UGLY PROOFS

and

BOOK PROOFS

Joel Spencer
Tournament $T$ on $n$ players

Ranking $\sigma$

$fit = \text{NonUpsets} - \text{Upsets}$

Erdős-Moon (1965): There exists $T$ for all $\sigma$

$$fit(T, \sigma) \leq n^{3/2} \sqrt{\ln n}$$

Proof: Random Tournament

JS (1972, thesis!): For all $T$ there exists $\sigma$

$$fit(T, \sigma) \geq cn^{2/3}$$

Proof: Random Sequential

Rank on Top or Bottom
JS (1980): For random $T$ for all $\sigma$

$$\text{fit}(T, \sigma) \leq cn^{3/2}$$

Proof: Ugly

de la Vega (1983): Gem

Level 1: Top half against bottom half.

$$\binom{n}{n/2} \text{ “different” } \sigma; n^2/4 \text{ games}$$

All 1-fit $\leq c_1n^{3/2}$

Level 2: 1–2 or 3–4 quartile games.

$< 4^n \text{ “different” } \sigma; n^2/8 \text{ games}$

All 2-fit $\leq c_2n^{3/2}$

Level 3: 1–2, 3–4, 5–6, 7–8 octile games.

All 3-fit $\leq c_3n^{3/2}$

$$\sum c_i \text{ converges}$$
Six Standard Deviations Suffice

$A_1, \ldots, A_n \subseteq \{1, \ldots, n\}$

$\chi : \{1, \ldots, n\} \rightarrow \{-1, +1\}$, $\chi(A) := \sum_{a \in A} \chi(a)$

JS (1985): There exists $\chi$

\[ |\chi(A_i)| \leq 6\sqrt{n}, \text{ all } 1 \leq i \leq n \]
\( b_i := \) roundoff of \( \chi(A_i) \) to nearest \( 20\sqrt{n} \)

\[ \vec{b}(\chi) = (b_1, \ldots, b_n) \]

(Boppana) \( b_i \) has low entropy

Subadditivity: \( \vec{b} \) has low \((n\epsilon)\) entropy

\[ \Rightarrow \text{Some } \vec{b} \text{ appears } 1.99^n \text{ times} \]

\[ \vec{b} (\chi_1) = \vec{b} (\chi_2) \text{ and differ in } \Omega(n) \text{ places} \]

On the shoulders of Hungarians:

Set \( \chi = (\chi_1 - \chi_2)/2 \)

\( \Omega(n) \) colored, \(|\chi(A_i)| \leq 10\sqrt{n} \)

Iterate …
ASYMPTOTIC PACKING

$k + 1$-uniform hypergraph (e.g. $k = 2$)

$N$ vertices

$\text{deg}(v) = D$

Any two $v, w$ have $o(D)$ common hyperedges.

$N, D \rightarrow \infty$, $k$ fixed

Conjecture (Erdős-Hanani) There exists a packing $P$ with $|P| \sim N/(k + 1)$

Rödl (1985): Yes!

Continuous Time

Birthtime \( b(e) \in [0, D] \)

Packing \( P_t \), Surviving \( S_t \)

\[
\Pr[v \in S_t] \rightarrow f(t) = (1 + kt)^{-1/k}
\]

History \( H = H(v, t) \):

- \( v \in e, \ b(e) \leq t \Rightarrow e \in H \)
- \( e \in H, \ e \cap f \neq \emptyset, \ b(f) < b(e) \Rightarrow f \in H \)

History determines if \( v \in S_t \)

History is whp treelike and bounded
History $\sim$ Birth Process

Time backward $t$ to 0
Start with root “Eve” ($v$)
Birth to $k$-tuplets Poisson intensity one
Children born fertile
Survival determined bottom up
Menendez Rule: If all $k$ of birth survive, mother is killed

$f(t) := \Pr[\text{EveSurvives}]$

$f(t + dt) - f(t) \sim -f(t) \cdot dt \cdot f^k(t)$

$f'(t) = -f^{k+1}(t)$

$f(t) = (1 + kt)^{-1/k}$