://datasketches.github.io/>



Conclusions

- Finding the frequent items is one of the most studied problems in data streams
 - Continues to intrigue researchers (for better or worse)
 - Many variations proposed (for weighted or negative updates)
 - Algorithms have been deployed in Google, AT&T, elsewhere...
 - New variants continue to be suggested
- Other streaming primitives have been similarly engineered
 - E.g. Bloom Filters, Hyperloglog (Heule et al '13), Quantiles
 - More general sketches that can handle deletions and insertions
- Areas for more work:
 - Allow easier composition of algorithms
 - Adapt to new models (parallel, distributed, FPGA/GPU)