Why do students rush to calculus?

by Anoop Ahluwalia and Joseph G. Rosenstein[1]

This article reports on a survey conducted in the spring of 2011 with Rutgers students who had taken the Advanced Placement (AP) calculus course and exam in high school and attempts to answer the question: Why do students take the AP calculus course and exam and what are the consequences of their doing so?

Introduction

In an earlier study, "Rush to Calculus"[2] [4], Rosenstein used high school and college transcripts of a cohort of Rutgers students to conclude that not too many (only 1 out of 16) students who took a full year of calculus in high school earned "advanced placement" and continued with their acceleration during their first year at the university. Although the article shared results from a single university, it is plausible that similar trends hold across the nation. Contrary to "Rising Above the Gathering Storm" [3], a report of the National Academy of Sciences, that advocates for increasing the number of students taking calculus in high school as a means to generate future STEM (Science, Technology, Engineering and Math) majors, we conjectured that this rush to calculus (i.e., taking calculus in high school) does not necessarily increase a student's motivation to pursue STEM majors and may indeed have the opposite effect. We contend that it is imperative to first investigate the reasons why students take AP Calculus, what are their expectations from the course, how does the course impact their future decisions and what do they ultimately do with this advanced placement. Only then can one make informed recommendations about attempting to increase the number of AP calculus students.

Methodology

We conducted an online survey in the spring of 2011 with the cohort of undergraduate students who entered the School of Arts and Sciences (SAS) at Rutgers University-New Brunswick[3] in the fall of 2007 and whose scores on the AP Calculus exam had been reported to Rutgers; in the survey we asked them a number of questions related to their reasons for taking AP Calculus, their undergraduate programs, and their intended careers. The Office of Institutional Research (OIR) at Rutgers provided us with the list of students in this cohort and their email addresses; the OIR also provided us with the students' AP Calculus scores and a list of all the college math courses that they had taken and their grades in these courses.[4]

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[2] "Rush to Calculus" by Joseph G. Rosenstein has been submitted for publication in Research in Collegiate Mathematics Education.

[3] SAS is the largest school at Rutgers-New Brunswick, with 3849 entering students in 2007; there were 625 students in the School of Engineering, 673 in the School of Environmental & Biological Sciences, 208 in the School of Pharmacy, and 153 in the Mason Gross School of the Arts. The engineering students are discussed in Section 5.

[4] We are very grateful to Tina Grycenko of the Rutgers' Office of Institutional Research and Academic Planning for gathering the data for this study and for providing us access to the necessary information.

A total of 531 students who had entered SAS in the fall of 2007 had taken the AP Calculus exam in the spring of 2007 and had reported their AP exam scores to the university.[5] Of these 531 students, 478 were still at Rutgers and their email addresses were provided to us by the Office of Institutional Research. These 478 students were invited to participate in an online survey of about 25 questions at the end of the spring 2011 semester. They were given a week to respond to the survey and those that had not yet completed the survey received two reminders during that week to do so. To provide the students incentive to participate in the survey, those completing every question in the survey were to be entered into a raffle for an I-Pad 2.

Before conducting the survey with all 478 students, we first piloted the survey by inviting 55 students chosen randomly to complete the survey. After we reviewed the responses of the 17 students who completed the survey, we slightly modified some questions in the survey instrument and added a few more questions to it. Next, all of the remaining 423 students were invited to participate. In total, 194 students actually completed the survey[6]; the high participation rate (40.6%) was presumably due, at least in part, to the popular incentive we provided.

To analyze the survey responses, we divided the participants into three groups as follows.

A. Group A consisted of the students who had earned credit for both Calculus 1 and 2 (score of 4 or 5 on the BC exam)[7],
B. Group B consisted of the students that received credit for Calculus 1 (score of 4 or 5 on the AB exam or on the AB portion of the BC exam) but not for Calculus 2, and
C. Group C consisted of the students that did not receive college credit for Calculus 1 (score of 3 or below on the AB exam).

Of the 194 students who completed the survey, 37 (19.1%) were in Group A, 58 (29.9%) were in Group B and 99 (51.0%) were in Group C.

To determine whether the sample of 194 students was representative of the entire cohort of 478 students who had taken the AP exam and were invited to participate in the survey, we examined the information provided by the Office of Institutional Research and found that, if all 478 students had participated in the survey, the number and percentages of students in the three categories would have been 64 (13.4%) in Group A, 129 (27.0%) in Group B, and 285 (59.6%) in Group C. Not surprisingly, the students in Groups A and B who were more successful in AP Calculus tended to participate in a survey on AP Calculus at a higher rate than the students in Group C who were less successful in AP Calculus. However, the rate of their participation was only slightly higher so that the differences are relatively small and the overall percentages of the

[5] Although there may have been some students who had not reported their AP scores to the university, their number was likely small, since the reason students take the AP exam is to enhance their chances for admission. There were likely some transfer students who had taken the AP exam who were not among the 478 students eligible for participation in the study but, for such students, we would not have had complete information about math courses taken in college.

[6] The 17 students who completed the initial version of the survey therefore did not respond to all of the questions in the final version of the survey and so, for a few questions, there were fewer than 194 responses.

[7] There are two Advanced Placement calculus exams, known as the AB exam and the BC exam. Students who take the high school equivalent of a one-semester college calculus course take the AB exam, whereas those who take the equivalent of a two-semester college calculus course take the BC exam; approximately 60% of the BC exam addresses topics on the AB exam.

478 students in the three categories correspond reasonably well to the percentages in the three categories of the 194 students who completed the survey.

In the questionnaire, we asked students to provide the following information, and we report on their responses in this article.

- Why they took the AP Calculus course and exam.
- Whether taking AP Calculus increased their interest in math and/or encouraged them to pursue studies leading to a career in math, science, or engineering.
- What math course they actually took in their first semester in college.
- Why they were unsuccessful in getting AP credit (if that was the case).
- How many math courses they took in college.
- Whether taking math courses in college increased their interest in math and/or encouraged them to pursue careers in math, science, or engineering.
- What were their majors and what were their intended careers.
- Whether they thought that they had benefited from taking AP Calculus in high school.

We also asked them several questions whose responses are not included in detail in this article: when they took Algebra 1[8], and why they stopped taking math courses (if they did).[9]

Motivation for this Study

In the previous study, "Rush to Calculus" [4], Rosenstein examined the high school and college transcripts of students who entered Rutgers College in the fall of 2003 to test the following conjectures that he had made on the basis of an informal study he had conducted a number of years earlier:

1. About half of the students who take a full-year calculus course in high school take an AP Calculus course
2. About half of the students who take an AP Calculus course take the AP Calculus exam
3. About half of the students who take the AP Calculus exam receive advanced placement
4. About half of the students who receive advanced placement continue their acceleration during their first year in college.

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[8] We anticipated that a substantial portion of the students in Groups A and B would have taken Algebra 1 in 8th grade, but that many fewer students in Group C would have done so. It turned out that there was little difference between the three groups and overall, about 74% had taken Algebra 1 in the 8th grade, about 13% in the 9th grade, and about 13% didn't remember. Not surprisingly, then, of the 25 students who said that they had taken Algebra 1 in the 9th grade, the percentages in Groups A, B, and C, respectively (20%, 32%, and 48%) were similar to the percentages of the 194 students in the survey in the three groups (19.1%, 29.9%, and 51.0%). Although one cannot draw conclusions from such a small sample, it seems that taking Algebra 1 in the 8th grade does not have much influence on one's performance on the AP Calculus test. This corroborates the results from the earlier study "Rush to Calculus" where similar conclusions were made using the students' high school and college transcripts – that is, although taking Algebra 1 in the 8th grade "benefits the best students by smoothing their path to AP Calculus, there is no evidence that it benefits all students."

[9] Since the students were now completing their undergraduate careers, their response to this question was usually "I completed the requirements of my major." Had we asked them a few years earlier why they stopped taking math their responses might have been substantive rather than administrative, and therefore more interesting.

If these conjectures were accurate, then only 1/16 (or 6.25%) of the students who take an accelerated math program in high school continue their acceleration during their first year in college. Rosenstein found that all four of these conjectures were indeed approximately correct:

- 123 out of 217 students (56.7%) who took a full year of calculus in high school took AP Calculus,[10]
- 59 out of 123 students (48.0%) who took AP Calculus took the AP Calculus exam,
- 162 out of 332 students (48.8%) who took the AP Calculus exam received advanced placement[11], and
- 66 out of the 162 students (40.1%) who received advanced placement continued with their acceleration.

This enables him to conclude that the percentage of students who take a full-year calculus course in high school who then continue with their acceleration through their first year at Rutgers is the product of these fractions, that is, (59/217) x (66/332), or 5.4%.

That is, of the students in the 2003 cohort, only 5.4% continued their acceleration during their first year at Rutgers.

Rosenstein's study could not address questions like "why did the students take the AP calculus course and the AP calculus exam?" and "what impact did taking this course have on their choice of majors and careers?"

The purpose of this study, and the focus of this report, is addressing those questions.

The present study cannot address the first two conjectures since it does not involve examination of high school transcripts; however, as we will see, the data from the present study corroborates the earlier study's finding with respect to the last two conjectures.

With respect to the third conjecture, we have already reported that 285 of the 478 students (59.6%) would have been in Group C, so that only 40.4% of the students who took the AP exam received credit for Calculus 1. However, all of the 531 students in the database should be included in this calculation. It turns out that of the additional 53 students who were no longer at Rutgers, 37 would have been in Group C, so that total number and percentage of students in Group C in the 2007 cohort would have been 322 out of 531 or 60.6%. In any case, approximately half of the students in this cohort who took the AP exam did not receive any college credit for calculus; this conclusion is consistent with both the conjectures and conclusions of Rosenstein's earlier study.[12] [13]

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[10] In conducting the earlier study, Rosenstein examined the high school transcripts of almost 400 students. Of these students 217 took a full year calculus course in high school.

[11] The cohort of Rutgers students who had taken the AP Calculus exam, provided for the earlier study by the Rutgers Office of Institutional Research, consisted of 332 students.

[12] In the earlier study, which dealt with students who entered Rutgers *College* in the Fall of 2003, 162 out of 332 students, or 48.8%, received credit for Calculus 1 and/or Calculus 2, so that 170 out of 332, or 51.2% would have been in what we have called Group C. Why the discrepancy between these percentages – 51.2% in Group C for students entering Rutgers in 2003 vs. 60.6% in Group C for students entering Rutgers in 2007? Prior to 2006, the different colleges at Rutgers University had different entrance requirements, with Rutgers *College* – whose students populated the earlier study – having the highest requirements, but after 2006 the requirements for admission to Rutgers University were standardized – so that the 2003 study involved a more select group of students than the

The findings of the present study with respect to the fourth conjecture are discussed in Section 4.

Results

Section 1: The reasons why students took the AP Calculus *course*

In this question, we asked students to read the fifteen statements in Table 1 describing why some students might take an AP Calculus course in high school and asked them to rate each statement according to 5 possible choices: strongly agree, agree, somewhat agree, disagree and strongly disagree.

To facilitate analysis of the students' responses to these fifteen statements, we have rearranged them thematically in Table 1; in the survey itself they were ordered more randomly and the categories ("educational," "pragmatic," etc.) were of course not given. We summarize the students' responses by reporting the percent of students that "strongly agree" or "agree" to the stated reasons for taking an AP calculus course in high school. The syntax of entries in Table 1 is as follows: the sum of "strongly agree" and "agree" is written first followed by a parenthesis that has only the "strongly agree" percent. For example, an entry "70 (50)" would mean that 50% "strongly agree" and 20% "agree," yielding the total of 70%. In the right-most column, numbers of students appear rather than percentages. The reason for this notation is that although for some statements the three groups seem to be close in preferences (that is, the total of "strongly agree" and "agree" are comparable), looking at just "strongly agree" percents reveals meaningful differences (as in the first row of Table 1).

Statement	**Group A N=37**	**Group B N=58**	**Group C N=99**	**All N=194**
Intrinsic rationales				
I really liked math when I was in high school	94.6 (83.8)	89.6 (51.7)	67.7 (41.4)	155 (103)
I wanted to learn more higher level mathematics	83.8 (51.4)	79.4 (46.6)	45.4 (22.2)	123 (69)
I enjoy challenging math courses	89.2 (64.9)	79.0 (43.9)	52.0 (25.5)	130 (74)
Educational rationales				
I wanted to start off in college with a higher level course	83.8 (70.3)	84.5 (58.6)	74.7 (44.4)	155 (104)
I wanted to be better prepared for college courses	78.4 (48.6)	74.2 (48.3)	72.7 (39.4)	145 (86)

2007 study. Of course, it is also possible that more students took the AP exam in 2007 than in 2003, and the additional students were not among those who performed well on the exam.

[13] It should be noted that students who intended to be engineers were not enrolled in Rutgers College and therefore were not part of this study. Of the 216 students in that cohort who took the AP Calculus exam, 134 (or 62.0%) rec'd advanced placement in mathematics, so that 82 students (or 38.0%) would have been in Group C. (The 2007 cohort of entering engineering students did much better on the AP exam than their 2003 counterparts.) If we combine the cohort in this study with the cohort of engineers, we get 404 out of 747 (or 55.6%) receiving no advanced placement in mathematics.

I planned to major in a math related subject	67.6 (51.4)	45.6 (36.8)	25.2 (13.1)	77 (53)
Pragmatic rationales				
AP Calculus looks good on college applications	78.4 (56.8)	82.7 (51.7)	80.8 (48.5)	158 (100)
Taking AP courses would enable me to save money by graduating from college in three years	21.6 (8.1)	20.7 (12.1)	24.2 (13.1)	44 (23)
Taking AP course would boost my GPA	45.9 (24.3)	32.8 (19.0)	30.3 (14.1)	67 (35)
Social rationales				
My friends were taking AP Calculus	81.1 (51.4)	53.5 (25.9)	69.7 (37.4)	131 (72)
My parents wanted me to take AP Calculus	51.4 (21.6)	39.7 (19)	46.5 (19.2)	89 (39)
It was expected of me	78.4 (51.4)	62.1 (32.8)	61.2 (24.5)	126 (63)
Default rationales				
I had to take some math course in my senior year	62.2 (54.1)	48.3 (29.3)	70.6 (42.9)	121 (80)
My math teachers or counselors suggested that I should take it	73.0 (37.8)	69.0 (41.4)	75.5 (35.7)	142 (74)
Negative rationales				
I wanted to avoid taking math in college	05.4 (2.7)	05.1 (1.7)	30.3 (18.2)	35 (20)

Table 1: *Student responses to 15 possible reasons for taking an AP Calculus* ***course***

Among the interesting observations that can be made from Table 1 are the following:

- Not surprisingly, the students with the strongest performance on the AP Calculus Exams (Group A) had the most positive attitudes towards mathematics. They really liked math in high school, wanted to start off college with higher-level math course, wanted to learn higher-level math and enjoyed challenging math courses[14].
- In Group B although only about half of them "strongly" liked math in high school (compared to over 80% in Group A), with respect to the other intrinsic rationales their percentages were comparable to those of Group A, that is, many of them wanted to start

[14] Examples of two Group A students who fit this description but followed different paths are "Z" and "M": The AP Calculus course encouraged Z greatly towards STEM majors as "I enjoyed the math classes in high school and I actively pursued learning higher level math, I decided I wanted to major in math when I went to college", which Z did along with a minor in Computer Science. Z plans to pursue a professorship in mathematics and is glad to have taken AP calculus in high school and started college with honors Calculus 3. Z shares that "Challenging yourself is what the study of mathematics at the high school level is all about … Pursuing and taking the AP Calculus exam is just striving for excellence at the high school level." Z took 18 math courses at college and earned grades of A or B in all of them. Taking AP Calculus also encouraged M toward STEM majors, as math had always been M's favorite subject. However, after completing the calculus sequence and linear algebra with grades of A and B+, M took three more advanced courses (Intro to Mathematical Reasoning – grade C+, Intro to Abstract Algebra – grade B+, and Advanced Calculus – withdrew) and, as a result, realized that "I couldn't do upper level mathematics, (e.g., topology) even though I am very good at solving problems with calculus and basic algebra". M majored instead in Materials Science and Engineering.

college with a higher-level course, wanted to learn higher-level math and enjoyed challenging math courses[15].

- In Group C, less than half (41.4%) "strongly" liked math in high school (compared to 51.7% in Group B and 83.8% in Group A) but many of them wanted to start college with a higher-level course. Fewer than half of the Group C students wanted to learn higher-level math and roughly half of them enjoyed challenging math courses. Approximately 71% of Group C agreed/strongly agreed that they took AP calculus because they *had* to take some math in their senior year.[16] Group C had the least favorable attitude towards mathematics and pursuing the subject further.
- In Group C, as many students as 30% of those who took the AP Calculus course wanted to avoid taking math in college versus only 5% from each of Group A and B. Group C was significantly more motivated to take AP calculus in order to avoid college level mathematics[17].
- While a high percentage (67.6%) of Group A students planned to major in a math related subject in college, fewer than half (45.6%) in Group B and barely a quarter (25.2%) in Group C planned to major in a math related subject.[18] It is interesting that a substantial percentage of the students in groups B and C took AP calculus even though they did not plan to major in a math related subject and were, therefore, not seeking to be "advanced" in mathematics. On the other hand, it is interesting that the number of students in each of the three groups that agreed or strongly agreed that they "planned to major in a math-related subject" was 25 or 26; that is, the potential number of math-related majors from each of the three groups is about the same.[19]
- In all three groups more than half of the students said that it was expected of them to take AP calculus in high school[20]. It is striking that so many students felt that they were "expected" to take the "advanced" math course.

[15] An example of a Group B student: "B" enjoyed challenging math courses and planned to major in a math-related field. Taking the AP course encouraged B somewhat toward a STEM career; B shared that "After finding I received a 5 (on the AP exam) and an 800 on my PSAT … notified me that I was a Math person, so at first I tried Comp Sci as a minor and then switched it to Math." B "was fortunate to have great math teachers in high school." B later started in business and dropped mathematics as a major but is thankful that the AP course facilitated finishing the business degree a year early. B took two courses in math (Calculus 2 and Linear Algebra) and did well in them.

[16] This suggests that had there been a good alternative course to AP Calculus, they might well have taken it.

[17] An example of a Group C student: "A" took AP calculus because A had to take some math in the senior year and because A wanted to avoid taking math in college. AP calculus discouraged A a great deal from STEM careers as "Math never was my strong suit, and after taking the AP exam I realized I did not want to get involved in a career path in which math was essential". In college A took a remedial math course followed by Math for Liberal Arts (in which A's grade was A). A does not recommend students to take the AP Math course unless they are good math students as the course is "incredibly difficult". A plans to teach history at the secondary level.

[18] In Section 5 we discuss the question of how many of these students actually majored in math-related subjects; that is not easy to determine, since the notion of "math-related subject" is somewhat subjective.

[19] Referring to the row labeled "I planned to major in a math-related subject" in Table 1, both 67.6% of the 37 students in Group A and 25.2% of the 99 students in Group C are 25, and 45.6% of the 58 students in Group B in 26. It is striking that the number of students in all three Groups A, B, and C that "planned to major in a math-related subject" is 25 or 26.

[20] This finding is also consistent with a conjecture in "Rush to Calculus" [4] that the exception had become a norm, that is, whereas AP Calculus was originally introduced to permit the best students to obtain "advanced placement" in mathematics, it was now expected that students should routinely take AP calculus.

- Most of the students in all groups thought that AP Calculus looks good on college applications and helps them better prepare for college.
- None of the three groups thought that saving money (by spending fewer years in college) was an important reason for taking the course and fewer than half of the students in each group thought that taking the course would boost their GPA.
- Friends and parents were more of an influence for Group A in their decision to take AP calculus than for Groups B and C, but the effects of teacher/counselor intervention were comparable in all three groups.
- The students were all very aware that "AP Calculus looks good on college applications," but for students in Groups A and B, the "intrinsic rationales" played the strongest role in their taking AP Calculus, whereas for students in Group C, the "college applications" rationale for taking AP Calculus earned the greatest percentage of agreement.

It is clear from looking at the observations above that the three groups had very different reasons for taking AP calculus in high school. While students in Group A were more inclined to take higher level math courses and pursue a math-related major, many students in Group C wanted to be done with math quickly and thought that AP Calculus would help them achieve that goal.

In the earlier study [4], Rosenstein conjectured that "the two most popular reasons [for students to take AP Calculus] would be that they or their parents believe that taking AP courses will help them get into better colleges and that taking AP Calculus was expected of them – that the exception had become the norm. But when such a study is conducted we may also find that the acceleration strategy has produced a lot of negative attitudes about mathematics." To what extent are these conjectures correct?

- Clearly the rationale that scored highest among the students in all three groups together was "AP Calculus looks good on college applications."
- For the students who were successful in gaining "advanced placement" (that is, the students in Groups A and B), "taking AP Calculus was expected of me" was overshadowed by their positive attitudes toward mathematics, but for the students who were not successful (that is, Group C), the social rationales and default rationales in Table 1 were more prominent – so that we can say that those students took AP Calculus because it was expected of them. Indeed, 72 of the 99 students in Group C strongly agreed with one or more of those rationales – that is, essentially every student in Group C, except for those 25 who planned to major in a math-related subject, strongly agreed with at least one of the social and default rationales, and on average strongly agreed with at least two of them (since these 72 students amassed 157 strong agreements with these five rationales).
- A substantial percentage of students in Group C had developed negative attitudes toward mathematics – 30.3% of these students agreed that they "wanted to avoid taking math in college."

Section 2: The reasons why students took the AP *exam*

We asked students to read several statements describing why students take the AP Calculus *exam* in high school and asked them to rate each statement according to 5 possible choices: strongly agree, agree, somewhat agree, disagree and strongly disagree. Although it might seem that the previous question would be sufficient, in the analysis of high school transcripts in the earlier study it was found that only about half of students who take the AP Calculus course actually take the AP Calculus exam. For this study, we did not have information about students who took the AP Calculus course but did not take the exam; as a result we are unable to determine how many students who entered SAS in 2007 took the AP Calculus course or, therefore, what percentage of the students who took the AP Calculus course also took the AP Calculus exam.

In Table 2 we summarize responses by reporting the percent of students that "strongly agree" or "agree" to the stated reasons for taking AP calculus *exam* in high school[21]. The syntax of the entries in this table is same as the previous table with percents for "strongly agree" written in parenthesis.

Statement	**Group A**	**Group B**	**Group C**
Intrinsic rationales			
I enjoyed studying math and taking math exams.	86.5 (59.5)	67.3 (41.4)	43.4 (21.2)
Educational rationales			
I wanted to start off at college with Calc 2 or Calc 3.	78.4 (56.8)	81.1 (48.3)	29.3 (10.1)
I thought that I would learn the course material better if I planned to take the exam.	59.4 (37.8)	41.4 (20.7)	52.5 (22.2)
Pragmatic rationales			
Colleges give extra consideration to applicants who are planning to take AP exams.	56.7 (18.9)	48.3 (22.4)	41.4 (18.2)
Taking the AP exam and getting advanced placement would enable me to save money by graduating from college in three years.	21.6 (10.8)	22.4 (15.5)	25.3 (16.2)
Social rationales			
My friends were taking the AP Calculus Exam.	62.1 (35.1)	48.3 (20.7)	48.4 (24.2)
My parents wanted me to take the AP Calculus Exam.	54 (21.6)	41.3 (17.2)	44.5 (18.2)
Default rationales			
My math teachers or counselors suggested that I should take it.	67.5 (40.5)	68.9 (31)	70.7 (27.3)
Negative rationales			
I wanted to take as few math courses as possible in college.	21.6 (8.1)	13.8 (5.2)	45.4 (23.2)
Requirements			
All students in AP Calculus were required to take the exam.	29.7 (27.0)	31.0 (22.4)	39.4 (30.3)

Table 2: Student responses to 10 possible reasons for taking the AP Calculus ***exam***

Among the interesting observations that can be made from Table 2 are the following:

- About 80% of the students in Groups A and B (Group A more "strongly") wanted to start college with Calc 2 and Calc 3 but very few students in Group C wanted to do the same.

[21] Again, the statements in the table are arranged thematically although in the questionnaire they were presented in a more-or-less random order without, of course, the italicized characterizations in this table.

This reflects, again, that many students in Group C were not looking for an "advanced placement" in math when they reached college.

- As expected, Group A students enjoyed studying math and taking math exams the most followed by Group B and then Group C.
- Group B students were significantly less positive in responding that taking the AP exam was a good way to learn the course material; no reason for this discrepancy is apparent.
- Group A students were more likely than the other groups to agree that "Colleges give extra consideration to applicants who are planning to take AP exams." It is not clear that this belief corresponds to reality; indeed, Rutgers admission personnel reported that this is not the case.[22]
- Significantly more students in Group C wanted to "take as few math courses as possible in college". This reflects that Group C students were less inclined towards a math-intensive major and perhaps viewed AP Calculus as a means to avoid future math courses. It is striking that, although 30.3% gave this as a reason for taking the AP *course* in the previous question, this percentage rose by half to total 45.4% when the students were asked to explain why they took the AP Calculus *exam*. Clearly the AP course served to discourage these 15 students from continuing to take math courses.
    - Referring back to the discussion at the end of Section 1, the confirmation of the conjecture in the previous study that "the acceleration strategy has produced a lot of negative attitudes about mathematics" is more pronounced here, as 45 out of the 99 students in Group C wanted to "take as few math courses as possible in college."
- Parents and friends were a bigger influence on Group A for taking the exam than the other two groups; but teachers and counselors had comparable effect for all three groups. It is possible that Group A students more often had peers and parents that encouraged their mathematical pursuit and, hence, benefitted from a support system that Group B and Group C students perhaps lacked.
- None of the groups considered saving money by graduating in three years as an important factor for taking the AP Calculus exam.
- If we compare the responses in Table 2 to the statement "I wanted to start off at college with Calc 2 or Calc 3" with the responses in Table 1 to the comparable statement "I wanted to start off in college with a higher-level course," we find a major difference between both Groups A and B on the one hand and Group C on the other. About 80% of the students in both Groups A and B responded "agree" or "strongly agree" to both of those statements. However, although 72 of the 99 students in Group C responded "agree" or "strongly agree" to the first statement (as to why they took the AP *course*), only 29 responded "agree" or "strongly agree" to the second statement (as to why they took the AP *exam*).[23] That is, as a result of taking the AP course, 43 out of the 72 students, or 59.7%, of the Group C students who intended to start college with a higher-level math course had lowered their expectations so that by the end of the school year, when the AP

[22] It may be that other colleges and universities do give extra consideration to those students who plan to take AP exams; this, however, is unlikely since the scores on AP exams taken by high school seniors, and the information as to whether they have actually taken the exams, arrive well beyond the date when admissions decisions are made.

[23] It is conceivable, though rather unlikely, that some students may have considered the statement in Table 1 as referring to higher-level courses in general (and not particularly in math) whereas the statement in Table 2 ("I wanted to start off at college with Calc 2 or Calc3") is specifically related to math.

calculus exam was given, they no longer expected that they would earn advanced placement.[24]

We asked students in Group C to indicate why they thought they didn't get a higher score on the AP Calculus exam. We gave them eight possible responses and allowed them to select more than one response if they found it relevant to their experience. The two most popular reasons that Group C students picked, from the eight reasons provided[25], were "I didn't study for the exam" (51 out of 99, or 51.5%) and "Senioritis" (40 out of 99, or 40.4%). Interestingly, more than half of the students said that they did not study for the AP exam. This is in line with the decreased expectations described above: only 29.3% of the Group C students indicated that they were taking the exam in order to obtain advanced placement. The next two popular reasons that students picked were that the course was not taught well (26, or 26.3%) or that they thought they had a better score (20, or 20.2%). Interestingly, 15 students chose the option, "I wasn't too interested in the advanced placement".

The issue of whether students were required to take the AP exam also leads to some interesting conclusions:

- Almost half of the students in each group said that the AP exam was required; that is, if the students who responded "somewhat agree" to this statement were included, then 43.3% of Group A, 41.3% of Group B, and 46.5% of Group C, and altogether 86 out of 194 students, or 44.3%, agreed with the statement that "All students in AP Calculus were required to take the exam."[26] This is surprising because, based on past information, it had seemed that many fewer schools in New Jersey (from which most Rutgers students graduate) required students who took AP courses to take AP exams.[27] It is important to note that a major study of the role of Advanced Placement courses as a criterion for admission to the University of California [2] concluded that "while student performance on AP examinations is strongly related to college performance, many students who take AP courses do not complete the associated AP exams, and merely taking AP or other honors-level courses in high school is not a valid indicator of the likelihood that students will perform well in college."
- It is also surprising that the 108 students who elected to take the AP exam even though they were not required to do so, did no better than the 86 students who were required to take the exam. That is, one might have expected that students who did not anticipate doing well in the AP exam would not have taken the exam if they weren't required to take the exam, so that a higher percentage of the students who elected to take the exam

[24] The number of students in Group C whose expectations were lowered was 43 out of 99. If we apply this proportion to the 285 students in the cohort of 478 students who were in Group C, we would conclude that 124 students, or 25.9% of all 478 students were adversely affected in this way by taking the AP Calculus course.

[25] In addition to the five options mentioned above, students could also pick from the following choices: "Does not apply because I received credit for Calculus 1"; "I had already gotten into college, so it didn't matter"; "I don't know" or "Other – please write your response in the comment box below".

[26] It is puzzling that anyone would respond "somewhat agree" to what seems to be a factual statement, but 15 students did. It is natural to assume that they meant that they were indeed required to take the exam but they were not motivated to take the exam by the requirement; but there may be other explanations.

[27] We cannot conclude that almost half of New Jersey schools require their AP students to take the AP exams, since those which do require students to take AP exams are likely to be over-represented in our sample of 194 students.

would have been in Groups A and B. However, as can be seen from Table 3, that was not the case.[28]

	# and % in Group A	# and % in Group B	# and % in Group C
86 students required to take the AP exam	16 (18.6%)	24 (27.9%)	46 (53.5%)
108 students not required to take the AP exam	21 (19.4%)	34 (31.48%)	53 (49.1%)
Total of 194 students	37 (19.1%)	58 (29.9%)	99 (51.0%)

Table 3: Comparison of students required to take the AP exam with those who were not

Why did the students who were not required to take the AP exam take it? One might reasonably expect that these students who took the AP Calculus exam voluntarily would have expressed more agreement with the first nine options in Table 2 (i.e., all but "it was required") than those who were required to take the exam. However, if one constructs two variants of Table 2, one restricted to those who took the exam voluntarily and another restricted to those who were required to take the exam, that doesn't happen. It is striking that in almost every case, the level of agreement of the students who were *not* required to take the exam with the 9 possible reasons in Table 2 *are weaker than* the level of agreement of the students who said that they *were* required to take the exam. In particular, in the subgroup of students who were not required to take the exam:

- significantly fewer students expressed agreement or strong agreement with the following reasons: their friends were taking the exam, their parents wanted them to take the exam, their teachers or counselors suggested that they take the exam, they would learn the course material better, colleges give extra consideration to applicants who are planning to take AP exams;
- fewer students in this group expressed agreement or strong agreement with the following reasons: they wanted to start college with Calc 2 or Calc 3, they enjoyed studying math and taking math exams.

For those in Groups A and B who were not required to take the AP exam, the most likely reasons they took the exam were that they wanted to start college with Calc 2 or 3 or that they enjoyed studying math and taking math exams, whereas for those in Group C who were not required to take the AP exam, the most likely reason they took the exam was that their math teachers or counselors suggested that they take it.

[28] If we delete from the first row of Table 3 the 18 students who agreed only "somewhat" that taking the AP exam was required, then the conclusion would be somewhat weakened, since the entries would be 11 (16.2%) for Group A, 18 (26.5%) for Group B, and 39 (57.4%) for Group C. But given the most likely interpretation of their "somewhat agree" response provided above, this emendation is likely inappropriate.

Section 3: The impact of AP calculus on students' choice of a major

We asked students to describe the extent to which taking AP calculus and its prerequisites in high school encouraged or discouraged them from pursuing studies leading to a career in math, science, or engineering. The responses are summarized in the table below with the percent of students in each category listed first, followed by the actual number of students inside the parentheses. Students were offered the opportunity to provide a written explanation of their response to this question and a substantial number (139 out of 194, or 71.3%) responded. The number of students in each category who provided comments is indicated in the subscript; in each cell of Table 4, a substantial percentage of the students shared a comment.

Statements	**Group A**	**Group B**	**Group C**
They encouraged me a great deal toward such careers.	35.1 (13)$_{10}$	32.8 (19)$_{14}$	14.1 (14)$_{7}$
They encouraged me somewhat toward such careers.	32.4 (12)$_{8}$	29.3 (17)$_{11}$	22.2 (22)$_{18}$
Taking those courses didn't make much of a difference.	32.4 (12)$_{11}$	36.2 (21)$_{17}$	49.5 (49)$_{31}$
They discouraged me somewhat from such careers		1.7 (1)$_{1}$	9.1 (9)$_{6}$
They discouraged me a great deal from such careers.			5.1 (5)$_{4}$

Table 4: Did taking AP Calculus encourage students toward careers in mathematics, science, and engineering?

About 2/3 of the students in both Groups A and B responded that taking these courses encouraged them toward careers in math, science, or engineering, and about 1/3 indicated that taking these courses made no difference. These fractions are reversed for Group C; that is, only 1/3 said that taking AP Calculus encouraged them toward careers in math, science, or engineering, and 2/3 indicated that taking these courses made no difference or even served to discourage them from such careers.

If the three groups are combined, we see that altogether 97 out of 194 or 50.0% of the students reported that the AP calculus course made no difference or in some cases discouraged them from a major in math, science, or engineering. This is somewhat disappointing, for one could reasonably expect that every student taking AP Calculus would become more positively oriented toward careers in math, science, or engineering. However, it is possible that many students responded this way because they were already committed to pursue such careers and the AP course in particular did not make a contribution to their existing interest in such careers. It is also possible that the students, who lowered their expectations as a result of taking AP Calculus, as we saw in Section 2, in effect took themselves out of the math, science, and engineering pipeline as a result.

One might instead focus on the opposite conclusion, that for the other half of the students AP Calculus served to encourage them to major in math, science, or engineering. However, the fundamental question is whether students took AP Calculus because they were already intending to major in math, science, or engineering or whether it was a result of taking AP Calculus that they decided to major in these disciplines.[29]

We do have some information about this because, as noted above, we encouraged students to share the reasons for their responses to this question through written comments, and a

[29] We could have asked that question directly in the survey, but it is not clear how reliable the responses would be.

substantial percentage of the students provided such comments. Following is a summary of their comments.[30]

In the paragraphs that follow, we will use the acronym "STEM" (science, technology, engineering, mathematics) instead of the phrase "math, science, and engineering." It should be noted that STEM does not mean the same as "math, science, and engineering" and that the acronym STEM was never used anywhere in the survey. We use it here simply to avoid repeating over and over the longer phrase "math, science, and engineering" that was used in the survey.

Of the 42 students in Groups A and B who said that they were "encouraged" or "strongly encouraged" toward STEM careers by AP Calculus and provided comments,

- 19 were clearly on the STEM track already (Examples: "I always wanted to be a physics major." "I love math and science; I knew I was going into science with a strong math.")
- 5 gave no indication of whether they were on the STEM track already (Examples: "After I saw how fun math could be, I decided I wanted a career in it." "Math definitely set me toward the hard sciences." "Led me to my major in computer science.") [31]
- 1 indicated that he was not previously intending to be a STEM major (Example: "After finding I received a 5 and an 800 on my PSAT when I was by far not the smartest in the AP course notified me that I was a Math guy, so at first I tried Comp Sci as a minor and then switched it to Math. Only after starting my business, did I drop Math.")[32]
- 5 (of whom 1 was counted in the first bullet as well) gained confidence in their mathematical abilities as a result of their success in AP Calculus (Examples: "It made me realize my strengths." "Scoring 4 on the AP exam gave me more confidence in my math ability and my potential to strive in the field.")
- 6 (of whom 1 was counted in first bullet as well) spoke of the positive influence that teachers had in encouraging them toward STEM careers (Examples: "Many of my teachers described their career paths and the usefulness of the material in many different careers, so I became interested in it." "Teacher made AP Calc very interesting.")
- 2 indicated that when they became a STEM major in college, having had AP calculus helped them
- 4 wrote comments that suggested that their response of "encouraged" was inaccurate (Example: "Calculus 1 is somewhat interesting, however not enough to determine my future life.")
- 2 wrote comments that were unresponsive to the question

Of the 28 students in Groups A and B who said that taking those courses didn't make much of a difference in their career intentions in high school (along with 1 student who said that it discouraged them) and provided comments:

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[30] Since some comments fit into several categories, the breakdown of the comments does not necessarily add up to the total number of comments submitted for a question.

[31] As can be seen from the examples of students' comments provided in this summary, it is not always clear how a student's comment should be interpreted. We interpreted the comments on this bullet as saying that the student was already intending to major in a STEM discipline but that, as a result of taking AP Calculus, decided to focus in a particular STEM area.

[32] Of all the comments received, this one was the closest to saying that "I decided to major in a STEM area as a result of AP Calculus."

- 13 were already intending to be STEM majors (Examples: "I always knew I would be majoring in biology and other sciences." "I already knew that I wanted to study math with or without the courses." "I knew I wanted to do premed.")
- 5 were already intending not to be STEM majors (Examples: "I already had planned to major in Finance." "Don't have any interest in it anyway.")
- 7 gave no indication whether they were intending to be STEM majors (Examples: "I had always known what I wanted to do so none of the courses I took made any impact on my decision to pursue my chosen career." "Math did not have a large influence on my choices.")
- 1 spoke of the positive influence that teachers had in encouraging students toward STEM careers
- 1 noted that STEM careers were not discussed
- 1 wrote comments that were unresponsive to the question

Of the 24 students in Group C who said that they were "encouraged" or "strongly encouraged" toward STEM careers by AP Calculus and provided comments,

- 13 were clearly on the STEM track already (Examples: "I was already taking AP Computer Science so I knew I wanted to major in Computer Science." "I liked the math courses I was taking and I decided I wanted to major in something either math or science related."
- 4 gave no indication of whether they were on the STEM track already (Examples: "They introduced me to higher math concepts and I realized I really like figuring out math problems. Led me to later deciding to be a math major." "AP Calculus created an interest in me in math and I enjoyed that.")
- 3 gained confidence in their mathematical abilities as a result of their success in AP Calculus (Example: "Success in AP calculus allowed me to see that I could have success in a math/science related college major and career.")
- 1 (also counted in the second bullet) spoke of the negative influence that teachers had in encouraging them toward STEM careers
- 1 (also counted in the second bullet) spoke of the positive influence that teachers had in encouraging them toward STEM careers
- 1 noted that STEM careers were not discussed
- 1 indicated that when they became a STEM major in college, having had AP calculus helped them
- 2 wrote comments that were unresponsive to the question

Of the 30 students in Group C who said that taking those courses didn't make much of a difference in their career intentions in high school and provided comments:

- 5 were already intending to be STEM majors (Examples: "I was not a big fan of math. I only took it because it was expected of me and also I wanted to become a doctor at the time." "I have always wanted to go into the science field and taking AP courses was already part of the plan.")
- 13 were already intending not to be STEM majors (Examples: "I decided to pursue a career in business before I entered college." "I loved math but didn't want to major in it." "I was never really interested in pursuing a career in math, science, or engineering so taking the course did not change my opinion.")
- 9 gave no indication whether they were intending to be STEM majors (Examples: "I had no idea what I wanted to do, I just took AP courses because I could." "I took all advanced

classes, so I was encouraged to pursue various fields." "I took the course just to take it; it didn't matter.")

- 1 spoke of the negative influence that teachers had in encouraging them toward STEM careers
- 1 gained confidence in their mathematical abilities as a result of their success in AP Calculus
- 1 wrote comments that were unresponsive to the question

Of the 10 students in Group C who said that they were "discouraged" or "strongly discouraged" toward STEM careers by AP Calculus and provided comments,

- 8 were discouraged by the difficulty of the material (Examples: "I was really good at algebra and geometry, calculus was a lot harder for me and discouraging." "I was considering engineering but I had trouble with calculus so figured I shouldn't do engineering.")
- 2 spoke of the negative influence that teachers had in encouraging them toward STEM careers (Example: "My teacher for Algebra 2 and Calc was a joke.")
- 1 wrote comments that were unresponsive to the question

We repeat that the acronym "STEM" was not used in the survey, which spoke only of "careers in mathematics, science, and engineering," and that we are only using "STEM" in this discussion of the students' responses because of its brevity.

Although not every student submitted a comment, the patterns that we see in the comments are reasonably clear:

- Students in Group A and Group B who indicated that they were "greatly encouraged" or "somewhat encouraged" shared that they loved math, that they were going into a math-related major, and that taking the AP course further strengthened their interest in math and STEM majors. Some of the students claimed that the AP course helped them solidify their math-intensive choices by helping them discover their personal strengths in mathematics. The students who said that the course did not make a difference mostly shared that they already knew what their major would be and, consequently, taking the AP Calculus course did not encourage or discourage them from their choices.
- In Group C slightly more than one-third of the students said that they were encouraged towards a major in math, science, or engineering by taking the AP course. In this subgroup of students, some students mentioned that that they liked math and challenging courses, others stated that the course made them enjoy math even more and some shared that the AP experience helped them choose a STEM major. From the subgroup that said the course made no difference**,** some mentioned that they didn't like math but took AP Calculus either because it was expected of them or because they were able to. This is an interesting finding as some of the students felt that they were "expected" to take the course even though they did not have a strong interest in pursuing mathematics as a subject. Another subgroup of students said that they knew what their majors were going to be (whether math related or not) and therefore taking AP Calculus did not make a difference.
- For a great number of students in this survey (one third approximately in groups A and B and approximately half in Group C) taking the AP calculus course made no difference towards their decisions to follow a STEM career path.

- Although many students indicated that the AP calculus course reinforced their intentions to become STEM majors, as the summary above shows, there were essentially no students who indicated that they planned to become STEM majors because of the AP Calculus course.
- These conclusions are significantly contrary to the widely held notion that taking AP calculus course serves as a means to expand the STEM pipeline.

Several students in each group mentioned that teachers influenced their major or career decisions positively or negatively. This is an important observation as for certain students the teachers greatly affected their motivation to continue as a STEM major or to turn away from STEM careers. It is fortunate that some students have teachers for AP Calculus who inspire them toward mathematics, and it is unfortunate that some districts assign unqualified teachers to teach AP Calculus courses.

It is also interesting to note that some students mentioned that STEM careers were not really discussed in the AP course. Perhaps this is a simpler drawback that could be fixed if more teachers were trained and encouraged to discuss STEM majors in the AP Calculus course. Although the number of students that mentioned lack of career discussions in their AP courses is small, we think it still reflects an underlying belief that AP Calculus teachers and students do not necessarily view the course as a stepping-stone towards a STEM major.

How many of the 194 students in our study began college with the intention to major in "math, science, or engineering"? We can reasonably assume that all those who said that they were encouraged toward STEM careers (see Table 4) can be counted in that group – 25 in Group A, 36 in Group B, and 36 in Group C, for a total of 97. However, many of those who said that the AP Calculus course did not encourage them in that direction gave as their reason that they already planned STEM majors. There were 13 such students in Groups A and B and 6 such students in Group C, among those who wrote comments. If we assume that the same proportion holds among all students as among those who wrote comments, these numbers would rise to 16 and 9, so that about 25 students altogether would say that AP Calculus didn't make a difference as they were already planning to pursue a STEM track. So the total number of students intending to be STEM majors is at least 97+25, or 122. There were also about 22 students (after adjusting proportionally) who said that AP Calculus didn't make a difference, but didn't make clear why that was the case, so it is possible that at least some of them, perhaps even half of them, were intending to be STEM majors. So we can reasonably conclude that between 122 and 133 of the 194 students in our sample, approximately 2/3 of them, were intending to be STEM majors when they entered college. We will see in Section 5 how many of these students actually majored in a STEM field.

We also asked students to describe the extent to which taking AP calculus and its prerequisites in high school affected their interest in math as a subject. Their responses to this question and the explanatory comments that they shared were similar to their responses and comments on the question of whether taking AP Calculus encouraged them toward STEM careers. We therefore summarize them only briefly here. In Groups A and B a large number of students (83.8% and 77.0%, respectively) said that taking math in high school had a positive impact on their interest in math but fewer than half of the students in Group C (48.5%) said the same. If all three groups are combined, 70 of the 194 students (36.1%) indicated that taking the AP calculus course had no effect or negative effect on their interest in math. It is encouraging that a majority of students (63.9%) reported that the AP calculus in high school increased their interest in math. However, as we discuss the following set of survey questions, it becomes apparent that many of the students who stated that they liked math beyond the high school years did not necessarily continue in math or choose a STEM major. Moreover, for Group C more than half (51.5%) of the students claimed that the course made no impact or a negative impact on their interest in math. This provides a partial answer to a question that David Bressoud asked in [1] about the impact of AP Calculus on students' perception of mathematics[33]. It is apparent that the perceptions that students had regarding mathematics were not really affected by their AP calculus experience.

In their comments, those students who said the course made no difference explained that they either never really liked math or that AP calculus was not really a requirement for their choice of major. In the subgroup of Group C students who were discouraged to some extent after high school, students either mentioned bad teachers or the difficulty levels of the subject as factors that lowered their math interest. As with their responses to the previous question, the students' responses here often implied that a good teacher was more influential in increasing their interest in math than the course material itself.

Section 4: The math courses taken and completed by students at the University

In the next set of questions we asked students to tell us what math course they took during their first semester at Rutgers. Each group was asked to respond in a separate question, since each group had different possible responses, but we summarize all three groups together here.

Group A: On the basis of their scores on the Advanced Placement BC exam, the students in Group A were all eligible to take Calculus 3 in their first semester at Rutgers. The courses that these students actually took can be summarized as follows:

- Of the 37 students in Group A, 22 took Calculus 3 in their first semester at Rutgers.
- An additional 7 students took Linear Algebra, which can be considered an appropriate follow-up to Calculus 2.
- Thus 29 of 37 (78.4%) in Group A continued with their acceleration during their first semester at Rutgers.
- Of the remaining 8 students, 2 took Calculus 2 in their first semester at Rutgers and 6 took no math course in their first semester at Rutgers.

Students in group A were given four options – "took Calc 3," "took Calc 2," "took no math," and "does not apply"[34] and were asked to explain their choice if they did not take Calc 3. Both of the students who took Calculus 2 in their first semester said essentially that although they got credit for two semesters of calculus, they felt unprepared for Calculus 3. The 6 students who took no math in their first semester gave various explanations for why they took no math – 2 said that they wanted to explore other subjects, 1 said that they didn't need it, 1 said that they

[33] The question that Bressoud posed is: "What are the most successful strategies for these students (students taking AP Calculus) and how do these choices (taking AP Calculus) affect their perception of mathematics and ability to complete the mathematical training needed for their chosen careers?" This is a very involved question and this study did not attempt to outline the successful strategies for the students that take AP Calculus.

[34] Students in Group A should have been offered a fifth response "took Linear Algebra," but this was not done since we were unaware that taking Linear Algebra before Calc 3 was a common practice. Of the 7 students who took Linear Algebra, 3 checked "took Calc 3" and explained that they really took Linear Algebra, and 4 checked "does not apply" and explained that they took Linear Algebra in the first semester at Rutgers.

could take it later, 1 said that they took Calculus 3 in the second year, and 1 just said "I took other courses".

Group B: On the basis of their scores on the AP exam[35], the students in Group B were all eligible to take Calculus 2 in their first semester at Rutgers. The courses that these students actually took can be summarized as follows:

- Of the 58 students in Group B, 45 took Calculus 2 in their first semester at Rutgers
- 1 student took Calculus 1 again
- 1 took another math course
- 8 took no math in their first semester
- 3 chose "Does not apply".
- Thus 45 out of 58 or 77.6% of students in Group B reportedly continued with their acceleration during their first semester at Rutgers.

Students were given the five options indicated in the bullets above and were encouraged to explain their choices if they did not take Calculus 2. The student who took Calculus 1 again explained that his high school did not offer Calculus 2 and, as a result, he had no math in his senior year and felt unprepared for Calculus 2. The student who took another math course said that he wanted to explore other fields that applied mathematical reasoning. Of the eight students who took no Math in first semester, 2 said they didn't need Calculus for their major, 2 said they took courses relevant to their majors in their first semester, 1 said they were not assigned a math course, 1 decided to take Calculus 2 in the second semester, and 1 didn't like the time at which the class met. One of the students who did take Calculus 2 in the first semester commented that the AP course was a big mistake.[36]

Interestingly, 77.6% of the students in Group B continued with their acceleration in math by taking the course that follows the one for which they received credit, and this is very close to the 78.4% of Group A students that continued with their acceleration (according to the responses that students shared in the survey). However, very different percentages surface when we actually consider whether the students passed these courses in their first semester. This information was not obtained from the student questionnaire – the survey did not ask students whether they passed their first math course – but from the information provided by the Rutgers Office of Institutional Research. From the Rutgers data, we learn that 44 students actually took Calculus 2 in the first semester (one didn't remember having started with Calculus 1); of those students, 8 failed the course and 1 withdrew from it. So the actual number who continued their acceleration in their first semester at Rutgers was 35 out of 58, or 60.3%.

When we do a similar analysis for Group A, we find that of the 32 students who took a math course in their first semester, 20 students took Calculus 3 (including 5 who took the honors version of the course), 9 students took Linear Algebra, 2 students took Calculus 2, and 1 student

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[35] Most of these students took the AB exam, but some may have taken the BC exam, scored less than 4 on the exam as a whole, but scored 4 or 5 on the AB portion of the BC exam, which constitutes 60% of that exam.

[36] This student, "L", liked math but wanted to "strongly" avoid taking math in college. L's parents, friends and counselors encouraged L to take the AP course but it did not make a difference towards L's plans to be a premed major. L did not enjoy challenging math courses and never wanted to major in a math related subject. L found AP calculus and Rutgers math courses to be "Too difficult". L only took Calculus 2 at Rutgers (C+) and majored in psychology with plans to attend graduate school. L recommends the AP course to other students as it can help them take fewer math courses at college level.

took Calculus 1. All 32 students passed these courses in their first attempt with a grade of C or better. Of the 9 students that took Linear Algebra in their first semester all but one student subsequently took 251, and we shall not count that exceptional student as "staying accelerated"[37]. In total we have 28 (20 + 8) students out of the total 37 or 75.7% of Group A students who continued with their acceleration in math during the first semester at Rutgers. As such, Group A was much stronger than Group B in terms of successfully completing math courses during the first semester and maintaining their advanced placement.

What percentage of the students in Groups A and B continued their acceleration throughout their first year at Rutgers – that is, successfully passed the next two courses in the Calculus sequence? Answer: 22 of the 37 students in Group A and 23 of the 58 students in Group B. That is, only 45 of the 95 (47.4%) students who received advanced placement credit actually continued their acceleration during the first year at Rutgers. This agrees with the findings in the previous study [4] that only about half of the students who receive advanced placement in mathematics continue with their acceleration through their first year in college.[38] [39]

Group C: On the basis of their scores on the AP exams, the 99 students in Group C were not eligible for advanced placement, and would therefore be expected to repeat Calculus 1 in their first semester at Rutgers. The courses that these students actually took can be summarized as follows:

- 60 students said they took Calculus 1 in their first semester.
- 14 said that they took Pre-Calculus in their first semester.
- 9 said they took another math course.
- 6 said they took no math in their first semester.
- 10 responded "does not apply" to this question.[40]

As this group did not earn Advanced Placement credit for Calculus, it is not meaningful to discuss whether they continued with their acceleration in their first semester at college – their acceleration certainly stopped. However, it is interesting to see which courses they actually took and how they fared in these courses. From the Rutgers data, we find that of the 99 students in Group C, 88 students took a math course in their first semester at Rutgers[41]:

[37] We are defining "staying accelerated" for Group A to only include students that subsequently took Calculus 3

[38] In the previous paper [4], we noted that this information addressed one of questions raised by David Bressoud in [1]. Over 52% of the students who received college credit for calculus studied in high school didn't go beyond the next math class. (In the 2003 cohort, it was almost 60%.)

[39] A much higher percentage of engineering students who received advancement continued with their acceleration through their first year (95 out of 134, or 70.9%), which is not surprising because taking math courses in the first year was required in order to continue in the engineering school.

[40] Given the wording of this question – "If you did not receive AP credit for Calc 1, what math course did you take during your first semester at Rutgers? " – those correctly choosing the answer "does not apply" would have received credit for Calculus 1 although they did not receive a 4 or 5 on the AP Calculus exam. Some of them perhaps took Calculus 1 during the summer before their first year at Rutgers, and received credit in this way, although it is clear that some who responded "does not apply" did not fully understand the question.

[41] The other 11 students all took at least one math course at Rutgers, 7 beginning with a calculus course and 4 taking a math for liberal arts course.

- 22 students took the calculus course at Rutgers intended for majors in mathematics, physical sciences, and engineering; all but four passed the course with a C or better (2 Ds and 2 Fs),
- 43 students took the calculus course at Rutgers intended for majors in the biological sciences, economics, and liberal arts; all but one passed the course with a C or better (1 D),
- 9 students took the pre-calculus course at Rutgers; all but one passed the course with a C or better (1 F),
- 10 students took lower level courses; all but two passed with a C or better (1 D and 1 F),
- 3 took Calculus 2 (and 1 failed the course), and
- 1 took a math for liberal arts course.

It is quite impressive that of the 65 students in Group C who took a Calculus 1 course in their first semester at Rutgers, all but 5 (92.3%) passed the course with a grade of C or better, since the percentage of all students who pass Calculus 1 with a grade of C or better is likely not more than 60%. So there may be some truth to the assertion that taking the AP Calculus course *and* exam helps students succeed in calculus in college; as noted earlier, the 2005 California study [2] shows that that benefit does not result from taking the AP Calculus course without taking the exam. But it may be that taking any calculus course in high school would have the same benefit, as long as the student had to take a comprehensive final exam in the subject.

It should also be noted that 22 of the 65 students took the calculus course intended for those majoring in math, physical sciences, and engineering, and that 43 took the calculus course which is designed for students who intend to take only one semester (or perhaps two semesters) of calculus in order to fulfill the requirement of their major. Thus the course taken in their first semester at Rutgers by 2/3 of these 65 students was a much lower level course than the AP Calculus course they took in high school, since AP Calculus is intended for students who are planning to continue in mathematics.

On the other hand, it is quite impressive that 19 of the 99 students in Group C were unable to succeed in gaining admission to Calculus 1 in Rutgers' placement test, and were placed into Precalculus or even Intermediate Algebra. That is, almost 20% of the students in Group C, who had taken a year of AP Calculus in high school, were considered by Rutgers as being unprepared to take Calculus 1. Since 60% of the students who took the AP exam were in Group C, we can conclude that almost 12% of all those who took the AP exam were unprepared for calculus at Rutgers even after taking a full-year AP Calculus course in high school.[42] One may surmise that they were not prepared for AP Calculus in the first place, and that should have been detected by their high schools; they should have been strongly discouraged from taking AP

[42] Bressoud asked how many students that take AP calculus continue but encounter "insurmountable difficulties" as they transition to college classes. Although it is not clear how we can concretely define "insurmountable difficulties", we can answer this question in part by noting that 30% of the students have to take a prerequisite course to calculus at Rutgers. Since most were successful, it is apparent that their difficulties were not "insurmountable." One student, "S", however, failed Algebra 2 three times. S really liked math and wanted to learn higher-level math and be better prepared for college courses. S had to take some math course in the senior year, so S took AP calculus because it looks good on college applications. S claimed to have been disadvantaged by taking the AP course because S's "basics were not strong enough."

Calculus and should have instead taken courses that better prepared them for a calculus course in college.

We turn now from considering the math courses that the students in our survey took initially to considering what math courses they took over their undergraduate years. With the data provided to us by the Office of Institutional Research at Rutgers, we were able to count the number of math courses that students took and successfully completed (that is, with a grade of C or better) at Rutgers over the four-year period. We had the students' grades for all semesters except the spring 2011 semester[43]. Table 5 has a summary of math courses completed by the students in all three groups. The percentages are written first followed by the actual number of students in parentheses.

Number of math courses completed (with a C or better)	Group A	Group B	Group C	Group C (# of courses Calc 1 or higher)
0	2.7 (1)	5.2 (3)	1.0 (1)	15.2 (15)[44]
1	16.2 (6)	31.0 (18)	55.6 (55)	56.6 (56)
2	13.5 (5)	15.5 (9)	21.2 (21)	9.1 (9)
3	5.4 (2)	8.6 (5)	4.0 (4)	4.0 (4)
4	5.4 (2)	6.9 (4)	6.1 (6)	5.1 (5)
5	5.4 (2)	10.3 (6)	2.0 (2)	2.0 (2)
6 or 7	10.8 (4)	8.6 (5)	4.0 (4)	4.0 (4)
8 or 9	18.9 (7)	10.3 (6)	3.0 (3)	3.0 (3)
10 or more	21.6 (8)	3.4 (2)	1.0 (1)	1.0 (1)
Total	37	58	99	99

Table 5: How many math courses did students take at Rutgers?

The following observations are noteworthy from the table above:

- In Group A, 23 out of 37, or 62.2%, of the students successfully completed 4 or more math courses at Rutgers, including at least one course beyond the calculus/linear algebra/differential equations sequence, and may have majored in a math-intensive discipline.[45]
- In Group B, 23 out of 58 or 39.7% of the students successfully completed 4 or more math courses at Rutgers (compared to 62.2% in Group A). This reflects that perhaps fewer than

[43] As we did not have the grades for spring 2011, we counted the courses they took in spring 2011 as "successfully completed". This is a fair assumption as the students who did not do well in their math courses would not have signed up for a math course as they were approaching the end of their college years.

[44] Of the 15 students who completed no calculus course, 7 completed pre-calculus but not calculus, 7 completed a "math for liberal arts" course, and 1 completed no math course at all (but failed Algebra 2 three times!).

[45] Since there are five courses in the calculus/linear algebra/differential equations sequence and they received AP credit for two courses, if a student took 4 math courses at Rutgers, at least one had to be beyond that sequence.

half of these students may have majored in a math-intensive discipline that requires completion of the courses through differential equations.

- In Group C, only 17 out of 99 or 17.2% of the students successfully completed 4 or more math courses at Rutgers, and only 8 completed 6 or more courses (including at least one course beyond differential equations), reflecting that very few of the students in this group chose a math-intensive major. Only 28 out of 99 or 28.3% of the students in Group C completed two or more courses at the level of Calculus 1 or higher.
  - Thus for the other 71 of the 99 students (71.7%) in Group C, those who completed Calculus 1 and no subsequent math course, the only math they took in college was a repeat of material that they took in high school.
  - Moreover, only 5 of these 71 students completed the calculus course intended for students in math, engineering, or the physical sciences; the courses taken at Rutgers by the other 66 of the 71 students were at a lower level than the courses they had taken in high school.
    - 15 of the 66 students only completed a precalculus course
    - The other 51 completed the calculus course for biology, economics, psychology, and liberal arts majors, a course that is at a much lower level than the AP Calculus course.[46]

This information about students in Group C is consistent with the previously discussed attributes of Group C, whose students didn't really like math or wanted to get done with math as quickly as possible.

The data of what math courses the students took each semester also enable us to determine what percentage of students who received AP credit continued their four-year high school acceleration during their first year at Rutgers – by taking at least one course during each semester during their first year.

- 14 out of the 37 students in Group A (or 37.8%) did not continue their acceleration through both semesters of their first year at Rutgers – that is, they did not successfully complete at least one math course each semester.[47] [48]
- 33 out of the 58 students in Group B (or 56.9%) did not continue their acceleration through both semesters of their first year at Rutgers – that is, they did not successfully complete at least one math course each semester. [49]
- Altogether 47 out of the 95 students in Groups A and B, the students who received AP credit for Calculus 1 (and possibly Calculus 2) did not continue their acceleration. This is very close to the estimate in the fourth conjecture that was investigated in the previous

[46] Of these 51 students, 9 were placed into a pre-calculus course and completed their calculus courses only after completing a pre-calculus course.

[47] Of these 14 students, 4 took no math courses, 9 took a math course only in the fall semester, and 1 took a math course only in the spring semester.

[48] Though the 14 students discussed here and the 23 students discussed on the previous page total 37, the two sets are not quite complementary; there were two students who completed four or more math courses, but did not continue their acceleration during their first year in college.

[49] Of these 33 students, 13 successfully completed no math course, 13 successfully completed a math course only in the fall semester, and 7 successfully completed a math course only in the spring semester.

study – that only half of the students who receive advancement placement as a result of their scores on the AP exams continue their acceleration through their first year at Rutgers.

Section 5: The majors and intended careers of the students

We asked students "What subject (or subjects) are you majoring in at Rutgers?" and provided a space in which they could write their response. We also gave them the yes/no question, "Are you planning a career in math, science, engineering, or technology?" and followed that up with "Please indicate in the box below the career (or careers) you are planning to pursue." The purpose of these questions was to determine the percentage of the students in the survey who had certain majors and certain career plans, and to link these responses to their responses to other questions in the survey.

Table 6 shows the number of students in each of Groups A, B, and C who majored in the indicated disciplines. Note that these are not the disciplines in which they intended to major; these are the majors which these fourth-year students were actually completing. It should be noted that many students listed more than one major; in such situations, if one of a student's majors was a math, science, or engineering discipline, the student is listed under that major. A student who had two majors in these areas, A and B, is listed under "A and B." If a student's second major was in a different area that was indicated if that combination of majors occurred relatively often.

Major	Number of students in Group A	Number of students in Group B	Number of students in Group C	Total
Math and …	**16**	**11**	**7**	**34**
Math	4	3	5	
Math and Physics	3	1		
Math and Computer Science	3	1		
Math and Statistics		1		
Math and Economics	4	3	2	
Math and other non-STEM	1			
Biomathematics	1	2		
Statistics	**2**	**1**		**3**
Physical Sciences	**8**	**11**	**4**	**23**
Physics	3	3		
Chemistry	3	4	1	
Chemistry and MBB			1	
Engineering	1	2	2	
Engineering and Physics	1	1		
Biomedical Engineering		1		
Subtotal Math-Intensive Majors	**26**	**23**	**11**	**60**
Computer Science	**1**	**5**	**6**	**12**
Biological Sciences	**6**	**16**	**18**	**40**
Genetics	3	2	1	
Biological Sciences	1	4	8	

CBNeuroscience and Physics	1			
Animal Science Ecology	1			
Cell Biology & Neuroscience		6	7	
Molecular Bio & Biochemistry		4	2	
Subtotal Math, Sci, Eng Majors	**33**	**44**	**35**	**112**
Other Majors	**4**	**14**	**63**	**81**
Accounting	1		1	
Finance and/or Economics	1	3	11	
Business			6	
Psychology	2	4	12	
Health-related		2	9	
Other		5	24	
Unreadable Response			**1**	**1**
Total	**37**	**58**	**99**	**194**

Table 6: What were the majors of the students in Groups A, B, and C?

The data in Table 6 are summarized in the first two rows of Table 7, which also provides percentages for each of the indicated categories. The final row in Table 7 comes from Table 1 which summarized the students' responses to the question of why they took the AP Calculus course in high school.

	Group A (37 students)	Group B (58 students)	Group C (99 students)	Total (194 students)
Majored in math, science, engineering	33 (86.4%)	44 (75.9%)	35 (35.4%)	112 (57.7%)
Majored in math-intensive subject	26 (70.3%)	23 (39.7%)	11 (11.1%)	60 (31.4%)
Planned to major in a math-related subject	25 (67.6%)	26 (45.6%)	25 (25.2%)	76 (39.0%)

Table 7: What were the majors of the students in Groups A, B, and C (continued)?

Group A: 33 of the 37 students in Group A, or 86.4%, were majors in math, science, or engineering, and 26 out of 37 or 70.3% were majors in math-intensive subjects (including the physical sciences). These are impressive outcomes. These results reflect the positive math attitude and encouraging high school AP experience that this group had shared in the preceding questions. It is appropriate to generalize at this point and say that overall, Group A students were strong math students who continued to do well in mathematics and science courses. However, as we noted in Section 3, we cannot conclude that Group A students majored in these subjects because of their experiences with the AP course. In fact, these students were already geared towards math-intensive majors because they enjoyed the challenge and wanted to learn higher level mathematics. Surely, for some of these students, taking the AP course increased or reinforced their interest in mathematics; however it is plausible that these students were going to major in math or science even in the absence of their AP Calculus experience.

Group B: 44 of the 58 students in Group B, or 75.9%, were majors in math, science, or engineering (compared to 86.4% in Group A). This is rather impressive proportion of Group B students in math, science, or engineering. However, whereas among the Group A students 27 out of 37 or 73.0% majored in math-intensive subjects (including the physical sciences), only 25 out of 58 or 43.1% of the Group B students did the same.

So, it is plausible that although a high percentage of Group B students intended to major in math, sciences, and engineering, many were inclined more towards the biological sciences which required fewer math courses than the math-intensive majors like math, physics, statistics, and engineering. This explanation also is consistent with Table 5 that indicated that only 39.7% of Group B students completed 4 or more math courses at Rutgers.

Group C: 35 of the 99 students in Group C, or 35.4%, were majors in math, science, or engineering (compared to 75.9% in Group B and 86.4% in Group A). The number of students in Group C who had math-intensive majors is 11 or 11.1% (compared to 39.7% in Group B and 70.3% in Group A).

As with Group B, the small number of math-intensive majors in Group C is consistent with Table 5 that indicated that only 15 (15.4%) of Group C students completed 4 or more Calculus 1 or higher math courses at Rutgers. Moreover, the percentage of Group C students who majored in math, science, or engineering was sharply less than the corresponding percentage of Group B students. This is also consistent with Table 5 that indicated that only 28.3% of Group C students had successfully completed two semesters of calculus.

Although only 8 of the 194 students majored in engineering, an area in which the United States has a critical shortage,[50] the survey did not include students in the College of Engineering. *[Note to editor: We anticipate getting data from Rutgers OIR in the next few weeks that will tell us how many of the students in the College of Engineering took AP Calculus and their math courses and grades at Rutgers.]* However, the data from this study shows that only a small number of students who did not intend to become engineers moved in that direction, so increasing the number of students taking AP Calculus does not seem to be having an effect on the "T" and "E" components of the STEM pipeline.

The first row of Table 7 indicates that 112 of the 194 students in all three groups majored in math, science, or engineering. At the end of Section 3, it was estimated that the number of students who intended to major in math, science, or engineering (based on Table 4) was between 122 and 133. Thus a substantial percentage, between 84.2% and 91.8%, of those who intended to major in math, science, or engineering actually did.[51]

The third row of Table 7 contains information that is presented in Table 1. These are the students who "agreed" or "strongly agreed" with the statement that they took the AP calculus course because they planned to major in a math-related subject. Interestingly, the number of students in Groups A and B who actually majored in math-intensive majors corresponds

[50] "The president and industry groups have called on colleges to graduate 10,000 more engineers a year and 100,000 new teachers with STEM majors," cited in "Why Science Majors Change Their Minds (It's Just So Darn Hard)" by Christopher Drew, New York Times, November 4, 2011.

[51] Among the 216 engineering students who took the AP exam, all but 51 ultimately completed a course in differential equations or advanced calculus for engineering with a grade of C or better, thereby completing the mathematics requirements of their programs. So it is quite possible that 76.4% of the engineering students who took the AP Calculus exam graduated (or will graduate) as engineers.

surprisingly well to the number who said that they took the AP Calculus course because they planned to major in math-related subjects.[52] However, among the Group C students, there is a sharp decline from the number of students who intended to major in a math-related subject to the number who actually did, presumably because a substantial percentage of the students in Group C were dissuaded from math by their experience in the AP Calculus course so that by the time they took the exam they no longer believed that they would be successful in mathematics.

The comparison in the paragraphs above is complicated by the fact that the terminology that we have been using in the survey and this report is neither well defined nor consistent. Thus, we have used the following five descriptions of majors and careers:

- "Math-related major" (in survey question about reasons for taking AP calculus course)
- "Math-intensive" (defined here to be major in math, statistics, or physical sciences)
- "Math, science, and engineering" (in survey question about influence of taking AP calculus)
- "Math, science, engineering, or technology" (in survey question about intended careers)
- "STEM" (the currently fashionable term)

When we place them next to one another, we can see that the second is broader than the first, the third is broader than the second, and that the fourth is broader than the third. From a linguistic perspective, it would seem that the fifth is equivalent to the fourth, since it is an acronym for the fields listed in the fourth. However, the acronym STEM has a decidedly different character so that, for example, careers in medicine are often not listed as STEM careers although they clearly fall under "science." Moreover, some careers related to finance may involve a great deal of mathematical modeling and so might be considered "math-intensive" even though they don't fall under STEM. Moreover, we cannot assume that the students answering the question "Are you planning a career in math, science, engineering, or technology?" have the same understanding of that question that we do. We present the data and analysis that appear above and below keeping in mind the imprecision of these terms.

When asked "Are you planning a career in math, science, engineering, or technology?", the students in Groups A, B, and C responded as follows:

	Group A (37 students)	Group B (58 students)	Group C (99 students)	Total (194 students)
Yes	31 (83.8%)	47 (81.0%)	46 (46.5%)	124 (63.9%)
No	6 (16.2%)	11 (19.0%)	53 (53.5%)	70 (36.1%)

Table 8: What percentage of students in Groups A, B, and C were planning careers in math science, engineering, or technology?

Although the percentage of students in Groups A and B who responded "Yes" corresponds roughly to the percentage of students in those groups who majored in math, science, or engineering, 10% more students in Group C responded "Yes" than the number who majored

[52] It is of course possible that their reporting on why they took the AP Calculus course may have been influenced by the fact that they did have math-intensive majors, but then the same influence should have been observed in Group C. It would of course have been better if we had conducted the first part of the survey when the students were first-year students, but unfortunately we were unable to transport ourselves back to that time.

in these areas, perhaps because they understood the term “math, science, engineering, or technology” more broadly.

Table 9 provides information about the careers of the students in Groups A, B, and C.

Career	**Number of students in Group A**	**Number of students in Group B**	**Number of students in Group C**	**Total**
Doctorate, perhaps leading to professorship in …	**11**	**5**	**3**	**19**
Math or Applied Math	5	1	1	
Physics	3	1		
Chemistry or Chemical Engineering	1	1	1	
Biomathematics	1			
Biology		2		
Computer Science				
Economics or Psychology	1		1	
Medical Doctorates	**6**	**16**	**13**	**35**
Medical Doctor		12	10	
Dentist		4	2	
Veterinarian			1	
Scientist	**8**	**10**	**7**	**25**
Scientist	5	3	2	
Engineer		5	3	
Mathematician	1		1	
Computing	2	2	1	
High School Teacher (Math or Science)	**2**	**5**	**3**	**10**
Math	1	3	3	
Physics	1			
Chemistry		1		
Biology		1		
Health	**1**	**5**	**14**	**19**
Public Health			3	
Nursing		1	1	
Med Tech			1	
Psychologist	1	1	3	
Exercise and sports, physical therapy		2	3	
Other		1	3	
Business	**8**	**9**	**18**	**35**
Finance	4	2	8	
Accounting or Actuary	3	3	1	
Business	1	1	6	
Business (Technology)		3	3	
Education (non-Math/Science)			**6**	**6**
Other	**1**	**1**	**13**	**15**

Undecided**		**1**	**7**	**8**
Did not answer		**6**	**15**	**22**
Total	**37**	**58**	**99**	**194**

Table 9: What careers are the students in Groups A, B, and C intending to pursue?

Group A: Although 31 of the 37 students in Group A (83.8%) indicated that they intended to pursue careers in math, science, or engineering (where business, psychology, linguistics, and economics are excluded), only 26 listed careers in those categories. There is clearly a discrepancy between what the students regard as "math, science, engineering, or technology" and what is commonly understood as falling under these categories.[53] The number of students in Group A who intend to pursue math-intensive careers is 19 out of 37 (51.4%).

Group B: Although 47 of the 58 students (81%) in Group B indicated that they intended to pursue careers in math, science, or engineering, only 41 out of the 52 (78.8%) listed careers in these areas, including 21 in the health professions, which are often not considered in those categories. Of the 41 students, 19 (36.5%) intend to pursue math-intensive careers.

Group C: 25 out of the 83 (30.1%) students in Group C who responded intend to pursue careers in math, science, or engineering. Of these, 12 (14.5%) intend to pursue math-intensive careers.

We summarize these results in Table 10:

	Group A (37 students)	Group B (58 students)	Group C (99 students)	Total (194 students)
Intends career in math, science, engineering	31 (83.8%)	47 (81.0%)	46 (46.5%)	124 (63.9%)
Lists career in math, science, engineering	26 (70.3%)	41/52 (78.8%)	25/83 (30.1%)	92/174 (52.9%)
Lists career in math-intensive subject	20 (54.1%)	20/52 (38.5%)	12/83 (14.5%)	52/174 (29.9%)

Table 10: Summary of intended careers of all students in study

The discrepancy between the percentage of Group C students who indicated that they were pursuing careers in math, science, or engineering (first row of Table 10) and the percentage of Group C students who listed careers in math, science, or engineering (second row of Table 10) can be explained by observing that the health-related careers that many students indicated they were pursuing are usually not considered careers in math, science, or engineering (or STEM careers) since the science requirements for those careers are often very modest.

Section 6: How did taking AP Calculus in high school benefit students? Would they recommend it?

In this question we asked students to explain how they benefited, or they didn't benefit, from taking AP Calculus in high school. The pilot group of students did not get to respond to

[53] The student who wrote "linguistics" said "I figure linguistics is like a science anyway." The student who plans to get a Ph.D. in economics also wrote "career in economic research." These answered "Yes" to the initial question as did several students who intended to pursue careers in finance or actuary.

this question. All but one student from Groups A and B responded very positively to having taken the AP course at high school. These students said the course helped them get ahead in college courses and finish requirements for their specific majors sooner. Some mentioned that the AP course helped them get ready for math courses at Rutgers. The one person who said "no benefit" shared that he had to repeat the course at Rutgers even though he had learned it well in high school.

In Group C, 17 of the 89 students (19.1%) who answered this question said that they did not benefit from taking AP calculus in high school. Some of these students shared that they did not learn much through the course or that the course was not taught well at their high school; others mentioned not being interested in the course or feeling unprepared for the college courses after the AP course. Among the Group C students who said they benefitted from the AP course the most popular reason (53 or 59.6%) was that the course helped them get ready for college and in specific helped them do well in Calculus 1. Other students gave the following reasons: they enjoyed the challenge as it generated interest in math for them, they were happy to receive college credits and needed to take fewer courses, and it helped them on Rutgers' placement exam

Next, we asked students if they would recommend that current high school students take the AP Calculus course and exam. Everyone in Group A and B, and all but 4 students in Group C said yes and several comments were posted. Again, the pilot group did not get a chance to respond to this question. In essence, the students said that the course saved them time by giving them a jump start at college courses and preparing them for higher level math, science or related courses. Some students mentioned that taking Calculus at high school was easier than taking it at a university and others mentioned saving money or "getting it out of the way" as advantageous. Some students in Group C indicated that only students that like math or are good at math should be encouraged to take the AP course.

To revisit the questions raised by Bressoud, he had asked in [1] how many students in retrospect felt that they have made good use of their time by taking AP Calculus. The answer is that most of the students think it was beneficial for them to have taken the AP course but their reasons vary. There were clearly some students in Group C that saw the course as a means to avoid mathematics in college or to reduce the number of math courses they had to take in college, and there were many Group C students who used the AP course as preparation for a terminal calculus course that was at a lower level than the AP course. Furthermore, as this study illustrates, the students' perception of "benefitting" from the AP Calculus course did not always translate to their future success in mathematics or a pursuit of STEM careers.

Conclusions

This report highlights several trends and reasons for why students take AP Calculus course and exam in high school and what they do with this advanced placement later on. We summarize some of the salient findings below.

- Approximately half of the students in this survey who took the AP exam did not receive any college credit for calculus and only half of the students who received advanced placement as a result of their scores on the AP exams continued the acceleration through their first year at Rutgers. This corroborates the results from the earlier study, "A Rush to Calculus" [4], and confirms that only a handful of the students who are accelerated throughout high school are able to reach and maintain their advanced placement through the first year of college.

- Group A and Group B students largely had “intrinsic rationales” for taking AP Calculus as they enjoyed mathematics and wanted to learn higher-level mathematics; whereas, students in Group C routinely took the course for the “college applications” reasons and were significantly more motivated to take AP calculus in order to avoid college level mathematics (30% in Group C versus only 5% in Groups A and B).
- While a high percentage (67.6%) of Group A students planned to major in a math related subject in college, fewer than half (45.6%) in Group B and barely a quarter (25.2%) in Group C planned to major in a math related subject. This reflects that half of the Group B students and three-quarters of the Group C students were not seeking to be “advanced” in mathematics and yet took the AP Calculus course. It is plausible that if these students had options to take another math course at high school that better suited their level of mathematical understanding and their major and career plans, they would have (and should have) taken it instead of the AP Calculus course. In fact, 70.6% of the students in Group C agreed or strongly agreed that they took AP calculus because they *had* to take some math in their senior year.
- It appears that, as a result of taking the AP Calculus course, up to 60% of the Group C students who intended to start college with a higher-level math course had lowered their expectations so that by the end of the school year, when the AP calculus exam was given, they no longer expected that they would earn advanced placement.
- About 2/3 of the students in both Groups A and B responded that taking the AP Calculus course encouraged them toward careers in math, science, or engineering, and about 1/3 indicated that taking these courses made no difference. These fractions are reversed for Group C; that is, only 1/3 said that taking the AP Calculus course encouraged them toward careers in math, science, or engineering, and 2/3 indicated that taking these courses made no difference or discouraged them from such careers.
- For a great number of students in this survey (one third approximately in groups A and B and approximately half in Group C) taking the AP Calculus course made no difference towards their decisions to follow a STEM career path.
- Although many students indicated that taking the AP Calculus course reinforced their intentions to become STEM majors, there were essentially no students who indicated that they planned to become STEM majors because of the AP Calculus course. This finding is significantly contrary to the notion implicit in [3] that taking AP calculus course serves as a means to expand the STEM pipeline.
- In Group C almost 20% of the students who had taken a year of AP Calculus, were considered by Rutgers as being unprepared to take Calculus 1. Overall, 12% of all students (as Group C made up 60% of this cohort) who took the AP exam were unprepared for calculus at Rutgers even after taking a full-year AP Calculus course in high school. These students were not prepared for AP Calculus in the first place, and should have been steered away from AP Calculus.
- In Group A, 86.4% were majors in math, science, or engineering and 70.3% were majors in math-intensive subjects (including the physical sciences); in Group B, 75.9% were majors in math, science, or engineering and 43.1% majored in math-intensive subjects. A

high percentage of Group B students were inclined towards the biological sciences that required fewer math courses. In contrast, among students in Group C, only 35.4% were majors in math, science, or engineering and only 11.1% had math-intensive majors.

- Strikingly, only 10 of the 194 students intend to be high school teachers of math (7) or science (3), an area in which the United States has a critical shortage,[54] compared to the 35 students who intend to be medical doctors. Certainly increasing the number of students taking AP Calculus is not having the desired effect of increasing the number of students intending to teach in high school.
- Among the Group C students, there was a sharp decline from the number of students who intended to major in a math-related subject to the number who actually did, presumably because a large percentage of these students were put off from math by their experience in the AP Calculus course.
- In Group A 19 out of 37 (51.4%) students that shared comments intend to pursue math-intensive careers whereas in Group B 19 out of 41 students (36.5%) and in Group C only 12 out of 83 (14.5%) intend to do the same. This again ties in with the low motivation of students in Groups B and C to pursue careers that need and require the AP Calculus course. At the end of this long haul of rushing through Calculus, very few engineers, scientists and mathematicians are being added to the nation's STEM pipelines.
- Interestingly, most of the students responded that they benefitted from taking AP Calculus in high school and that they would recommend it for future students. However, some of the students cited AP calculus as a medium that allowed them to take fewer math courses at college and avoid bad teaching experiences at the college-level.

Recommendations

The two most important findings of this study are that, first, as with the previous study, a very small percentage of those who are accelerated throughout high school maintain that acceleration through their first year at college and, second, that there is no evidence that encouraging more students to take AP Calculus will expand the STEM pipeline.[55]

Yet the curriculum in many schools is organized to accelerate students into AP Calculus – beginning with the policy of expecting more (or even, all) students to take Algebra 1 in the 8th grade – often taught by middle school teachers who lack the appropriate mathematical background or credentials.

Moreover, more and more students are encouraged to take AP Calculus, including those students who struggled to complete Algebra 2, Geometry, and Precalculus in the 9th, 10th, and 11th grades.

The driving force behind the practice, as seen from the students' responses and the recommendations of their teachers and counselors, is that colleges routinely favor students for admission if they have AP Calculus on their transcripts.

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[54] According to an article by Christopher Drew in the New York Times of 11/4/11 entitled "Why Science Majors Change Their Minds": "The president and industry groups have called on colleges to graduate 10,000 more engineers a year and 100,000 new teachers with majors in STEM — science, technology, engineering and math."

[55] An important exception must be noted. There are many schools in the nation (perhaps even in New Jersey) in which AP Calculus is not offered at all, and we may lose from the STEM pipeline those students from such schools who would belong to Groups A and B.

The basic recommendation is “STOP these practices.”

Many detailed recommendations follow. Some are similar to the recommendations in the previous study, but are modified by what we have learned from the student surveys. But none of these will be implemented until colleges change their admissions policies and practices – that is, they should not “routinely” favor students with AP Calculus.

What we have learned is that students, teachers, counselors, and principals all believe that taking AP Calculus enhances the possibility of admission to college. As a result, many students take AP Calculus who do not benefit from that course. Ideally, colleges should announce that an application for admission will only be enhanced by taking AP courses if the students take the exams and score 4 or 5 on them. Unfortunately, that is impossible since the students are typically admitted to college well before the AP exams are given. However, we encourage colleges to adopt a policy of the following type in evaluating the AP credentials of applicants for admission.

Recommendation to Colleges on Admission Policy:

With each student’s application to admission to college, the high school must submit a document indicating, for each of the previous five years, the number of students who took an AP Calculus course, the number of students who took the AP Calculus exam, and the grades of those students on the AP Calculus exam. Each college should modify their admissions policy so that it gives extra weight for taking the AP Calculus course only to those students whose high schools can report that they treat the course seriously, not just as a means for enhancing college admissions. For example, a college might decide to give extra weight if 90% of the students taking the AP Calculus course in that high school took the AP Calculus exam, and that 75% of the students who took the exam received a grade of 4 or 5.

Recommendations on curriculum and entry to AP Calculus:

- Students should take Algebra 1 in 8th grade only if they have teachers who have the appropriate background and certification in teaching mathematics.
- Students should be discouraged from taking any course if they did not achieve at least a B (and ideally a B+) in the previous course.[56]
- Students should be encouraged to take an AP Calculus course only if they
    - received an A or B+ in the Precalculus course,
    - indicate that they like mathematics and wish to be challenged,
    - intend to take more advanced math courses in their first year in college, and
    - are strongly considering a career that requires a substantial number of college mathematics courses – i.e., mathematics, statistics, and the physical sciences.[57]

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[56] If parents insist on their child’s taking the next course, as they may be able to do in many locations, then the student should be required to take an accompanying course, for as long as necessary, to remediate his or her deficiencies.

[57] One has to question on an individual level why many Group C students took the AP course when they already knew they were not going to be a STEM major down the road. Also, one has to ponder at a policy level if it is it worthwhile to spend resources teaching calculus to students similar to those in Group C.

Otherwise, AP Calculus is not the ideal course for them.

- School counselors should receive training that will enable them to identify students who should take the AP Calculus course and to convincingly discourage students who should not take AP Calculus. This includes
  - students who are not prepared for AP Calculus – recall that almost 12% of the students who had completed a full-year AP Calculus course were placed at Rutgers into Intermediate Algebra or Precalculus
  - students who are intending to major in a subject that has limited mathematical prerequisites (including biology) – recall that only 76 of the 194 students agreed or strongly agreed that they took the AP Calculus course as "I planned to major in a math related subject".
  - students who will require extra help or private tutoring to keep up with the material in the course.
- Students and parents should be made aware that the course is challenging, intended for math-intensive majors and not necessarily going to lead to an advanced placement in mathematics or save them time in college.
- All students who take the AP Calculus course should be required to take the AP exam.

- AP Calculus should not be the "default" mathematics option for seniors, that is, the course that they take automatically after they complete Precalculus.[58] Since all college-bound students should take a math course in their senior year of high school, schools should design and offer other attractive and interesting courses that seniors can take after completing Precalculus. These courses should include material that reinforces algebra and geometry skills, should include probability, statistics, and discrete mathematics, and should build on the Precalculus course (including some calculus topics) so that students completing these courses will be prepared for college math courses.

In this study we found no evidence that increasing the number of students taking AP Calculus expands the STEM pipeline. If that will not expand the STEM pipeline, what will? The problem is not that there are too few students in the STEM pipeline, but that the pipeline is too leaky, that is, students who were previously attracted to math (and similarly for science) have decided not to pursue careers in those areas. The national focus should not be on recruitment, but on retention of students already in the STEM pipeline; this applies at all grade levels, but perhaps particularly at the college level.[59] [60] [61]

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[58] It appears that treating AP courses as "default" options is not unique to mathematics. Indeed, it seems that many students take several AP courses in their senior year because their high schools do not offer anything else. Although it is conceivable that it is appropriate for a student to take, simultaneously, AP courses in chemistry, mathematics, computer science, and English, it is rather unlikely that the student feels equally passionate about all four. It would make sense for high schools to require students to choose one subject (or at most two subjects) in which they take the AP courses.

[59] According to an article by Christopher Drew in the New York Times of 11/4/11 entitled "Why Science Majors Change Their Minds": "Studies have found that roughly 40 percent of students planning engineering and science majors end up switching to other subjects or failing to get any degree. That increases to as much as 60 percent when

But even at the high school level, there are important strategies for encouraging students to consider STEM careers:

- Implement appropriate and early interventions to cultivate and enhance younger students' interest in mathematics, science and engineering. For example, students can be exposed to field trips related to STEM careers or be given an opportunity to talk to real scientists, mathematicians and engineers to discuss the challenges and rewards of such careers.
- Interestingly, in all three groups a number of students claimed that the teachers were far more influential than the course itself in terms of motivating them to pursue or not pursue a STEM major. Teachers of science and mathematics, and particularly the teachers of AP Calculus, should be trained to discuss career options and real life applications of the ideas that students learn in the course.
- Ensure that students have positive experiences in their math classes; that students get the assistance that they need to succeed in mathematics; and, that they are not pushed ahead at a pace that will cause them to avoid mathematics in the future.

Future studies

We did not get a chance to speak to the students in our survey at length about their individual experiences, as the sample size was rather large. Perhaps an interview with some individuals could have helped us further understand the reasons why students take the AP Calculus course and exam and why they choose to pursue or not pursue STEM majors. Also, it might be a worthwhile future study to interview high school students that are currently enrolled in AP Calculus course and then follow them through their four-year college experience to trace the factors that most significantly contribute towards their motivation and successful completion of STEM majors.

To conclude, there is a pressing need to find other creative and effective initiatives to help recruit future engineers, mathematicians and scientists rather than merely increase the number of transcripts that have AP Calculus on them.

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pre-medical students, who typically have the strongest SAT scores and high school science preparation, are included, according to new data from the University of California at Los Angeles."

[60] On the other hand, the current economic situation may have had a major impact on the number of students who are pursuing STEM careers. David Bressoud reports, based on data from the UCLA's Higher Education Research Institute, that: "There has been a strong upward trend toward mathematics, the sciences, and engineering over the past decade. It has recently accelerated. In the past five years, the number of students intending to major in Mathematics has risen by 31%, in the Physical Sciences by 37%, in Engineering by 44%, and in the Biological Sciences by 67%. Most of this growth has occurred in just the past three years, since 2007. This is most dramatic within Engineering, which went from 102,000 freshmen intending to major in this discipline in the Fall of 2007 to 156,000 in Fall 2010."

[61] It should also be noted that, as reported in the analysis of Table 7, a substantial percentage of the students who participated in this survey, between 84.2% and 91.8%, of those who intended to major in math, science, or engineering actually did. This observation should not be taken to imply that taking AP Calculus results in staying in the STEM pipeline, but rather that those who intend to stay in the STEM pipeline may be more likely to take AP Calculus.

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