Assignment Four.

April 25, 2008

NOTE: Due Date for submission is Thursday, 01st May.

Problems

1. A set of polynomials $f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}) \in \mathbb{F}[x_1, \ldots, x_n]$ are said to be $\mathbb{F}$-linearly dependent if there exist constants $a_1, \ldots, a_m \in \mathbb{F}$ such that

   $$a_1 \cdot f_1(\mathbf{x}) + a_2 \cdot f_2(\mathbf{x}) + \ldots + a_m \cdot f_m(\mathbf{x}) = 0.$$

   Devise a randomized polynomial-time algorithm that given polynomials $f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})$ in the form of arithmetic circuits, tests whether $f_1, \ldots, f_m$ are $\mathbb{F}$-linearly dependent.

2. $\mathbb{Q}$ as usual denotes the field of rational numbers. Design a polynomial-time algorithm that given a bivariate polynomial $f(x, y) \in \mathbb{Q}[x, y]$ determines if it has the property that for all $y \in \mathbb{Q}$, there exists a $x \in \mathbb{Q}$ such that

   $$f(x, y) = 0.$$

   (Hint: Use the appropriate version of Hilbert's Irreducibility theorem for bivariate polynomials over rational numbers. You need not prove the Hilbert Irreducibility theorem here.)