

DIMACS Center  
Rutgers University

**DIMACS-DIMATIA-Rényi Collaboration on Discrete Mathematics and Theoretical  
Computer Science**

**Annual Report**

April 2004

## **Ia. Participants from the program**

### **Participants who spent 160 hours or more**

PI: Fred Roberts, DIMACS  
Gyula O. H. Katona, Rényi Institute  
Jaroslav Nešetřil, DIMATIA

### **Working Group I Algebraic and Geometric Methods in Combinatorics**

#### **Organizers:**

Jaroslav Nešetřil, DIMATIA  
Jan Kratochvíl, DIMATIA

#### **Graduate Students:**

Manuel Bodirsky, DIMATIA, Berlin  
Stephen Hartke, DIMACS-Rutgers  
Jan Kára, DIMATIA, Charles University, Prague  
Tamas Kiraly, Rényi Institute  
Daniel Král, DIMATIA, Charles University, Prague  
Martin Marés, DIMATIA, Charles University, Prague  
Jana Maxová, DIMATIA, Charles University, Prague  
Attila Por, Rényi Institute  
Robert Šámal, DIMATIA, Charles University, Prague  
Nick Weininger, DIMACS-Rutgers

#### **Undergraduate Students:**

Zdeněk Dvořák, DIMATIA, Charles University, Prague  
Dan Krasner, UC Berkeley and DIMACS  
Ida Svejdevrora, DIMATIA, Charles University, Prague

### **Working Group II Extremal Combinatorics**

#### **Organizers:**

Gyula O. H. Katona, Rényi Institute  
Ervin Györi, Rényi Institute  
Dezső Miklós, Rényi Institute

#### **Graduate Students:**

Bill Cuckler, DIMACS-Rutgers  
Paul Ellis, DIMACS-Rutgers  
Stephen Hartke, DIMACS-Rutgers

## **Working Group III Graph Colorings and their Generalizations**

### **Organizers:**

Fred Roberts, DIMACS-Rutgers  
Stephen Hartke, DIMACS-Rutgers

### **Graduate Students:**

Daniel Král, DIMATIA  
Trevor Bass, DIMACS-Rutgers  
Bill Cuckler, DIMACS-Rutgers  
Jin Jinji, Kyung Hee University  
Klay Kruczek, DIMACS-Rutgers  
John Villalpando, Clemson University and Gonzaga University  
Nick Weininger, DIMACS-Rutgers  
Teresa Xiaohua Jin, University of South Carolina

### **Ib. Participating Organizations**

Telcordia Technologies: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning.

AT&T Labs - Research: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

NEC Laboratories America: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning.

Lucent Technologies, Bell Labs: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

Princeton University: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning.

Avaya Labs: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning.

HP Labs: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning.

IBM Research: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning.

Microsoft Research: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

DIMATIA: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

Rényi Institute: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

### **1c. Other Collaborators**

The project involved scientists from numerous institutions in numerous countries. The resulting collaborations also involved individuals from many institutions in many countries.

## **II. Project Activities**

### Overview

DIMACS, the Center for Discrete Mathematics and Theoretical Computer Science, headquartered at Rutgers University, DIMATIA, the Center for Discrete Mathematics, Theoretical Informatics, and Applications, at Charles University in Prague, Czech Republic, and the Alfred Rényi Institute of Mathematics of the Hungarian Academy of Sciences in Budapest, Hungary are engaged in three-way international research collaboration. These three distinguished research centers form three of the most important centers in the international community of researchers in discrete mathematics and theoretical computer science. It is thus natural that they have combined their research strengths with the creation of multinational “working groups” in research areas where the three centers have major strength and where focused collaboration has the likelihood of leading to major scientific advances and increasing the involvement of outstanding junior researchers in international collaborations.

The working groups are devoted to Extremal Combinatorics, Graph Colorings and their generalizations, and Algebraic and Geometric Methods in Combinatorics. Three meetings were held during the past year, one for each of the working groups. Each included a workshop and a smaller, more informal meeting.

### **Working Group on Algebraic and Geometric Methods in Combinatorics**

The kick-off meeting of the project was the meeting of the working group on Algebraic and Geometric Methods in Combinatorics. It was held December 2-6, 2002, in Nova Louka, Czech Republic. This was an informal program providing lots of opportunities for discussion and the development of new collaborations. Some of the formal presentations included the following:

- Jaroslav Nešetřil (DIMATIA) – On constructions of graph duals
- Jiří Matoušek (DIMATIA) - Geometric lower bounds for graph invariants
- David Galvin (DIMACS-Microsoft) - Counting H-colourings of a regular, bipartite graph
- Jan Kratochvíl (DIMATIA) – Locally constrained homomorphism

Graduate students and postdoctoral fellows also gave talks, such as:

- Manuel Bodirsky (Berlin, DIMATIA) - Surjective Homomorphism Problems
- Tamas Kiraly (Rényi) - Some new results on the edge-connectivity of (hyper)graphs

### **Working Group on Extremal Combinatorics**

This group organized a very successful meeting in Budapest, April 10-16, 2003. The participants included junior researchers from Rutgers, Prague (and Budapest of course), as well as senior combinatorialists from the U.S., Canada, Japan, Denmark, Great Britain, Italy, and the Czech Republic. The group used this opportunity to celebrate the 60<sup>th</sup> birthday of Béla Bollobás, a world leader in extremal combinatorics.

Talks included the following:

- Richard Anstee (UBC) - Small Forbidden Configurations
- Zsolt Katona (SZTAKI) - Multiply intersecting families of sets
- Zsolt Tuza (SZTAKI) - Coloring mixed hypergraphs
- Gyula O. H. Katona (Rényi Institute) - Béla's families of subsets
- Miklós Simonovits (Rényi Institute) - How many graphs?
- Zoltán Füredi (Rényi Institute) - Turán type problems
- Ervin Gyóri (Rényi Institute) - Extremal problems related to cycles
- Norihide Tokushige (University of the Ryukyus) - The maximum size of 3-wise 2-intersecting Sperner families
- Ákos Kisvölcsy (Rényi Institute) - Flat Antichains
- Péter L. Erdős (Rényi Institute) - Splitting property and property B
- Vince Grolmusz (Eötvös Loránd University) - From Ramsey-graphs to fast matrix multiplication
- Gábor Tardos (Rényi Institute) - Forbidden submatrices
- Gyula Y. Katona (Budapest University of Technology and Economics) - m-path cover saturated graphs
- Géza Tóth (MIT) - How many string graphs are there?

- Tamás Fleiner (Eötvös Loránd University) - On a lemma of Scarf
- Annalisa De Bonis (Rényi Institute) - Optimal two-stage algorithms for group testing problems
- Gábor Wiener (Budapest University of Technology and Economics) - Approximately separating systems
- Stephen Hartke (DIMACS-Rutgers) - The Voter model with confidence levels
- Krisztián Tichler (Rényi Institute) - Extremal theorems for databases

### **Working Group on Graph Colorings and their Generalizations**

The first meeting of this working group was held October 13-15, 2003 at DIMACS Center, Rutgers University, Piscataway, New Jersey. It was organized by Fred S. Roberts, DIMACS, and DIMACS graduate student Stephen Hartke, Rutgers University.

Among the highlights of the talks, Gábor Simonyi, in his talk on local chromatic number and Sperner capacity, described an interesting and new technique relating local chromatic number to the traditional Sperner capacity, which was very difficult to compute using previous technology. Daniel Král, a graduate student at Charles University, in his talk Coloring Powers of Chordal Graphs and  $L(2,1)$ -labeling, gave the proof of a general result that led to the  $L(2, 1)$  labeling of powers of chordal graphs.

One of the main themes of this meeting was  $L(2,1)$  labeling, including presenting the main conjectures in this area. It was addressed in the following talks:

- Peter Fishburn (AT&T Labs) -  $L(2,1)$ -colored trees of max degree 3 and Span 5
- Denise Sakai Troxell (Babson College)  $L(2,1)$  - labeling of products of two cycles
- Renu Laskar (Clemson University) - Irreducible no-hole colorability of some classes of graphs
- John Villalpando (Clemson University and Gonzaga University) - Irreducibility of  $L(2,1)$ -colorings and the inh-colorability of unicyclic and hex graphs
- Daphne Liu (California State University) - Multi-level distance labeling

Closely related were the talks of Jerrold R. Griggs, University of South Carolina, Real number channel assignments with distance conditions and Teresa Xiaohua Jin, University of South Carolina, Real number graph labeling for the triangular lattice and the square lattice.

Fred Roberts, DIMACS, related generalizations of graph colorings to biology in his talk Consensus list colorings of graphs and physical mapping of DNA.

Additional talks were given by:

- Zsolt Tuza (Computer and Automation Institute, Hungarian Academy of Science)
  - Choosability problems for  $(d,s)$ -colorings
- Alexander Soifer (DIMACS-Rutgers) - Coloring unit distance graph of the plane

- Vadim Lozin (DIMACS-Rutgers) - Between 2- and 3-colorability
- Nathan Segerlind (Institute for Advanced Study) - Using hypergraph homomorphisms to guess three secrets
- Vladimir Gurvich (DIMACS-Rutgers) - Kernel-solvable graphs, core-solvable cooperative games, and list coloring conjecture
- Jan Kratochvíl (DIMATIA) - Partial covers and distance constrained labellings of graphs
- Fanica Gavril (DIMACS-Rutgers) - Minimum covering by cliques of perfect filament graph
- Ervin Györi (Rényi Institute) - Adjacent vertex distinguishing edge-colorings

Jiří Fiala, Charles University, gave the final formal talk of the session, On locally constrained graph homomorphisms, presenting many open problems. The meeting concluded with a comprehensive open problem session. Some of the open problems presented are discussed in another section of the report.

### III. Project Findings

#### New collaborations and research directions

From the list of papers included in this report, it is clear that many fruitful collaborations developed among the participants from the three institutions, DIMACS, DIMATIA, and Rényi. A description of a selection of these is included.

Jiří Matoušek, DIMATIA, and Imre Bárány, Rényi Institute, developed a fractional Helly theorem for convex lattice sets and also developed an application to art galleries, where you must decide where to place guards to protect the collection. The Helly theorem states that given any finite set of convex sets in Euclidean  $d$ -space, if any  $d+1$  or fewer subsets has a nonempty intersection then the set has a nonempty intersection. We call  $d+1$  the Helly number of the set of  $d$ -dimensional convex sets. For convex sets in the space of integer lattice points in Euclidean  $d$ -space, the Helly number is much larger, namely  $2^d$ . Matoušek and Bárány showed that there is a sense in which this number can be considered an anomaly and that for more global Helly-type properties, the Helly number is still  $d+1$ . Technically, their result is a fractional Helly theorem and settles a conjecture made by Alon, Kalai, Matoušek, and Meshulam. The Helly number of convex lattice sets has applications in integer programming, geometry of numbers, crystallographic lattices, computational complexity in lattices, and indivisibilities in economy.

Pavel Valtr, DIMATIA, and Gyula Károlyi, Eötvös Loránd University, collaborated on point configurations in  $d$ -space without large subsets in convex position. They found a lower bound for the Erdős–Szekeres number in higher dimensions. Namely, in two different ways they constructed, for every  $d$  at least 2 but less than  $n$ , a configuration of  $n$  points in general position in  $d$ -dimensional Euclidean space containing at most  $c_d(\log n)^{d-1}$  points in convex position. (Points in  $d$ -dimensional Euclidean space are in convex position if none of them lies in the convex hull of the others.)

Pavel Valtr, DIMATIA, and Imre Bárány, Rényi Institute, solved a problem on planar point sets with a small number of empty convex polygons. Combining results and methods of

combinatorial geometry, probability theory, and number theory, they constructed a set of  $n$  points in general position in the plane with only  $\approx 1.62n^2$  empty triangles,  $\approx 1.94 n^2$  empty quadrilaterals,  $\approx 1.02 n^2$  empty pentagons, and  $\approx 0.2 n^2$  empty hexagons. This improves previous bounds given in several papers.

Problems posed during meetings of working groups were solved by collaborations formed during the meetings. Jiří Fiala, DIMATIA and Jan Kratochvíl, DIMATIA, solved the problem of the computational complexity of  $L(2,1)$ -labeling regular graphs posed by Renu Laskar, Clemson University, during the colorings meeting at DIMACS in October. An  $L(2,1)$ -labeling of a graph assigns each vertex of a graph a number from 1 to  $k$  such that any two adjacent vertices have numbers that are at least 2 apart and any two vertices connected by a path of two edges have numbers at least 1 apart. This labeling has applications in the assigning of radio frequencies to transmitters to avoid interference. A regular graph has vertices whose degrees are all equal.

The graduate student Daniel Král obtained the asymptotics for the question posed during this same meeting of  $L(2,1)$ -colorings of powers of chordal graphs and has submitted a paper for publication. A graph is chordal if it contains no induced cycle of length four or more. The chordal graphs have been well-studied because of their desirable algorithmic characteristics. Many problems that are intractable in the general case are solvable by fast algorithms in the chordal case.

Jiří Fiala and Jana Maxová, DIMATIA, solved a question asked by Maxová during the first meeting of the algebraic and geometric methods group on the simultaneous existence of locally simple and locally constrained graph homomorphisms. Besides generating a paper, this result was also reported by Jiří Fiala during the 2nd European Symposium on Combinatorics, EUROCOMB '03, in Prague in September 2003. They showed that the simultaneous existence of a locally surjective and a locally injective graph homomorphism between a connected graph  $G$  and a connected, finite graph  $H$  implies that all such homomorphisms are in fact locally bijective.

Fred Roberts, DIMACS, and Peter Fishburn, AT&T Labs, obtained useful results on no-hole  $L(2,1)$ -colorings. An  $L(2,1)$ -coloring has no holes if every value of the color set is used. They found a new proof determining which trees have no-hole  $L(2,1)$ -colorings.

Continuing with the theme of  $L(2,1)$ -labelings of products of graphs, Christopher Schwarz, The Schwarz Group, and Denise Sakai Troxell, Babson College, collaborated on the  $L(2,1)$ -labelings of products of two cycles. The  $\lambda$  number of a graph is the minimum number of labels needed for an  $L(2,1)$ -labeling of the graph. Schwarz and Troxell showed that the  $\lambda$  number of the Cartesian product of any two cycles is 6, 7, or 8. In addition, they completely characterized the products of two cycles with  $\lambda$  number exactly equal to each one of these values.

One of the best examples of the results of the long term collaboration of DIMACS, DIMATIA, and Rényi is the following book:

J. Nešetřil and P. Winkler (editors), **Graphs, Morphisms and Statistical Physics**, *AMS DIMACS Series 63*, 2004.



Though this work is primarily the result of a prior grant, this project allowed the authors to include some additional papers and put the finishing touches on the project.

Each of the meetings of the working groups included both informal and formal discussions of new research directions and specific problems needing to be solved. Here is a selection of these open problems:

J. Kratochvil, Z. Tuza, and M. Voigt asked if every planar graph is  $(4,1, 2)$ -choosable. That is, given a planar graph  $G$ , is there a set of 4-sets, such that each vertex can be assigned a set in such a way that the intersection of the sets of two connected vertices has at most two elements. It is known that the answer is yes if the intersection is required to have at most one element and the answer is no if the intersection can have as many as 3 elements. But the case for 2 elements remains open.

Fanica Gavril, inspired by Renu Laskar's work, asked if for a graph property  $P$  and a graph  $G$ , there is a maximal matching  $M$  in  $G$  such that the subgraph induced on  $M$  has property  $P$ . For example, property  $P$  could be "being a tree."

Gyula Katona raised the following two problems at the April 2003 Extremal Combinatorics meeting in Budapest:

Let  $X$  be a set of  $n$  elements and let  $A$  be a set of  $k$ -element subsets of  $X$ . The shadow of  $A$ ,  $\sigma(A)$ , is the set of  $(k-1)$ -element subsets of elements of  $A$ . The value of  $|\sigma(A)|$  for given  $n$ ,  $k$  and  $|A|$  is well known as the shadow theorem. Are there analogous results for the deep shadow of  $A$ ,  $\delta(A)$ , defined to be the set of  $(k-1)$ -element sets that are subsets of at least two distinct elements of  $A$ ?

Now let  $A$  be a set of subsets of  $X$  of various cardinalities. Let  $p_i(A)$  be the number of  $i$ -element members of  $A$ . The vector  $(p_0, p_1, p_2, \dots, p_n)$  is called the profile vector of  $A$ . Let  $\mathbf{A}$  be a class of families satisfying a certain property. Consider the set of all profile vectors of families belonging to  $\mathbf{A}$ . The set of extreme points of the convex hull of this set of profile vectors is denoted by  $\text{EXTRA}(\mathbf{A})$ . This set of points is known for many classes (properties) such as Sperner, intersecting, intersecting Sperner, etc. Previously  $\text{EXTRA}(\mathbf{A})$  had not been determined for any property involving intersection sizes greater than 1. What is  $\text{EXTRA}(\mathbf{A})$  for the class of 2-intersecting (or 2-Sperner) families? The work of Ahlswede and Khachatryan on  $k$ -uniform,  $l$ -intersecting families perhaps provides an approach to this problem.

Fred Roberts posed several open problems on generalizations of graph colorings:

1. Let  $G = (V,E)$  be a graph and  $T$  a set of nonnegative integers. In radio frequency assignment, we seek an assignment of a positive integer  $f(x)$  to each vertex  $x$  of  $G$  so that for all  $x,y$  in  $V$  with  $\{x,y\}$  in  $E$ ,  $|f(x)-f(y)|$  is not in  $T$ . Such an assignment  $f$  is called a  $T$ -coloring.  $T$ -colorings are especially interesting for graphs arising from points in Euclidean  $p$ -space where  $\{x,y\}$  is in  $E$  if and only if the distance between  $x$  and  $y$  is at most  $M$ , the so-called  $p$ -unit sphere graphs. We need good algorithms for finding  $T$ -colorings for 2-unit sphere graphs that minimize the "span"

between the smallest and largest channel  $f(x)$  used. The same problem is of interest for 3-unit sphere graphs.

2. Let  $G = (V, E)$  be a graph. A set coloring is an assignment of a set  $S(x)$  to each  $x$  in  $V$  so that if  $\{x, y\}$  is in  $E$  then  $S(x)$  has an empty intersection with  $S(y)$ . The order of a set coloring is the size of the union of the sets  $S(x)$  used and the score is the sum of the "sizes" of the sets  $S(x)$  (where "size" = cardinality or measure, as appropriate). The problem where the sets are all intervals on the real line arises in traffic phasing, task assignment, channel assignment, and other applications. Similar problems arise if the sets are each unions of at most two real intervals. In the latter case, the problems of finding optimal order and optimal score set assignments are open. The same problems when the sets are rectangles in the plane of a certain minimum measure arise in fleet maintenance problems and are also open.

3. Let  $G = (V, E)$  be a graph and  $T$  be a set of nonnegative integers. Given a list  $R(x)$  of positive integers at each vertex  $x$  of  $G$ , a list coloring is an ordinary graph coloring  $f$  in which  $f(x)$  always belongs to  $R(x)$ . A list  $T$ -coloring is defined analogously. Very little is known about list  $T$ -colorings. One open problem is the following. We say that  $G$  is  $T$ - $k$ -choosable if there is a list  $T$ -coloring for every assignment of lists  $R(x)$  where each list has size  $k$ . The  $T$ -choice number  $T\text{-ch}(G)$  is the smallest  $k$  such that  $G$  is  $T$ - $k$ -choosable. Tesman has found bounds on the  $T$ -choice number for chordal graphs when  $T = \{0, 1, \dots, r\}$  and shown that they are tight. It remains an open question to determine if the upper bound,  $(2r+1)(\chi(G) - 1) + 1$ , is tight for 1-unit sphere graphs. Virtually nothing is known about the generalization that combines list  $T$ -colorings and set  $T$ -colorings, and this topic of set list  $T$ -colorings presents many interesting questions.

#### **IV. Project Training/Development**

The meetings offered many students the opportunity to travel to and participate in international collaborations. The following students participated:

##### **Graduate Students:**

Trevor Bass, DIMACS-Rutgers  
Manuel Bodirsky, DIMATIA, Berlin  
Bill Cuckler, DIMACS-Rutgers  
Paul Ellis, DIMACS-Rutgers  
Stephen Hartke, DIMACS-Rutgers  
Jin Jinji, Kyung Hee University  
Jan Kára, DIMATIA, Charles University, Prague  
Tamas Kiraly, Rényi Institute  
Daniel Král, DIMATIA, Charles University, Prague  
Klay Kruczek, DIMACS-Rutgers  
Martin Marés, DIMATIA, Charles University, Prague  
Jana Maxová, DIMATIA, Charles University, Prague  
Attila Por, Rényi Institute  
Robert Šámal, DIMATIA, Charles University, Prague  
John Villalpando, Clemson University and Gonzaga University

Nick Weininger, DIMACS-Rutgers  
Teresa Xiaohua Jin, University of South Carolina

### **Undergraduate Students:**

Zdeněk Dvořák, DIMATIA, Charles University, Prague  
Dan Krasner, UC Berkeley and DIMACS  
Ida Svejdevrora, DIMATIA, Charles University, Prague

Daniel Krasner was a participant in DIMACS REU in 2001. This grant and the DIMACS-DIMATIA REU grant (NSF EIA-01-38973) have played a major role in the development of this undergraduate's career. Upon his graduation from Berkeley, he obtained a visiting appointment at DIMATIA, where he is currently. He will be starting graduate school at Columbia University in the fall.

The following talks were presented by students:

- Stephen Hartke (DIMACS-Rutgers) - The Voter model with confidence levels
- Manuel Bodirsky (Berlin, DIMATIA) - Surjective Homomorphism Problems
- Tamas Kiraly (Rényi Institute) - Some new results on the edge-connectivity of (hyper)graphs
- Daniel Král (DIMATIA, Charles University) - Chordal Graphs and  $L(2,1)$ -labeling
- John Villalpando (Clemson University and Gonzaga University) – Colorability of unicyclic and hex graphs

## **V. Outreach Activities**

## **VI. Papers/Books/Internet**

Bárány, I., and Matoušek, J., “A fractional Helly theorem for convex lattice sets,” *Adv. Math.* 174 (2003) 227-235.

Bárány, I., and Matoušek, J., “Berge's theorem, fractional Helly, and art galleries,” in preparation.

Bárány, I., and Valtr, P., “Planar point sets with a small number of empty convex polygons,” to appear in *Studia Math Hung.*

Brass, P., Károlyi, Gy., Valtr, P., “A Turán-type extremal theory of convex geometric graphs,” in B. Aronov et al. (eds.), *Discrete and Computational Geometry-The Goodman-Pollack Festschrift*, Springer-Verlag, Berlin, 2003, 275-300.

Erdős, P. L., Faigle, U., Hochstätter, W., Kern, W., “Note on the Game Chromatic Index of Trees,” *Theoretical Computer Science* (Special Issue on Algorithmic Combinatorial Game Theory) to appear.

Fiala, J., and Paulusma, D., “The computational complexity of the role assignment problem,” *ICALP'03*, LNCS 2719, Springer-Verlag, Berlin 2003, 817-828.

Fiala, J., and Kratochvíl, J., “Computational complexity of  $L(2,1)$ -labeling regular graphs,” in preparation.

Fiala, J., Král, D., Škrekovski, R., “Brooks-type theorem for generalized list  $T$ -Coloring,” submitted to *SIAM Journal of Discrete Mathematics*.

Fiala, J., and Maxová, J., “Cantor-Bernstein type theorem for locally constrained graph homomorphisms,” submitted to *European Journal of Combinatorics*.

Győri E., Nikiforov V., Schelp R. H., “Nearly bipartite graphs,” *Discrete Mathematics* to appear.

Hamburger, P., and Sali, A., “Symmetric 11-Venn Diagrams with Vertex Sets,” *Studia Sci. Math. Hung.* to appear.

Károlyi, Gy., and Valtr, P., “Point configurations in  $d$ -space without large subsets in convex position,” *Discrete and Computational Geometry* 30 (2003) 277-286.

Klazar, M., “Counting even and odd partitions,” *Amer. Math. Monthly* 110 (2003) 527-532.

Klazar, M., “Non  $P$ -recursiveness of numbers of matchings or linear chord diagrams with many crossings,” *Adv. Appl. Math.* 30 (2003) 126-136.

Král, D., “Coloring powers of chordal graphs,” submitted to *SIAM J. Discr, Math.*

Král, D., and Škrekovski, R. “A theorem about the channel assignment problem,” *SIAM Journal on Discrete Mathematics* 16 (2003) 426-437.

Loebl, M., Nešetřil J., Reed, B., “A note on random homomorphisms from arbitrary graphs to  $\mathbb{Z}$ ,” *Discrete Mathematics* 273 (2003) 173-181.

Nešetřil, J., and Tardif, C., “On maximal finite antichains in the homomorphism order of directed graphs,” *Discuss. Math.* 23 (2003) 325-332.

Nešetřil J., Valtr P., Solymosi, J., “A Ramsey property of planar graphs,” in **Towards a Theory of Geometric Graphs**, Vol. 342, J. Pach (ed.), AMS (2004).

Schwarz, C., and Troxell, D., “ $L(2,1)$ -labelings of products of two cycles,” DIMACS Technical Report 2003-33 (2003).

Wiener, G., “Recognition Problems and Communication Complexity,” *Discrete Applied Mathematics* to appear.

The main web site for DIMACS-DIMATIA- Rényi Collaboration on Discrete Mathematics and Theoretical Computer Science is

<http://dimacs.rutgers.edu/Research/partnership/>

## **VII. Other Products**

Main web site for DIMACS/DIMATIA/Renyi Working Group on Algebraic and Geometric Methods in Combinatorics <http://dimacs.rutgers.edu/Workshops/Algebraic/main.html>

Main web site for DIMACS/DIMATIA/Renyi Working Group on Extremal Combinatorics <http://dimacs.rutgers.edu/Workshops/Extremal/main.html>

Main web site for DIMACS/DIMATIA/Renyi Working Group on Graph Colorings and their Generalizations <http://dimacs.rutgers.edu/Workshops/GraphColor/main.html>

## **VIII. Contributions within Discipline**

This report has already addressed the many problems that have been solved and new areas of research and collaboration that have been developed. Probably the most important contribution of this project is the precedent it has set for international collaborations among the senior and junior researchers of the United States, the Czech Republic, and Hungary. While many of these collaborations are documented in this report, we expect that the ground breaking work of this project will lead to substantially more in the future.

## **IX. Contributions -- other Disciplines**

This is an inherently interdisciplinary project. Connections between computer science, mathematics, statistics, and other disciplines were brought to light. New results about  $L(2,1)$ -labeling of graphs have applications to the assignment of frequencies to transmitters that avoid interference. Fred Roberts' work on consensus list colorings of graphs has applications to biology, specifically to the physical mapping of DNA.

## **X. Contributions -- Human Resource Development**

One of the key components of the meetings was the strong participation of students and postdoctoral fellows. The meetings included five papers that were co-authored and/or presented by students. The meeting also provided substantial travel support for many students.

Stephen Hartke, a graduate student at Rutgers, attended the first meeting, gave a talk at the second meeting, and was a co-organizer of the third meeting. Stephen has this to say about the impact of this program on his career:

“It was great talking with non-local combinatorialists, especially in such strong traditions as the Czechs and the Hungarians, seeing what they do, and networking. It was also useful to see a bunch about a particular topic, including introductory talks (i.e., graph homomorphisms) and

recent research. Even though I haven't worked seriously (yet!) on any of the open problems I heard about, some of them are very interesting, and I certainly have thought about them.”

William Cuckler, a graduate student at Rutgers, attended the Extremal Combinatorics meeting in Budapest. He found the experience to be very valuable and had this to say about it:

“The talks gave me a very broad perspective of combinatorics. Many different topics were covered, expanding on what I learned in combinatorics classes. This broad perspective was especially helpful to me because I was a second year student trying to narrow my focus on a specific area of combinatorics. The topics that interested me the most were Turan-type problems (discussed by Dr. Furedi) and forbidden submatrix problems (discussed by Dr. Tardos). I read more about both topics after the conference and discussed them with Professor Komlos and other graduate students, including Paul Ellis and Stephen Hartke. The other graduate students and I also discussed many of the open problems that were presented. This discussion was particularly valuable to Paul Ellis and me because we learned a lot from Stephen Hartke who, as a more advanced student, often had greater insight into the problems.”

## **XI. Contributions to Resources for Research and Education**

## **XII. Contributions Beyond Science and Engineering**