The Rush to Calculus

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This article reports on the extent to which students continued their high school math acceleration in their first year at Rutgers, on the basis of a study of their high school and college transcripts. (Hint: Not too many did.)

Introduction

During recent years, the number of students taking calculus in high school has increased dramatically. Indeed, the number of students taking AP Calculus in high school is now slightly greater than the number of students taking first-semester calculus in colleges (including two-year colleges).[1] And an equal number of students take non-AP calculus courses in high school, so that nowadays altogether only about 1/3 of the students who take calculus see that subject for the first time in college.[2]

We need to be aware that two generations ago, essentially no one (including myself) took calculus in high school; indeed, most students began college with "analytic geometry," a topic which is now incorporated into high school precalculus courses. AP calculus was introduced in order to provide the very best math students with "advanced placement," a way to start their college careers a semester or two ahead and get to more advanced mathematics more quickly.

However, for a variety of reasons, providing this option for a few students has, over the past 25 years, been transformed into a policy of accelerating the curriculum so that large numbers of students could take advantage of this "advanced placement" option.[3] After all, shouldn't everyone have the opportunity to be "advanced"?

In order to get to calculus in high school, a student would have to complete four courses – Algebra I, Geometry, Algebra II, and Precalculus – by the end of the 11th grade. The most mathematically talented students did this by taking two courses in the same year, one in Geometry and one in Algebra. However, as the calculus rolls expanded, this was more often accomplished by starting with Algebra I in the 8th grade. Indeed, there are those nowadays who advocate that all students should take Algebra I in the 8th grade so that they will be able to take a calculus course in high school, and there are districts and states that have adopted this strategy, even though many teachers of mathematics at the 8th grade level have an inadequate mathematics background and lack the certification necessary to teach high school math courses.

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[1] According to data from the Conference Board of Mathematical Sciences (CBMS) survey in 2007, the number of students taking mainstream Calculus I courses in 4-year colleges and universities was about 200,000, and the number in 2-year colleges was about 50,000, whereas the enrollment in AP Calculus annually is well over 250,000. Reported by David Bressoud in "The Crisis in Calculus" [3].

[2] Based on data from NAEP, about 500,000 students studied calculus in high school in 2004-2005. Reported in "The Crisis in Calculus" [3].

[3] Between 1980 and 2008, the number of AP Calculus Exams increased almost 10-fold, from about 30,000 per year to nearly 300,000 per year.

In the face of the "calculus juggernaut[4]," it is important to consider whether the "acceleration strategy" is successful, that is, to describe the goals of this strategy and to determine whether it is achieving its goals.

The 2007 report by the National Academy of Sciences, "Rising Above the Gathering Storm" [8], sees offering more AP Calculus courses in high school as a major way of addressing the country's pipeline problem, that is, of encouraging more students to consider pursuing STEM careers (that is, careers in science, engineering, mathematics, and technology), and recommends training large numbers of high school math teachers to teach AP Calculus. This recommendation assumes that encouraging more students to take AP calculus will indeed increase the pipeline and ignores the fact that there are already large numbers of experienced teachers at the college level who are prepared to teach calculus.

This article considers the question of what problems the "rush to calculus" and the accompanying "rush to algebra" are intended to solve, and provides some data that show the extent to which they appear to address these problems. Although the data is based on high school and college courses taken by students at one university, it might well point to similar patterns throughout the country.

The data will also provide high school math teachers with some ammunition that they can use to respond to administrators and parents who are caught up in the rush to calculus.

Literature

The main source for information for what is already known about AP Calculus is a brief survey article in June 2009 by David M. Bressoud[5] entitled "AP Calculus: What We Know" [1] and his earlier (2007) article "The Crisis of Calculus," [3] both in the "Launchings" series of the Mathematical Association of America (MAA). Information from these articles was cited in the Introduction above.

A detailed summary of the main points of the Bressoud articles will be provided at the end of this paper, with comments on those points reflecting the findings of the present study. The data from the present study will also provide some answers to the questions that he raises in the 2007 article concerning the problems resulting from the calculus juggernaut. After noting that "we need a much better grasp of the extent and nature of the problem[6]," Bressoud asks a number of questions including the following:

a. Of the students who receive college credit for calculus studied in high school, how many never go on to the next mathematics class?
b. How many choose to retake the class they are entitled to skip?
c. How many students take calculus in high school but are deemed inadequately prepared to study calculus when they get to college?
d. Of the students who retake the courses they studied in high school, how many of them now succeed?

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[4] This term is used by David Bressoud in "The Crisis in Calculus" [3].

[5] David Bressoud was then the president of MAA; he is the DeWitt Wallace Professor of Mathematics at Macalester College in St. Paul, Minnesota.

[6] The problem, as described by Bressoud, is that "relatively few of these students earn recognition for college-level work in calculus" and "what is worse, many who arrive in college with a calculus course on their high school transcript are not even considered ready for calculus."

The data described in this article sheds some light on these and other questions Bressoud raises.[7]

Conjectures

Over 25 years ago, long before college admissions were handled electronically and long before IRB approval was needed[8], I physically went through the folders of what seemed like hundreds of Rutgers College first-year students to see what math courses they had taken in high school. I don't know what happened to the raw data that I collected, but my conclusions, which I remember clearly, served as conjectures for the present study. Taken together these conclusions implied that very few students who were accelerated into a calculus course in high school actually took advantage of that acceleration by earning and making use of advanced placement in mathematics. That didn't make much sense to me at the time, but I did not attempt to formalize the study or publish the data.

However, the aforementioned discussions of expanding calculus into high school and algebra into middle school reminded me of that study, and suggested that a more formal study of this type might be valuable and might reveal some interesting and useful information.

I embarked on the present study seven years ago, in 2005, with the following four conjectures and the conclusion that follows from them:

1. About half of the students who take calculus in high school take an AP course,
2. About half of those who take the AP Calculus course take the AP Calculus exam,
3. About half of those who take the AP Calculus exam receive grades of 4 or 5 (sufficient to get college credit at Rutgers), and
4. About half of those who receive grades of 4 or 5 actually continue on an accelerated path.

Conclusion: Only about 1 out of 16 students who are accelerated into a calculus course in high school earn and make use of advanced placement in mathematics.

Methodology

With the assistance and support of the Rutgers Office of Institutional Research[9], I was provided information in 2005 about two groups of students who were part of the entering class at Rutgers College[10] in the Fall 2003 semester – all 332 students who had taken the AP Calculus exam, and a group of 400 students selected randomly from that

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[7] It should be noted that the data for this study was gathered well before Bressoud's article was written, and that this study therefore did not set out to address the questions that were only raised several years later.

[8] Any research study that involves human subjects, even if they are not affected in any way by the study, requires approval from the institution's Institutional Review Board (IRB) for the Protection of Human Subjects in Research.

[9] I would like to thank Tina Grycenko of the Rutgers Office of Institutional Research and its Director, Robert Heffernan, for helping gain approval and cooperation for conducting this study, and for gathering and facilitating my use of the needed information.

[10] Each student entering Rutgers University – New Brunswick in the Fall 2003 semester was enrolled in a college. The largest of the colleges was Rutgers College. Other colleges were Cook College, Douglass College, Livingston College, and the Colleges of Engineering and Pharmacy. All non-professional colleges were subsequently combined into the School of Arts and Sciences.

entering class of 2130 students[11]. The information that I was provided, and on which this study is based, was the following:

a) electronic access to high school transcripts (for the second group);
b) the scores on the students' AP Calculus examinations (for the first group); and
c) the mathematics courses the students had taken at Rutgers during their first two years and the grades they had received (for both groups);

The information requested was what was needed to address the four conjectures above.

- The information in a) would address conjecture 1, since an examination of the transcripts would reveal the number of students who had taken calculus in high school and the number who had taken AP Calculus[12];
- The information in a) and b) would together address conjecture 2, since the high school transcripts would show which of the 400 students took AP Calculus and the scores on the AP Calculus exam would show which of them actually took the exam[13];
- The information in b) would address conjecture 3, since b) provides the scores on the AP Calculus exam; and
- The information in c) would address conjecture 4, since we could determine from students' courses and grades whether they continued their mathematical acceleration at Rutgers.

The information, however, would also make possible an analysis of the consequences of taking, or not taking, Algebra 1 in the 8th grade. Since I had not looked into this question previously, I did not have any specific conjectures in mind about the consequences of this analysis.

Conclusions

Conjecture 1:
About half of the students who take calculus in high school take AP Calculus.

I examined the high school transcripts of 400 students who entered Rutgers College in Fall 2003; in twelve cases, transcripts were not available or not informative[14], so the usable sample was actually 388 students. I found (see Table 1) that 217 of the 388 students (55.9%) had taken a full year of calculus in high school. Of these students, 123 had taken AP Calculus and an additional 94 had taken another calculus course.

[11] The sample size is sufficiently large so that the results obtained for this sample may be extrapolated to the entire group; this article does not discuss the confidence level of these extrapolations. The random sample of 400 students was generated by the Office of Institutional Research.

[12] It is of course possible that the high school transcript might "under-represent" the courses taken by students, for example, by failing to indicate that a calculus course was actually an AP Calculus course; this possibility is discussed later in the article.

[13] It is of course possible, but unlikely, that students could have taken the AP Calculus exam but not have reported their scores to Rutgers; this possibility is discussed later in the article.

[14] For example, math courses were not listed by content but by year – Math 1, Math 2, Math 3, and Math 4.

	Number	Percentage[15]
# of students	400	
# with transcripts	388	
# taking calculus	217	55.9%
taking AP Calculus[16]	123	31.7%
taking Honors calculus	36	9.3%
taking non-Honors calculus	58	14.9%

Table 1: How many took calculus in high school and how many took AP Calculus

The data in the bottom two rows of Table 1 are conservative; that is, a student was counted as having taken calculus only if the title of the course specifically said "calculus" and only calculus. Thus, for example, students who took "advanced math" or "math analysis" were not counted as having taken calculus, even if they had previously completed a course in precalculus. Many of these courses likely had a calculus component, and in some cases calculus may have been the major topic of the course; however, they were not counted in Table 1. Similarly, those who took courses named "trig and calculus," or other such combinations, were not included in Table 1, even though some of these courses may also have addressed mostly calculus.

As to Conjecture 1, of the 217 students who took a full-year calculus course in high school, 123 or 55.9% took an AP Calculus course, somewhat more than the 50% that was conjectured. This may be due, in part, to the conservative approach (see paragraph above) taken to determining which students took calculus courses. As noted above, the sample of 400 students from the class of 2120 students is sufficiently large that we can say that this percentage reflects the entire class – that is, we may conclude that of the class that entered Rutgers College in the Fall 2003 semester, about half of those who took calculus in high school took AP Calculus.

It is noteworthy that over half of all students entering Rutgers College in Fall 2003 have taken a full year of calculus in high school.

It should also be noted that 32 students took an AP Statistics course. Of these students, 11 also took an AP Calculus course; of the remaining 21 students, 6 also took a full-year calculus course. An additional 10 students took a non-AP full-year course in statistics, one of whom also took a full-year calculus course. Thus an additional 24 students, beyond the 217 taking a full-year calculus course, took a full-year statistics course.

Conjecture 2:
About half of those who take the AP Calculus course take the exam.

[15] Percentages in Table 1 were taken of the 388 students whose transcripts were actually available and informative.

[16] The number of students whose transcripts say that they took AP Calculus is fewer than the actual number. We know this because we found two students whose transcripts said simply "calculus" and who took the AP exam according to Rutgers' records. It is likely that there are also students who took the AP Calculus but not the AP exam whose transcripts say that they took "calculus." The number who took "Honors calculus" may be similarly underreported.

Since the high school transcripts do not indicate whether or not a student took the AP Calculus exam, this conjecture could not be tested by an examination of high school transcripts alone. However, I was provided with a list of all students who took the AP Calculus exam, or, more properly, of all students whose scores on the AP Calculus exam were reported to Rutgers. It is reasonable to assume that all students who took the AP Calculus exam did have their scores sent to Rutgers because when students sign up to take the exam they indicate where the scores should be sent and they anticipate (perhaps incorrectly) that their having taken the exam will be counted in their favor.

Altogether 332 of the students entering Rutgers College in the Fall 2003 semester took the AP Calculus exam. I compared the list of the 332 students who took the AP Calculus exam with the list of 123 students in Table 1 who took AP Calculus in high school and obtained the information in Table 2.

	Number	Percentage
# of students	123	
Did not take AP Calculus exam	64	52.0%
Took AP Calculus exam	59	48.0%

Table 2: How many of those who took AP Calculus course took AP Calculus exam.

As to Conjecture 2, 48% of the students who took the AP Calculus course took the AP Calculus exam. As noted above, the sample of 400 students from the class of 2120 students is sufficiently large that we can say that this percentage reflects the entire class – that is, we may conclude that of the class that entered Rutgers College in the Fall 2003 semester, about half who took an AP Calculus course in high school actually took the AP Calculus exam.[17]

It is possible that one can generalize this conclusion to all New Jersey students. Among New Jersey students who go to elite colleges it may be that a higher percentage of those who take the AP Calculus course take the AP Calculus exam, but this effect is likely outweighed by the larger number of New Jersey students who go to colleges whose ranking is lower than that of Rutgers and are less likely to take the AP Calculus exam. However, one cannot generalize this conclusion even further because it is quite possible that some states and some districts require students who take an AP Calculus course to take the AP Calculus exam, denying credit for the course if they do not fulfill this requirement.

Conjecture 3.
About half of those who take the AP Calculus exam receive grades of 4 or 5.

Table 3 records the scores on the AB version of the AP Calculus exam[18] of the 332 students who took the exam. Note that the highest score on the exam is 5 and the

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[17] That the sample of 388 students fairly represented the entire class of 2130 students can be seen in the fact that the 59 students in the sample of 388 students who took the AP Calculus exam is roughly the same proportion as the 332 students in the whole entering class of 2130 students who took the AP Calculus exam – that is 388/2130 (or 18.2%) and 59/332 (or 17.8%) are roughly the same.

[18] The AB course and exam is taken by students who intend to get advanced placement credit for one semester of calculus. Students who intend to get advanced placement credit for two semesters of calculus

lowest is 1. At Rutgers University, students who receive scores of 4 or 5 on the AP Calculus exam may receive 4 credits toward graduation (that is, for first semester calculus), and may start with a second semester calculus course; that is, they receive "advanced placement." Comparable universities likely have a similar policy.

Score on AB exam	# of students	% of students
1	40	12.0%
2	48	14.5%
3	82	24.7%
4	68	20.5%
5	94	28.3%
Total	332	100.0%

Table 3. Scores on AP Calculus Examination AB

As to Conjecture 3, we see that about half of the students who took the AP Calculus exam – 162 out of 332, or 48.8% – received grades of 4 or 5 and therefore received advanced standing[19] and 170 out of 332, or 51.2% did not receive advanced standing.

Students may also take the BC version of the AP Calculus exam and those whose scores are 4 or 5 would be eligible to receive 8 credits toward graduation (that is, for first and second semester calculus), and may start with a third semester calculus course.

Score on AB exam	# of students	# not taking BC	# taking BC exam	Score of 4 or 5 on BC exam	Score of 1, 2, or 3 on BC exam
1	40	40			
2	48	45	3		3
3	82	71	11		11
4	68	*49	19	**5	*14
5	94	*47	47	**45	*2
Total	332	252	80	50	30

Table 4. Scores on AP Calculus Examination BC

Table 4 notes that 80 (or 24.1%) of the 332 students who took the AP Calculus exam took the BC exam. Table 4 also records their scores on the BC exam.

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take the BC course and exam. A substantial portion of the BC exam covers AB material and so students who take the BC exam also receive an AB grade.

[19] This conclusion is derived from the data about all students who took the AP Calculus Examination. If we focus on the 59 students within the sample (of 388 students) who took the AP Calculus Examination (see Table 2), we would get similar results: 29 (or 49.2%) of these students received advanced standing.

The entries in Table 4 preceded by a single star indicate students who received credit for one semester of calculus. This includes students who scored 4 or 5 on the AB exam but did not take the BC exam (49 + 47 = 96 students) and students who were unsuccessful on the BC exam but nevertheless received credit for one semester of calculus (14 + 2 = 16 students). The entries preceded by a double star indicate students who received credit for two semesters of calculus (5 + 45 = 50 students).

As noted earlier, 162 students received advanced standing. From Table 4 we learn that 112 of these students received credit for one semester of calculus and 50 received credit for two semesters of calculus.

It is noteworthy that only 50 out of the 80 students (63.5%) who took the BC calculus course and exam received credit for two semesters of calculus, and quite disturbing that 14 out of these 80 students (17.5%) did not even receive credit for one semester of calculus.

Conjecture 4:
About half of those who receive grades of 4 or 5 actually continue on an accelerated path.

One of the principal reasons that students take AP Calculus in high school is, presumably, so that they can start with the second (or third) semester of calculus when they arrive at college; after all, that is the principal meaning of "advanced placement." My working definition of "continuing on an accelerated path" is thus that they take, and successfully complete, the next two courses in their first year at Rutgers.

For those who received credit for first semester calculus, "continuing on an accelerated path" means that they take second and third semester calculus in their first year at Rutgers. For those who receive two semesters of advanced standing, "continuing on an accelerated path" means that in their first two semesters at Rutgers they take at least two of three courses normally taken in the second year – third semester calculus, differential equations, and linear algebra.

Table 5 reports on how many of the 112 students who received one semester of advanced standing (see discussion after Table 4) actually continued on an accelerated path in their first year at Rutgers, and on how many of the 50 students who received two semesters of advanced standing actually continued on an accelerated path.

	4 or 5 on AB exam	4 or 5 on BC exam	Total
Continued on accelerated path	40	26	66
Did not continue on accelerated path	72	24	96
	112	50	162

Table 5. How many of those who succeeded in the AP Calculus exam continued their acceleration.

As to Conjecture 4, Table 5 shows that somewhat fewer than one half of those who received grades of 4 or 5 actually continue on an accelerated path – that is, 66 of the 162 students (40.7%) who received advanced standing continued with their acceleration, 40 of the 112 students who received one semester of advanced standing and 26 of the 50 students who received two semesters of advanced standing.

We pause here to review the first of David Bressoud's questions that appear at the beginning of this article: Of the students who receive college credit for calculus studied in high school, how many never go on to the next mathematics class?

This question doesn't take into consideration that the courses that students take in their first semester at college are generally courses that are assigned to them; thus students who pass the AB exam are automatically enrolled in Calc 2 and those who pass the BC exam are automatically enrolled in Calc 3. Although it is possible for students to change the courses to which they are assigned, most don't do that. Thus the appropriate formulation of the first question is: Of the students who receive college credit for calculus studied in high school, how many never **go beyond** the next mathematics class? That is more or less equivalent to the question "how many don't continue in an accelerated path during their first year?"

The answer to that question is that almost 60% of the students in this study who received college credit for calculus studied in high school didn't go beyond the next math class!

Conclusion:

Collecting together the findings concerning Conjectures 1-4, we can conclude that:

- 123 out of 217 students who took calculus in high school took AP Calculus,
- 59 out of 123 who took AP Calculus took the AP Calculus exam,
- 162 out of 332 students who took the AP Calculus exam received advanced placement,[20] and
- 66 out of the 162 students who received advanced placement continued with their acceleration.

This enables us to conclude that the percentage of students who take a full-year calculus course in high school who then continue with their acceleration through their first year at Rutgers is the product of these fractions, that is, (59/217) x (66/332), or 5.4%. .

It is rather striking that these conjectures, made 25 years ago about students of that era, are apparently still true[21]. After all, many more students are taking calculus in high school now than a generation ago, and one might expect that, as a result, different percentages would emerge.

[20] It should be noted that students who intended to be engineers were not enrolled in Rutgers College and therefore were not part of this study. However, the results regarding Conjecture 3 were the same for students entering the engineering school. Of the 130 who took the AB exam, exactly 65 (50.0%) received advanced placement, and, of the 45 who took the BC exam, exactly 24 (53.3%) received two semesters of advanced placement, a percentage that is lower than the 63.5% of Rutgers College students who were successful on the BC exam. On the other hand, a much higher percentage of engineering students who received advancement continued with their acceleration through their first year (74.7%), which is not surprising because taking math courses in the first year was required in order to continue in the engineering school.

[21] I would have wanted to state this conclusion more strongly but am unable to do so, for I do not have data from 25 years ago to back up the claim that the conjectures were true at that time.

Significant pause

The acceleration strategy does apparently benefit 5.4% of the students, or, rather, 5.4% of the students at Rutgers who took calculus in high school. For these students it was a benefit to take Algebra 1 in 8th grade, Algebra 2 and Geometry in the 9th and 10th grades, Precalculus in the 11th grade, and AP Calculus in the 12th grade, since this system enabled them to continue their acceleration in their first year in college and complete the calculus sequence one semester or one year ahead of their counterparts. How much they really benefited by this acceleration is not clear, and is beyond the scope of this article.

However, it is appropriate for us to ask whether the acceleration strategy benefits the other students or whether, on the other hand, it harms them in some way.

One way of addressing this issue is to look at each of the groups of students whose acceleration was interrupted and examine what happened to the students in each of these groups. The remaining 94.6% of the students are made up out of those who:

- Passed the AP exam but did not continue their acceleration.
- Took the AP exam, but didn't pass it.
- Took the AP course, but didn't take the AP exam.
- Took calculus, but didn't take the AP course.

The impact of acceleration on each of these groups of students will be discussed below, but it is important to mention that the 5.4% of students who we consider as having benefited from acceleration is not 5.4% of all students, but 5.4% of those Rutgers students who took calculus in high school. As we see from Table 1, this is 5.4% of 55.9% of Rutgers students, or 3.0% of Rutgers students.

When we compare the group of students who take the AP Calculus course and exam in high school, and do well enough to receive advanced standing in mathematics in college, with the entire cohort of high school students, the benefits of the acceleration strategy likely accrue to fewer than 1% of all high school students.

So the question of whether a school district should conceptualize and structure all of its mathematics programs so that they address the needs of 1% of their students is a significant policy issue. A calculus-driven curriculum is appropriate for a small percentage of students, but it is hardly desirable or beneficial for all students.

1. What happened to the students who passed the AP exam (with scores of 4 or 5), but did not continue their acceleration?

It was noted above (see Table 5) that 72 of the 112 students who scored 4 or 5 in the AB Exam did not continue their acceleration during their first year at Rutgers – that is, they did not take and successfully complete the second and third semesters of calculus as first year students. What math courses, if any, did they take? These courses are described in Table 6.

	4 on AB exam	5 on AB exam	Total
Continued on accelerated path	19	21	40
Completed second semester calculus and took no further math courses	25	15	40
Skipped math in Fall 2003 or Spring 2004, but continued to calc 3 subsequently	7	5	12

Got poor grades (D, F, W) in calc 2 and stopped	8	6	14
Completed calc 2, but did poorly in calc 3	1	1	2
Took calc 1 and calc 2	2	1	3[22]
Did poorly in calc 2, but subsequently passed two courses	1		1
Total[23]	63	49	112

Table 6: Courses taken by students who received one semester of advanced standing

What is striking about this information is that for 54 of the 112 students (48.2%), Calc 2 was their last math course – this includes 40 who passed Calc 2 and 14 who did poorly in Calc 2 – and that another 12 students (10.7%) decided to compensate for their high school acceleration by decelerating in college.[24]

These data undermine the belief that we may have had that most of the students take AP Calculus because of their positive attitudes toward mathematics and their desire to pursue mathematical careers. Why then do students take AP Calculus? Here is a list of possibilities:

i. because they want to learn more mathematics
ii. because they love math
iii. because they want advanced placement in college
iv. because they want to be better prepared for college courses
v. because it looks good on their college applications
vi. because they have to take a math course
vii. because it's expected of them
viii. because of peer pressure or parental pressure
ix. because of parental peer pressure
x. because they want to stop taking math sooner
xi. because they want to save money by graduating from college earlier

We would like to believe that all students take AP Calculus for one of the first three reasons, but in fact the data in Table 6 do not support that belief. This study did not involve interviews with students, so I don't know how many of the students took AP Calculus for each of the reasons given, but I would conjecture that the two most popular reasons would be that they or their parents believe that taking AP courses will help them get into better colleges and that taking AP Calculus was expected of them – that the

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[22] In response to the second of David Bressoud's questions listed at the beginning of this article, a very small percentage of the students who earn credits for Calc 1 turn down those credits and repeat the course.

[23] The 63 students in the first entry consists of the students in the two single-starred groups in the "4" row of Table 4. The 49 students in the second entry consists of the two single-starred groups in the "5" row of Table 4. The 112 students in the final entry consists of all those who were eligible to receive 4 credits for calc 1 and start at Rutgers with calc 2.

[24] The data in Table 6 is disaggregated by score on the AB exam to underscore that drop-out rate among the highest scoring students is (almost) equally substantial – that even among the students who got 5 on the AB exam, as many stopped after Calc 2 (15 + 6 = 21) as continued their acceleration (21) during their first year.

exception had become the norm. But when such a study is conducted we may also find that the acceleration strategy has produced a lot of negative attitudes about mathematics.[25]

One might reasonably ask why those who stopped after Calc 2 actually took that course. And the answer is likely to be that they were placed into the course as a result of their AP score and they never thought to question that placement. However, once they realized, as first-year college students do, that they now had substantial control over their lives, they said "that's enough math for me."

The data in Table 6 also undermine the assumption that increasing the number of students in AP Calculus will expand the STEM pipeline. It may well have the opposite effect. Indeed this data shows that a focus on retaining students who are already in the pipeline may be a better strategy than adding more students to a very leaky pipeline.[26]

Similar conclusions emerge from Table 7 which reports on the course taking of students who were successful on the BC Exam. As noted in Table 5, 24 of the 50 students who scored 4 or 5 in the BC Exam did not continue their acceleration during their first year at Rutgers – that is, they did not take and successfully complete the third semester of calculus and either linear algebra or differential equations as first year students. Here again we see that a substantial percentage, 17 out of 50 students (34.0%) dropped out of mathematics after at most one semester, and 5 out of 50 students (10.0%) opted for deceleration. The discussion above about student motivation also applies to many of the students who are the highest mathematical achievers in high school. Their success in high school mathematics courses unfortunately may not be indicative of a positive attitude toward mathematics or a desire to pursue a math-intensive major or career.

	4 on BC exam	5 on BC exam	Total
Continued on accelerated path	6	20	26
Completed calc 3 or linear algebra and took no further math courses	1	11	12
Skipped math in F03 or S04, but then continued to calc 3 and linear algebra		5	5
Got poor grades (D, F, W) in calc 3 or linear algebra and stopped	1	2	3
Took no math courses	1		1
Unsuccessful but now resuming	1	1	2
Took 103 (Math for Liberal Arts Students)	1		1
Total	11	39	50

Table 7: Courses taken by students who received two semesters of advanced standing

[25] Such a study was conducted in the Spring semester of 2011 by Anoop Ahluwalia and Joseph Rosenstein; the results of the study will be published in a companion article entitled "Why Do Students Rush to Calculus?"

[26] This only applies to schools where substantial numbers of students are already taking AP Calculus. It is important that the opportunity to take AP Calculus be available to students in all schools, particularly those in economically disadvantaged communities.

2. What happened to the students who took the AP exam, but didn't get advanced placement?

Of the 332 entering Rutgers College students who took the AP exam, 162 students (48.8%) received scores of 4 or 5 and received advanced placement in mathematics – these students were discussed in Tables 6 and 7 – and the remaining 170 students (51.2%) received scores of 1, 2, or 3. What happened to them?

Although all of these students had on their high school transcripts the prerequisites for a first-semester calculus course, entry into Calculus 1 at Rutgers is permitted only to those who score sufficiently high on a reliable calculus-readiness placement test that has been administered by the Rutgers department of mathematics for about 30 years. After taking this test, students are placed into calculus, precalculus (Math 115), two-semester precalculus (Math 111-112), algebra 2 (Math 026), or algebra 1 (Math 025)[27]. Experience has shown that students whose scores on the test are below the cut-off for a certain level are very unlikely to succeed in a course at that level without additional preparation.

Students who place into calculus have two options: Math 151 is the course taken by prospective math, physics, and chemistry majors and engineering students; and Math 135 is the course taken by prospective majors in economics and the biological sciences. Students planning to take math courses beyond second semester calculus are encouraged to take the Math 151-152 sequence or to transfer to Math 152 after taking Math 135. Students who take Math 135 and its successor, Math 136, understand that those are terminal courses, that is, they do not satisfy the prerequisites of any more advanced course in mathematics. In the language of "The Crisis of Calculus [3]," Math 135 and Math 136 are not "mainstream" calculus courses, since they do not "lead to upper division courses in the mathematical sciences."

Students who do not wish to take calculus may take Math 103, Mathematics for Liberal Arts, if they are placed out of algebra 2. They may also opt to take no math course in their first semester; however, relatively few students exercise this option since that would mean that they would have to change the list of courses to which they have been assigned.

Based on their performance on the placement test and their intended majors, the 170 students were enrolled in the following courses during their first semester at Rutgers:

Course	Number enrolled
151 (calculus for math, physical sciences, and engineering)	40
135 (calculus for biology, economics, and psychology majors)	91
115 (precalculus)	19
111 (two-semester precalculus)	8
026 (intermediate algebra)	3
103 (math for liberal arts)	9

[27] Credits for algebra 1 (025), algebra 2 (026), and the first semester of the two-semester version of the precalculus course (Math 111) do not count toward graduation.

TOTAL	170

Table 8. Initial math course taken at Rutgers by the 170 students who scored 1, 2, or 3 on the AP Calculus Exam

It is perhaps surprising that at least 31 who took a full-year AP Calculus course in high school (including one student who took the math for liberal arts course)did not achieve placement into calculus. This provides a partial response to the third of David Bressoud's questions at the beginning of this article: "How many students take calculus in high school but are deemed inadequately prepared to study calculus when they get to college?" Of the 170 students who took the AP Calculus test and did not score 4 or 5, 31 or 18.2% placed in precalculus or high school algebra courses. (This question will be discussed further in the section on students who took non-AP calculus courses in high school.)

It is also striking that the first college mathematics course taken by 130 of these 170 students is a lower level course than the AP Calculus course that they took in high school, which is intended to be comparable to first semester calculus courses at the level of Rutgers' Math 151[28]; students who take advanced level courses in high school should not be taking lower level courses when they get to college. Fully 39.2% (130 out of 332) of the students who took the AP Calculus exam started off in college at a lower mathematical level than they were at the beginning of their senior year in high school; this was true of 76.5% (130 out of 170) of those who were unsuccessful on the AP Calculus test.

How did these students fare in these courses? Table 9 presents the grades of the students who took calculus.

Grade	Math 135			Math 151		
	AP Score of 1 or 2	AP Score of 3	Total	AP Score of 1 or 2	AP Score of 3	Total
A	11	20	31	2	12	14
B+	10	10	20	2	5	7
B	9	10	19	2	3	5
C+	5	3	8	2	2	4
C	5	5	10	3	5	8
D	1		1			
F	2		2	1	1	2
Total	43	48	91	12	28	40

Table 9. Grades of the 131 students who scored 1, 2, or 3 on the AP Calculus Exam and were placed into calculus

What is striking in Table 9 is how well the students did in these courses. 88 of the 91 students in Math 135 scored C or better, and 70 of the 91 (or 79.5%) scored B or

[28] One problem may be that many high school AP calculus courses may, in fact, not be conducted at the high level that is intended by the College Board.

better. Similarly, 38 of the 40 students in Math 151 scored C or better (the other 2 students passed the course the following semester), and 26 of the 40 (65.0%) scored B or better.

The students who scored 3 on the AP Calculus Exam, not surprisingly, performed even better, with 40 out of 48 (83.3%) of the students who got a 3 on the AP Calculus exam scoring B or better in Math 135 and 20 out of 28 (71.4%) of the students who got a 3 on the AP Calculus exam scoring B or better in Math 151.

Moreover, it should be noted that the students who scored 1 or 2 on the AP exam also did quite well in Math 135 and Math 151, if they were placed into calculus. One may surmise that although they took the AP exam, they did not prepare for that exam.

One might be tempted to conclude that taking an AP Calculus course does indeed provide good preparation for college calculus courses. That conclusion would be premature; indeed, we shall shortly see that those who took the course but not the exam did not fare as well. What we can conclude, however, is that taking the AP Calculus exam does indeed give students an advantage. However, this is not an argument for acceleration, since it is equally true, for example, that students who repeat Algebra II would learn it better than those who take it just once. Moreover, it is not unexpected that after taking what is advertised as a higher level calculus course, students will do well on a lower level course.

The observation that students who take the AP exams for their AP courses are successful is consistent with the results of a major study[29] on the role of Advanced Placement courses as a criterion for admission to the University of California which concludes that "while student performance on AP examinations is strongly related to college performance, merely taking AP or other honors-level courses in high school is not a valid indicator of the likelihood that students will perform well in college."

3. What happened to the students who took the AP Calculus course, but did not take the AP Calculus exam?

This question is addressed together with the following question.

4. What happened to the students who did not take the AP Calculus course, but did take a year of calculus in high school?

Although the response to the previous two questions involved analysis of the dataset involving the 332 students who had taken both the AP Calculus course and the AP exam, in order to respond to these two questions, which discuss students who did not take both the AP Calculus course and the AP exam, we need to analyze instead the dataset involving the 400 randomly selected students.

Information about the math courses in which the 400 students first enrolled (no matter which semester) was obtained from the database of entering 2003 students provided by Rutgers, that information is recorded in Table 10.

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[29] "The Role of Advanced Placement and Honors Courses in College Admissions," Saul Geiser and Veronica Santelices, Center for Studies in Higher Education, University of California, Berkeley, 2004 (ishi.lib.kerkeley.edu/cshe), pp. 1-2.

Course	# who scored 4 or 5 on the AP exam	# who scored 1, 2, or 3 on the AP exam	# who took the AP course but not the AP exam	# who took honors calculus but not AP calculus	# who took calculus but not honors calculus[30]	# who did not take a full-year calculus course	# for whom no transcript was available	Total
Higher level courses	26					2	1	29
151	1	6	15	2	11	9	1	45
135	1	16	18	17	30	38	1	121
115 (precalc).		6	7	6	5	19	4	47
111 (precalc two-semester)		2	2	1	2	18	2	27
103 (math for liberal arts)		1	2	2	5	20	2	32
026 or 027 (intermed algebra)				3	3	39		45
023 or 025 (elementary algebra)				1		12		13
None[31]			20	4	2	14	1	41
TOTAL	28[32]	31	64	36	58	171	12	400

Table 10. Initial math course taken at Rutgers by the 400 randomly selected students

Connecting the data in Table 10 with the data in earlier tables:

- As noted in Table 1, 217 of these students took a full-year of calculus in high school.
- Of these 217 students, 123 took an AP Calculus course (they appear in Columns 1-3 of Table 10) and 94 took a non-AP calculus course (they appear in Columns 4-5 of Table 10).
- As noted in Table 2, of the 123 students taking the AP Calculus course, 59 took the AP exam – they appear in Columns 1-2 of Table 10 – and 64 did not – they appear in Column 3.

[30] We cannot be certain that these students were not in an honors calculus course. High school transcripts do not always correctly record the level of the courses that a student takes.

[31] The students listed here registered for no math class in any of their first two years at Rutgers.

[32] Two students who were eligible for advanced placement instead repeated Calculus 1; one took Math 151 and the other took Math 135.

The response to Question 3, “What happened to the 64 students who took the AP course, but not the AP exam?” is that:

- 33 out of these 64 students (51.6%) took a calculus course as their first math course in college.
  - 15 of these 33 students took the calculus course Math 151 intended for potential majors in math or the physical sciences; the other 18 took a lower level calculus course than the AP Calculus course they took in high school
  - of the 15 students who took Math 151, only four managed to complete the second semester of calculus; 3 completed Math 152 (one after three tries) and 1 completed the lower level calculus course Math 136.
- 20 of the 64 students (31.3%) who took the AP Calculus course but not the AP exam never took a math course in their first two years at Rutgers.
- 10[33])of these 64 students were placed into precalculus or high school algebra.
- The response to Question 4, “What happened to the 94 students who did not take the AP Calculus course, but did take a year of calculus in high school?” is that, parallel to the bulleted items above:
- 60 out of 94 of these students (63.8%) took a calculus course as their first math course in college, almost as high a percentage as those who took the AP Calculus exam but did not get advanced placement (22 out of 31 or 71.0%). Those who took a *non-honors* calculus course in high school, had a comparable percentage; 41 out of 58 (70.7%) started with calculus at Rutgers.
  - only 13 (13.8%) of these 94 students took the calculus course Math 151 intended for potential majors in math or the physical sciences; the other 47 took a course that may or may not have been at a lower level than the calculus course they took at high school.
  - of the 13 students who took Math 151, only five managed to complete the second semester of calculus; 2 completed Math 152 and 3 completed the lower level calculus course Math 136.
- 6 of the 94 students (6.3%) who took a non-AP Calculus course never took a math course in their first two years at Rutgers.
- 24[34] of these 94 students were placed into precalculus or high school algebra. This provides another partial answer to David Bressoud’s third question: At least 25.5% of the students who took a non-AP calculus course were placed into precalculus or high school algebra.

Before we draw comparisons between these two groups of students, let us look at the additional information in Table 11, which looks beyond the *first* math course taken by the students and considers what happens to them in their first two years at Rutgers:

	# enrolled who took	# enrolled who took	# who took	# who took	Total

[33] This includes the 7 students in Table 10 who took a precalculus course, the 2 students who took a two-semester precalculus course, and 1 who took no math but placed in the two-semester precalculus course.

[34] This includes the 11 students in Table 10 who took a precalculus course, the 3 students who took a two-semester precalculus course, the 7 students who took an algebra course, and the 3 students who took no math but placed into these courses.

	the AP exam but did not receive advanced placement	the AP course but not the AP exam	honors calculus but not AP calculus	calculus but not honors calculus	
Total	31	64	36	58	189
Took Calc 1[35]	27	37	22	46	132
Grade C or better in Calc 1	26	33	19	42	120
Took Calc 2	7	12	7	11	37
Grade C or better in Calc 2	4	7	5	10	26
Took no math courses		20	4	2	26

Table 11. Calculus performance by the students (out of the 400 randomly selected) who took a calculus course in high school but did not gain advanced placement.

Tables 10 and 11 paint a rather negative picture of what happened to students who were not successful in AP Calculus:

- Although most students in the sample of 400 students who scored poorly on the AP Calculus test took Calc 1 in college and succeeded in getting a grade of C or better, relatively few of them (7 out of 31 or 22.6%) continued to Calc 2. Altogether, only 4 of these 31 students successfully completed Calc 2 with a grade of C or better.
    - The results were better for the 170 students (see Table 3) in the entire cohort of students who took the AP exam but did not receive advanced placement; 52 of them (30.6%) took calculus 2 (38 the mainstream course and 14 the terminal course) and 41 of them successfully completed the course with a grade of C or better.
- Slightly over half of the students who take the AP Calculus course and did not take the AP exam take calculus in college, and most of those who do are successful. Only 1/3 of those who pass Calc 1 take Calc 2. Altogether, only 7 of these 64 students successfully completed Calc 2 with a grade of C or better.
- Fully 1/3 of the students who take the AP Calculus course but not the exam take no math in college. This is one of two significant signs of “mathematical fatigue”, the other being the substantial percentage of students in honors calculus courses who decided to stay away from Math 151, as if to say, “I’ll take the one semester of mathematics that I need to take, and that’s all the mathematics that I’ll ever take.”

[35] This includes those students who took Calc 1 in their first semester at college (which are discussed in the previous table), but also those students who took Calc 1 subsequently, typically because they were placed in and took a precalculus course before taking Calc 1.

On the other hand, those who do not take AP Calculus in high school, but take some other calculus course, seem to be much more successful in calculus than those who take AP Calculus but don't take the exam.

From all of these data it is reasonably clear that the preponderance of the students who took AP Calculus but did not receive advanced placement did not really benefit from taking AP Calculus in high school and would have been better served by a non-AP Calculus course. Perhaps enrolling such students in another course would have avoided the substantial number of students who dropped out of math after high school. Such a course could provide an introduction to the concepts and applications of calculus and, at the same time, strengthen the students' understanding and skill of precalculus topics; it could also provide applications of other areas of mathematics, such as probability, statistics, and discrete mathematics to show students how interesting and useful mathematics can be.

The data in these tables also enable us to respond to David Bressoud's fourth question: Of the students who retake the courses they studied in high school, how many of them now succeed? Of the 189 students who took a full year of calculus in high school, but did not receive AP credit, 132 took a calculus course and 120 of them (90.9%) succeeded. However, only 34 of them took the mainstream calculus course, so the course in which most of these students succeeded was at a lower level than the course they took in high school.[36] Moreover, only 26 of the 132 students who took a calculus course completed Calc 2 with a grade of C or better.

Algebra 1 – 8th vs. 9th grade

Let us move from examining the final course taken in high school to the first course. Many students are encouraged to take Algebra 1 in 8th grade. Do they benefit from this acceleration? We can make a strong argument that the 5.4% who gain advanced placement have benefited. But what about the other students?

How can one measure the impact of taking Algebra 1 in the 8th grade? One possibility is to look at the Rutgers course-taking patterns of the different groups of students. Table 12 looks at the first course taken at Rutgers by students who took Algebra 1 in the 8th grade vs. the 9th grade.

	Alg 1 in grade 8	Alg 1 in grade 9
151 and higher	54	16
135 (calculus)	58	57
115 (precalc)	20	21
111 (2-sem precalc)	6	18
103 (math lib arts)	11	19
algebra courses	20	38

[36] This statement assumes that the course that they took in high school was actually a college-level course. One concern that has been often expressed is that high school calculus courses (including AP Calculus courses) may not be equivalent to college-level calculus courses; indeed, the presidents of the Mathematical Association of American (MAA) and the National Council of Teachers of Mathematics (NCTM) issued a joint statement in 1986 urging strong standards for high school calculus courses (see Recommendations section below).

None	32	9
Total	201	178

Table 12: First course taken at Rutgers by the 400 students in the sample, disaggregated by the grade in which they took Algebra 1[37] [38]

Certainly one difference between the two columns in the table is that many more students who took Algebra I in the 8th grade were able to benefit by being prepared for Math 151 or higher courses in their first year at college; as we know, taking Algebra I in the 8th grade facilitates that possibility. But note that a reasonable number of students were able to start with Math 151 and higher courses even though they had not taken Algebra I in the 8th grade – they simply took Geometry at the same time as either Algebra 1 or Algebra 2. Nevertheless, we can argue that taking Algebra 1 in the 8th grade benefits the 54 out of 201 students (26.9%) who ended up starting at Rutgers with Math 151 or a higher course.

But what about the other students, the "average" students who were less concerned with acceleration? From this table we see that the Rutgers course-taking patterns of students who took Algebra 1 in the 8th grade was not significantly different from those who didn't take Algebra 1 until the 9th grade. (That is, the differences may be significant statistically, but they are not significant educationally.) Roughly the same numbers of students were placed into calculus (Math 135) and roughly the same numbers were placed into precalculus (Math 115). About twice as many of those who took Algebra 1 in the 9th grade were placed into non-credit-bearing courses, but that is because the students in this group were weaker mathematically than the students who took Algebra 1 in the 8th grade. The most striking difference between these two groups is that 32 of the students who took Algebra 1 in grade 8 did not take any mathematics course in their two years at Rutgers.

Again, the conclusion is that Algebra 1 in the 8th grade benefits the best students by smoothing their path to AP Calculus, but there is certainly no evidence here that it benefits the average students … and there is certainly no evidence here that it benefits all students. Nevertheless, as noted in the introduction, there are districts and states that have decided that all students should take Algebra 1 in the 8th grade … even though many of the 8th grade teachers do not have a strong mathematical background.

Conclusions

- Only a small percentage (5.6%) of the students who are accelerated into algebra 1 in grade 8 and AP calculus in grade 12 continue their acceleration in their first year in college.
- Those who take the AP Calculus exam but are not successful apparently do better in first semester calculus in college than those who take the AP Calculus course but not the exam.

[37] The two columns do not add up to 400 because in 21 cases it was not possible to determine from the high school transcripts what courses the students took in 9th grade.

[38] 5 of the students counted in the right column actually took Algebra 1 in the 10th grade.

- Those who take the AP Calculus course but don't take the exam don't do appreciably better in first semester calculus in college than those who take a non-AP calculus course.
- Taking AP Calculus seems to generate a good deal of fatigue with mathematics in high school students, since many stop taking mathematics as soon as possible.
- There appears to be no significant advantage of taking Algeba 1 in the 8^{th} grade, except for the top 25% of the students who now take that course in the 8^{th} grade.

Literature (revisited)

As noted at the outset, we now present a partial summary of Bressoud's 2009 article with relevant observations based on the study discussed in this article; these observations appear in open bullets.

- The number of students taking the AP Calculus has increased from 25,000 in 1979, to 74,000 in 1989, to 158,000 in 1999, and to 300,000 in 2009. Despite this dramatic increase in the number of students taking the AP Calculus Exam, the number of students enrolled nationwide in Calculus II in the fall semester "has remained essentially unchanged over the past two decades: 110,000 in 1990, 106,000 in 1995, 108,000 in 2000, and 104,000 in 2005."
    - These data are consistent with the findings of this report, which concludes that fewer than 6% of the students who take calculus in high school continue their acceleration in college. They support the conclusion that, if the purpose of the AP exam is to provide "advanced placement," then almost none of the additional students who take the AP exam actually gain or accept advanced placement, and that given the negative effects of the "rush to calculus" on most high school students, an appropriate national goal would be to scale back the number of students taking AP Calculus to 1990 levels.
- The College Board estimates that 70-75% of the students in AP Calculus course take the exam. Consequently, since 300,000 students took the AP Calculus exam in 2009, about 400,000-430,000 took an AP Calculus course in 2009.
    - The percentage estimated by the College Board is much higher than the numbers in the present study, where 59 out of the 123 students who took an AP calculus course in high school (or 48%) took the AP calculus exam. It is possible that this discrepancy may result from the fact that the students in the present study are mainly from New Jersey and the percentage is different in different states; one possible reason for this difference is that in some states and districts, students who take the AP calculus course may be required to take the AP exam.
- Since in 2004, 52% of students who took a calculus course took the AP Calculus exam[39], Bressoud notes that, assuming the percentage remains the same five years

[39] Bressoud notes that the 2004 data is based on a large-scale transcript analysis by the US Department of Education: Education Longitudinal Study of 2002 (ELS:2002), nces.ed.gov/surveys/ELS2002/ (2009).

later, of the 2009 graduating class, since 300,000 took the AP Calculus exam, about 575,000 took calculus in high school.[40]

- These data imply that of the 575,000 students who took calculus, between 400,000 and 430,000 took AP Calculus, which is again between 70-75%. Here again this is much higher than the results of the present study, where 123 out of 214 students (57%) who took calculus took the AP Calculus course. One possible reason for this difference is that in many states and districts, the only calculus option available to students is AP calculus, whereas non-AP calculus is available in most New Jersey high schools. Of course, it is also possible that the sample in the present study is too small or even that the College Board estimates are simply incorrect.
- Note also that in the 2004 data, 14.1% of graduating seniors had taken a course that was called "calculus." In the present study, which focused on college-bound students, and indeed those attending a first-class state university, the percentage of graduating seniors who had taken a course called "calculus" was 54.9% (214 out of 390). Comparing these two pieces of information is of course like comparing apples and oranges.
- The national data suggests that over half of students who take a course called "calculus" actually take the AP Calculus Exam. Our data suggests that for New Jersey students it is only one quarter, since we can assume that Rutgers bound students are more likely to take the AP Calculus exam than their non-Rutgers bound counterparts.

- The College Board document "Setting a Policy for AP Calculus" – intended to assist college faculty and administrators in setting "appropriate credit and placement policies for AP Calculus" – notes the following: "The AP Exam scoring rubric is established so that the <u>lowest</u> composite score that earns an AP grade of 5 is equivalent to the <u>average</u> score earned by college students who received grades of A in a comparable course. The <u>lowest</u> score that earns an AP grade of 4 is equivalent to the <u>average B</u>, and the <u>lowest</u> score that earns an AP grade of 3 is equivalent to the <u>average</u> C.
  - Since this grading policy has presumably been in place for a long time, the question is why don't Rutgers and other universities give advanced placement credit to students who receive grades of 3 on the AP exam, and why do they not allow such students to take the next calculus course. The likely answer is that students who scored 3 on the AP exam didn't do very well in Calculus 2, and therefore that the College Board's claim that "an AP grade of 3 is equivalent to the average C" applies only to lower-ranking institutions. Indeed, our experience has been that getting a grade of C in other colleges is not the same as getting a grade of C at Rutgers. [41]

[40] As noted in the initial sentences of this article, this is twice as many as the number of students taking first semester calculus in their first semester of college.

[41] A recent state policy decision that Rutgers has to accept courses taken at community colleges as equivalent to its own courses has reinforced this perspective; students who come to Rutgers having taken

- The College Board document (p. 3) shows that of the 185,992 AB examinees in 2005, 40.2% had scores of 4 or 5.
    - In the present study, 162 out of 332 students (or 48.8%) had scores of 4 or 5; it is not surprising that students entering a state university would have better scores than the entire cohort.
- A troubling aspect of the College Board document is the implication that students who take the AP course and exam do better in Calculus 2 than students who don't. Bressoud does not comment on this directly, but he does note about a 1998 study by Morgan and Ramist[42] that … "there is no attempt to control for the possibility that the population of students who earn AP credit for and are sufficiently confident to skip Calculus 1 are not completely comparable to the population of those who take and pass Calculus 1." Bressoud also notes that most of the studies have been funded through College Board or ETS, with the unstated implication that their conclusions should not be taken at face value.
    - The students in the present study who took a non-AP calculus course in high school actually were more successful in calculus in college than those who took the AP Calculus course but not the exam. Furthermore, although those who scored poorly on the AP Calculus exam generally passed Calc 1 in college, relatively few successfully completed Calc 2. And of those who took the AP Calculus course but not the exam, fully 1/3 took no math course in college.
- One of the main questions that Bressoud addresses is "Does it make sense for students who have done well in AP Calculus to skip Calculus I in college?" The six studies he cites suggest that AP calculus students who take advantage of the placement do at least as well (and significantly better if they earned a score of 4 or 5) in Calculus 2 compared to their counterparts who take the course again and choose not to use their AP credits.
    - As noted earlier, only 3 of the 112 students in the present study who scored 4 or 5 elected to decline the advanced placement and repeat Calculus 1. However, since 40 of the 112 completed Calculus 2 but took no further math courses and an additional 14 of the 112 took Calculus 2 but were unsuccessful, it is clear that half of the students who were "successful" in the AP Calculus exam essentially dropped out of math. It may have made sense for them to "skip Calculus I in college," but for those who are concerned about the STEM pipeline, it certainly didn't make sense. As recommended below, fewer students should be taking AP Calculus and, as Bressoud notes, "some students are better served by being allowed not to place as far ahead as they are entitled."

Calculus 1 (and Calculus 2) at a community college often have a very difficult time when they take the next course at Rutgers.

[42] Morgan, R. and L. Ramist. 1998 Advanced Placement Students in College: An Investigation of Course Grades at 21 Colleges. Educational Testing Survey Report No. SR-98-13. Princeton, NJ. www.collegeboard.com/press/releases/50405.html

Recommendations

This study shows that something is very amiss. Although originally intended as an opportunity for the most able students to have "advanced placement" in college, the AP Calculus course and exam has become the default option for students who need to impress colleges. (The same is likely true of the other AP courses.) This is damaging to most of the students, to the high school curriculum, and to the national interest … since so many able students leave the STEM pipeline.

The following recommendations are intended to alleviate the problem. The recommendations are addressed to schools, to parents, and to colleges. We begin with colleges because change cannot take place until colleges change how they view AP courses in the admissions process.

- **For colleges**
  - Colleges should not automatically weigh AP courses in an applicant's favor unless they are convinced that the applicant's high school has appropriate screening criteria for entrance into AP courses, requires students to take the AP exam, and has a track record for success in AP exam.
  - Colleges should institute a policy of the following form – that it will only take a student's AP course into consideration if at least xx% of the students at the respective high school who took that course in the past five years took the AP exam and scored 3 or better.
- **For students and parents**
  - Students and parents should be made aware that there are serious negative consequences of taking courses for which students are inadequately prepared – that they will not be successful in that course and/or will likely stop taking mathematics.
  - Students and parents should be made aware that colleges will no longer automatically reward students for taking AP courses if there is no reason to believe that such courses are beneficial to the students.
- **For schools and districts**
  - Schools and districts should introduce performance-based entrance criteria for high school courses; students should be strongly discouraged (if not barred) from taking courses for which they are unprepared or in which they are really not interested.
  - Enrolling in courses in the senior year solely to enhance college applications should be strongly discouraged.
  - Schools and districts should inform parents of the consequences of their children's taking courses for which they are unprepared.
  - Schools should provide other course options for seniors so that they are not in effect forced to take calculus; such courses could include an introduction to calculus, for those students who will need to take calculus in college, but for most students a more suitable course would be one that focuses on probability, statistics, and discrete mathematics, with a particular focus on their applications to real-world problems.

- Only the most able students should take Algebra 1 in the 8th grade, particularly since many 8th grade teachers are not certified to teach math and there is no evidence that taking Algebra 1 early benefits any but the most able students. An appropriate goal might be for each district to reduce 8th grade Algebra 1 classes to one-quarter of their present enrollment.

We close by noting and commenting on the 1986 recommendations of the presidents of the Mathematical Association of American (MAA) and the National Council of Teachers of Mathematics (NCTM), as paraphrased by David Bressoud[43], recommendations that he notes, "should be repeated and re-emphasized":

1. In spite of the pressures to take calculus while still in high school, students should never short-change their mathematical preparation in subjects such as algebra, geometry, or trigonometry. Solid mathematical preparation is far more important than exposure to calculus.
2. When calculus is taught in high school it should be a college-level course. This means that the goal of the course should be to give students the same breadth of topics and mastery of calculus obtained by students taking such a course in college. It means that the course should be taught with the expectation that students who perform satisfactorily will be able to place into the succeeding college calculus course.

It is clear that the situation has changed since 1986. Whereas then, it seems, most students taking calculus were seeking "advanced placement" and it was important that the calculus course they took in high school should properly prepare them for "the succeeding calculus course," that is no longer the primary motivation for taking calculus in high school. Many students now, evidently, take calculus in high school in preparation for taking calculus in college. Although the first recommendation above still applies to courses taken by these students, the second does not. However, it does still apply to AP Calculus; "the course should be taught with the expectation that students who perform satisfactorily will be able to place into the succeeding college calculus course."

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