Polynomial-Time Optimal Equilibria with a Mediator in Extensive-Form Games

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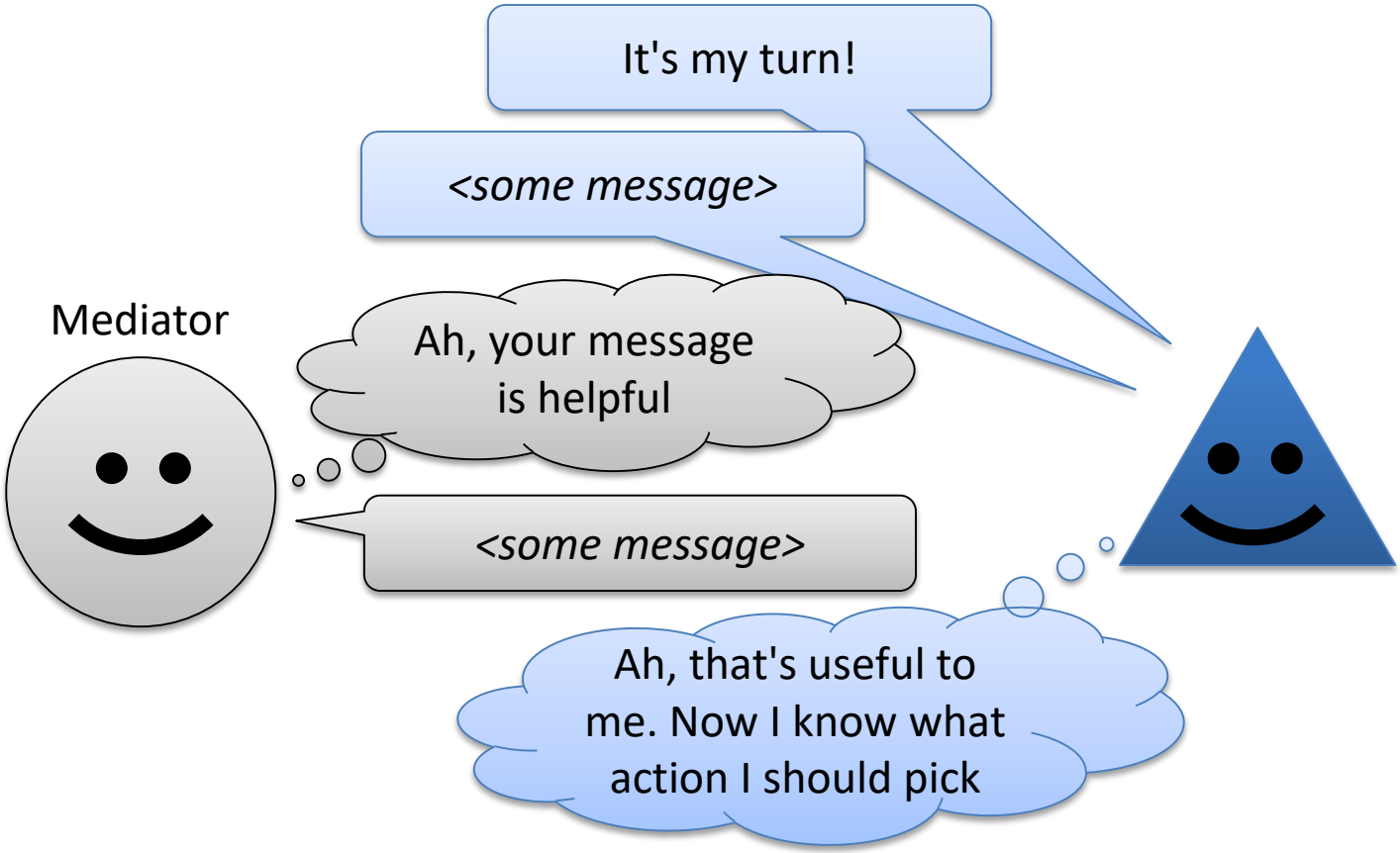
Communication Equilibria



A **communication equilibrium** is a **mediator strategy** $\boldsymbol{\mu}$, and a **strategy profile** $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)$ **for the players** such that, with the mediator's strategy held fixed, $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)$ is a Nash equilibrium for the players.

Forges (Econometrica 1986), "An approach to communication equilibria"

Polynomial-time Communication Equilibria

Theorem [Main result]: Given an extensive-form game Γ with N nodes, an optimal communication equilibrium can be computed in time polynomial in N.

Proof Sketch

1. Consider the augmented extensive-form game $\hat{\Gamma}$ in which the mediator is an explicit extra player
2. Prove a slightly stronger version of the revelation principle that ensures that $\hat{\Gamma}$ can be simplified to have size $O(N^2)$
3. find mediator strategy $\boldsymbol{\mu}$

   such that for all players i

   obedience is a best response for i if all other players are obedient

This can be written as a linear program!

□

Seemingly unrelated problems

- Optimal communication equilibria
- Optimal Bayesian persuasion (information design)
- Optimal automated mechanism design
- Optimal extensive-form correlated equilibria (EFCE)
- …and more!

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Takeaway 1: Several seemingly unrelated problems can in fact be grouped under the same umbrella and solved using very similar techniques

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Optimal EFCE is NP-hard! [von Stengel & Forges 2008, Chu & Halpern 2001]

Takeaway 2: The hardness of optimal equilibrium computation, at least among these equilibrium concepts, is driven by **the imperfect recall of the mediator**

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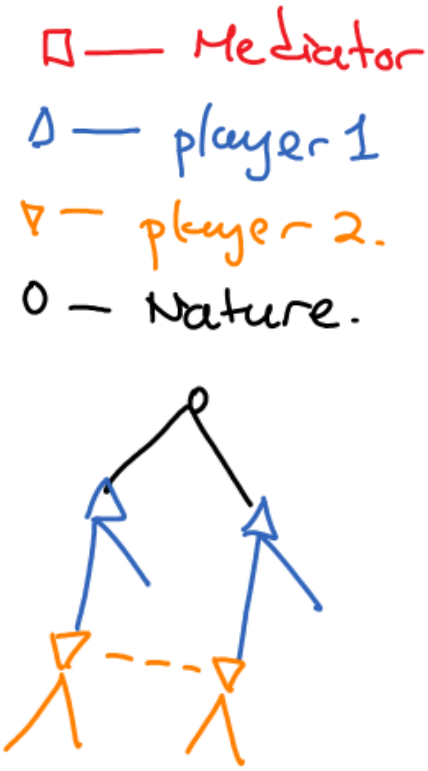
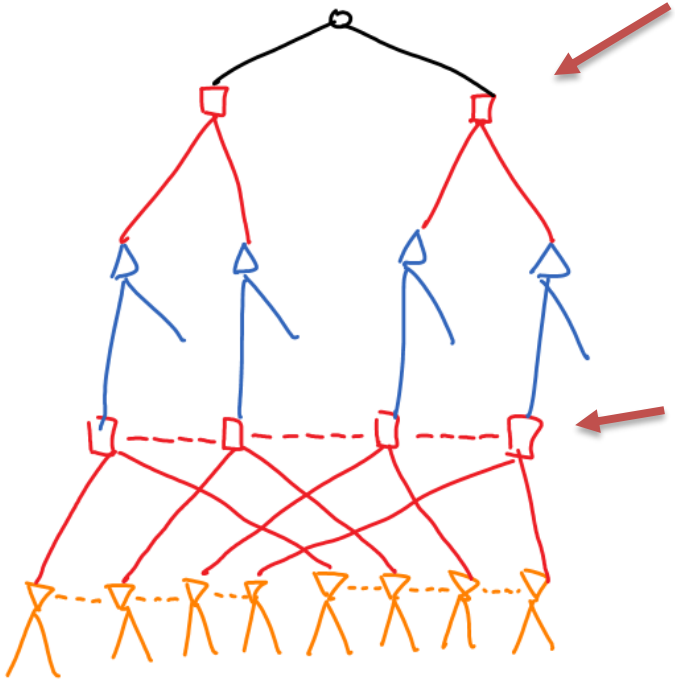
Extensive-form correlated equilibria via imperfect-recall mediators



Augmented game:



The mediator can give P1 a recommendation conditioned on P1's information

The mediator can give P2 a recommendation, but it can't condition that recommendation on P1's information

⇒ Imperfect recall!