On Ramsey number of sparse uniform hypergraphs

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For a $k$-uniform hypergraph $G$, the Ramsey number $R(G,G)$ is the minimum positive integer $N$ such that in every 2-coloring of edges of the complete $k$-uniform hypergraph $K^k_N$, there is a monochromatic copy of $G$. Say that a family $\mathcal{F}$ of $k$-uniform hypergraphs is $f(n)$-Ramsey if there is a positive $C$ such that $R(G,G) \leq C f(n)$ for every $G \in \mathcal{F}$ with $|V(G)| = n$.

Burr and Erdős conjectured that for every $d$, the families $\mathcal{M}_d$ of graphs with maximum degree $d$ and $\mathcal{D}_d$ of $d$-degenerate graphs are $n$-Ramsey. Recall that a graph is $d$-degenerate if each its subgraph has a vertex of degree at most $d$. Chvátal, Rödl, Szemerédi and Trotter proved the first conjecture.

The second conjecture is open. However, Kostochka and Rödl proved recently that $\mathcal{D}_d$ is $n^2$-Ramsey and then Kostochka and Sudakov proved that for every $\epsilon > 0$ and every positive integer $d$, the family $\mathcal{D}_d$ is $n^{1+\epsilon}$-Ramsey.

In this talk, we prove that for every $\epsilon > 0$ and for every fixed $k$ and $d$, the family $\mathcal{D}_d^k$ of $k$-uniform hypergraphs with maximum degree at most $d$ is $n^{1+\epsilon}$-Ramsey.