Modelling hospitalization, home-based care and individual withdrawal for people living with HIV/AIDS in high prevalence areas

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Outline of presentation

- Aim
- Objectives
- Introduction
- The Model
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- Numerical simulations
- Results and Discussion
Aim

Qualitatively study the dynamics of HIV/AIDS holistically from the current situation in developing countries
Objectives

To evaluate and quantify the efficacy of Community Home-Based Care (CHBC) for people living with HIV/AIDS (PLWHA)

To identify the most effective intervention strategy.

To validate the model with the actual data.
Introduction

- The number of PLWHA has increased over the last decade due to the availability of antiretrovirals (ARVs)
- Majority of people seek medical treatment at hospitals
- Shortage of staff and space at hospitals and other patient referral systems to care for HIV patients
- PLWHA - early discharge from the hospitals or not admitted at all
The model of care of PLWHA has shifted from hospital care to CHBC (Akintola, 2006).

In response, policies to promote home-based care of patients had to be implemented by hospitals, health departments and non-governmental organizations (NGOs).

What is home-based care?
Home-based care

- An approach to care provision that combines clinical services, nursing care, counselling, psycho-spiritual care and social support.

- (WHO) defines CHBC, as ‘the provision of health services by formal and informal caregivers in the home in order to promote, restore and maintain a person’s maximum level of comfort, function and health including care towards a dignified death.

- Not a replacement for hospital care- comprehensive continuation of prevention, care, treatment and support services.
Home-based care classification

- Preventive - further infections,
- Promotive - change of behaviour,
- Therapeutic - ARVS,
- Rehabilitative,
- Long-term maintenance and
- An extension of services
CHBC Objectives

To enhance the quality of life of PLWHA by

- facilitating health care from the hospitals to homes
- raising awareness of HIV/AIDS
- reducing overcrowding in health care facilities

This work is based on the first two authors' personal experiences about CHBC in Zimbabwe.
Model Formulation

- Model is a modification of a multi-strategy model by Nyabadza (2006)
- Model takes into account hospitalization of symptomatic AIDS patients and their release into CHBC
- Screened infectives seek more help from CHBC
- Force of infection $\lambda$, incorporates behaviour change, individual withdrawal from risky sexual activities and the efficacy of CHBC.
Model Formulation

- Adjustment factors to $\beta$, which are $\eta_i$, $i = 1, 2, 3$ intended to reflect the influence of pre- and post-counselling on biological and behavioural processes that pushes the risk of HIV transmission.

- The natural history of HIV infection is represented by the model diagram shown in Figure 1.
Figure 1: Model diagram showing movements of individuals between compartments.
Model Equations

\[
\begin{align*}
\dot{S} &= \Lambda - \lambda S - \mu S, \\
\dot{I} &= \lambda S - (\mu + \sigma + \gamma_1) I, \\
\dot{I}_s &= \sigma I - (\mu + \gamma_2 + \rho_1) I_s, \\
\dot{A} &= \gamma_1 I + \gamma_2 I_s - (\mu + \theta + \delta_1) A, \\
\dot{H} &= \theta A - (\mu + \rho_2 + \delta_2) H, \\
\dot{H}_b &= \rho_1 I_s + \rho_2 H - (\mu + \delta_3) H_b,
\end{align*}
\]

\[
\lambda = \beta e^{-m(\delta_1 A + \delta_2 H + \delta_3 H_b)} \left( \frac{I + (1-p)(\eta_1 I_s + \eta_2 A + \eta_3 (1-\phi) H_b)}{N} \right),
\]

\[
\Omega = \left\{ (S(t), I(t), I_s(t), A(t), H(t)), H_B(t) \in \mathbb{R}_+^6 : N \leq \frac{\Lambda}{\mu} = N^* \right\}.
\]
The exponential function that measures behavior change encompasses the use of condoms, serial monogamy, reduction in concurrent partnerships, resulting from observed mortality due to HIV/AIDS.

The response parameter $m$, measures how individuals respond to the increase or decrease of mortality due to HIV/AIDS.
Equilibrium Points

We begin our analysis by considering the case $m = 0$ and leave the case $m \neq 0$ for the numerical simulations.

The disease-free equilibrium given by,

$$E_0 = \left( \frac{\Lambda}{\mu}, 0, 0, 0, 0, 0 \right).$$

The endemic equilibrium point is given by

$$S^* = \frac{N^*}{R_e}.$$
Equilibrium Points

\[ I^* = \frac{\mu}{R_e(\mu + \gamma_1 + \sigma_1)}(R_e - 1) \]
\[ I_s^* = \frac{\sigma}{\rho_1 + \gamma_2 + \mu} I^*, \]
\[ A^* = \frac{(\gamma_1(\rho_1 + \gamma_2 + \mu) + \gamma_2\sigma_1)}{(\rho_1 + \gamma_2 + \mu)(\theta + \mu + \delta_1)} I^* \]
\[ H^* = \frac{\theta(\gamma_1(\rho_1 + \gamma_2 + \mu) + \gamma_2\sigma)}{(\rho_1 + \gamma_2 + \mu)(\theta + \mu + \delta_1)(\rho_2 + \mu + \delta_2)} I^* \]
\[ H_B^* = \left[ \frac{\rho_1\sigma}{(\rho_1 + \gamma_2 + \mu)(\delta_3 + \mu)} \right. \]
\[ + \frac{\rho_2\theta(\gamma_1(\rho_1 + \gamma_2 + \mu) + \gamma_2\sigma)}{(\rho_1 + \gamma_2 + \mu)(\mu + \delta_3)(\theta + \mu + \delta_1)(\rho_2 + \mu + \delta_2)} \left. \right] I^* \]
\[ \lambda^* = \frac{(\mu + \gamma_1 + \sigma_1)}{N^*_R} R_e I_1^*. \]
The effective reproduction number, $R_e$

\[
R_e = R_I + \frac{(1 - p)\sigma}{\sigma + \gamma_1 + \mu} R_{I_s} + (1 - p) \left[ \frac{\gamma_1}{\gamma_1 + \sigma + \mu} + \frac{\sigma \gamma_2}{(\sigma + \gamma_1 + \mu)(\gamma_2 + \rho_1 + \mu)} \right] R_A \\
+ (1 - p) \left[ \left( \frac{\sigma \rho_1}{(\sigma + \gamma_1 + \mu)(\rho_1 + \gamma_2 + \mu)} \right) \frac{\gamma_1 \theta \rho_2}{(\gamma_1 + \sigma + \mu)(\theta + \delta_1 + \mu)(\rho_2 + \delta_2 + \mu)} + \frac{\sigma \gamma_2 \theta \rho_2}{(\sigma + \gamma_1 + \mu)(\gamma_2 + \rho_1 + \mu)(\theta + \delta_1 + \mu)(\rho_2 + \delta_2 + \mu)} \right] R_{Hb},
\]
Contributions of infectious classes

where

\[ R_I = \frac{\beta c}{\gamma_1 + \sigma + \mu}, \quad R_{I_s} = \frac{\eta_1 \beta c}{\gamma_2 + \rho_1 + \mu} \]

\[ R_A = \frac{\eta_2 \beta c}{\theta + \delta_1 + \mu}, \quad R_{H_b} = \frac{\eta_3 \beta c(1 - \phi)}{(\delta_3 + \mu)} \]
Contributions of infectious classes
Analysis of $R_e$

- No intervention - $R_0$
- Screening and counselling - $R_{es}$: – This is a situation which is common in Zimbabwe especially among the poor.
- Hospitalization, Screening and counselling - $R_{esh}$
- Home-based care, Screening and counselling - $R_{eshb}$
- All the strategies - $R_e$. 
Numerical simulations of $Re$

Figure 2: shows the relationship between the reproduction numbers as $\beta$ changes.
Effect of parameters on $R_e$

(a) $0.992$ $0.992$ $0.992$ $0.996$ $0.996$ $0.996$

(b) $1$ $1$ $1$ $1.004$ $1.004$ $1.004$

$\rho_2$

$P$ and $\rho_2$ change

$\phi$ and $\rho_2$ change.

$p > 45\%$ and $\phi > 44\%$ contain the epidemic.

Figure 3: shows the effect of the parameters on the reproduction number as (a) $P$ and $\rho_2$ change (b) $\phi$ and $\rho_2$ change. $p > 45\%$ and $\phi > 44\%$ contain the epidemic.
Effect of withdrawal, $p$

Differentiating partially $R_e$ with respect to $p$ we obtain,

$$\frac{\partial R_e}{\partial p} = \frac{-\beta}{\mu + \gamma_1 + \sigma} \left\{ \frac{\eta_1 \sigma}{\gamma_2 + \rho_1 + \mu} + \frac{\eta_2}{\theta + \delta_1 + \mu} \left( \frac{\gamma_1 + \sigma \gamma_2}{\gamma_2 + \rho_1 + \mu} \right) + \frac{\eta_3 (1 - \phi)}{\mu + \delta_3} \left( \frac{\sigma \rho_1}{\rho_1 + \gamma_2 + \mu} + \ldots \right) \right\} < 0, \forall p.$$

- $R_e$ is a decreasing function of $p$. 
Effect of CHBC, $\phi$

Differentiating $R_e$ partially with respect to $\phi$ we obtain,

$$\frac{\partial R_e}{\partial \phi} = \frac{-\beta}{\mu + \gamma_1 + \sigma} \left[ \frac{\eta_3}{(\mu + \delta_3)} \left( \frac{\sigma \rho_1}{(\rho_1 + \gamma_2 + \mu)} \right. \right.$$

$$\left. + \frac{\gamma_1 \theta \rho_2}{(\theta + \delta_1 + \mu)(\rho_2 + \delta_2 + \mu)} \right] < 0, \forall \phi.$$

- $R_e$ is a decreasing function of $\phi$. 
Stability analysis of the equilibria

**Theorem 1** If \( R_e > 1 \), the system (1) has a unique endemic equilibrium given \( E_1 \) in \( \Omega \).

**Theorem 2** If \( R_e < 1 \), the disease-free equilibrium is globally asymptotically stable and unstable if \( R_e > 1 \).

A Lyapunov function of the form

\[
L = aI + bI_s + cA + dH + eH_b
\]  

(2)

where \( a, b, c, d, e \) are constant to be determined, was used.
Stability analysis of the equilibria

**Theorem 3** If $R_e > 1$, the endemic equilibrium is *locally asymptotically stable* and unstable if $R_e < 1$.

Employed the centre manifold theory as described in the works of Castillo-Chavez and Song, 2004 (Theorem 4.1).
Numerical simulations: Parameters

Integrate (1) by RK-4 in Matlab to determine the effects of various parameters on the dynamics of the disease rates vary.

First cases of HIV were recorded in 1985 in Zimbabwe (Zungu-Dirwayi et al, 2004) but take the initial time for our simulations - 1990 - data available from 1990.

Zimbabwe’s population \( \approx 10.156 \) million in 1990, with a life expectancy of 59 years (US Census Bureau,).
In 2007, the population ≈ 13.349 million and 51.8% of the population was aged 15-49 (≈ 6.915m) (UNAIDS/WHO).

In 1990, crude estimate of the initial population of adults aged 15 - 49, ≈ 5.26 m, ≈ 52% of the adult population.

One condition: prevalence of infection in 1990 must approximate the data in UNAID/WHO report.
Life expectancy dropped to 43 years in 2006 (UNAID/WHO) and average life expectancy of 50 years over the past decade,

17 years the average age of first sexual intercourse globally (American Sexual Behavior)

⇒ consider $\mu = 0.029$, corresponding to an average of 33 years of sexual activity (see also Blower et al., 2002 for Botswana).
$\Lambda$- recruitment rate of $S$ is chosen and calculated such that it is related to the growth rate of the country’s population varying from 0.6% between 2005 and 2009 (UNAIDS), 2.28% in 1990, 2% between 2002 and 2005 (Zimbabwe), -0.8% in 2007 (USB).

Here we consider an average growth rate of 1% so that $\Lambda = 52600$.

No treatment - the average time from infection to death due to the disease $\approx 10.5$ years (Hallet et al, 2009) → to between 18 and 20 years in the presence of treatment (Hallet et al, 2009)
Numerical simulations: Parameters

- Progression rates are generally difficult to measure as they vary between individuals.
- Give ranges between which these progression rates vary
- Infection and progression rates between compartments are estimated from published data
- The uptake rates into treatment are from 2% in 2004 to 18% in 2007, representing a mean rate of 0.053 per year (UNAIDS2). For Botswana, a value of 0.05 per year was chosen in similar way.
Numerical simulations: Parameters

- Treatment is offered to individuals in the AIDS classes on national ART programs, we take $\rho_2 = 0.053$ and estimate a much lower value for $\rho_1$.

- HIV-related bed occupancy vary from 5%-80% in many African countries (Guinness et al, 2000).

- An average value of 12 years is chosen for the progression of individuals who have been screened for HIV, to the symptomatic AIDS stage giving $\gamma_2 = 1/12$.

- Model parameters based on the progression and transmission rates given in (Hallett et al, 2009) for the HIV epidemic in Zimbabwe.
## Numerical simulations: Parameters

### Table 1: Parameter values used in the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>$(0, 1)$</td>
<td>[? , ?]</td>
</tr>
<tr>
<td>$c$</td>
<td>$(1, 5)$</td>
<td>(Nyabadza, 2006)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$0.029$</td>
<td>(Bloer et al)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$(0, 1)$</td>
<td>fitted</td>
</tr>
<tr>
<td>$p$</td>
<td>$(0, 1)$</td>
<td>fitted</td>
</tr>
<tr>
<td>$\eta_1$, $\eta_2$</td>
<td>$(0, 1)$</td>
<td>fitted</td>
</tr>
<tr>
<td>$\eta_3$</td>
<td>$(1, 3)$</td>
<td>fitted</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>$(0.01, 0.053)$</td>
<td>fitted</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>$0.053$</td>
<td>(UNAIDS)</td>
</tr>
</tbody>
</table>
Numerical simulations:

Figure 4: Prevalence data for Zimbabwe. Source (UNAIDS2)
Figure 5: (a) shows the prevalence fit to the UN-AIDS data for Zimbabwe. (b) show the incidence curve corresponding the prevalence curve of the model fit.
Figure 6: (a) shows projection of prevalence up to 2020. and (b) show the projected incidence up to 2020.
Mortality:

Figure 7: (a) show the mortality curve taken from UNAIDS, 2008 and (b) shows the mortality curve corresponding to the prevalence curve of the model fit.
Population Dynamics:

Figure 8: shows the changes in the population trends over 100 years for the parameter values that fit to the curve. $R_e = 2.4815$
Screening into CHBC:

Figure 9: shows the changes in the prevalence with increasing recruitment of the screened into the CHBC.
Changes in population trends

Figure 10: shows the changes in the population trends from 1990 to 2009 with (a) no intervention, (b) screening and counselling, (c) screening, counselling and home-based care, (d) screening, counselling and hospitalization, (e) screening, counselling, home-based care and hospitalization, and (f) screening, counselling, and hospitalization.
Infections and death Averted:

Figure 11: (a) shows the number of infections averted and (b) show the number of deaths averted.

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The paper highlights **four** major challenges associated with the HIV/AIDS epidemic in Zimbabwe.

- Hospitalization of AIDS sufferers,
- CHBC,
- Screening
- Behavior change driven by mortality.

CHBC class is likely to grow as it acts as the destination of many HIV infected individuals as a result of HAART- $\phi > 45\%$

At least 45% to withdrawal from risky sexual activities.
Latest data shows that Zimbabwe HIV rate continue to drop rapidly.

Overall HIV prevalence among pregnant women who attended antenatal clinics decreased from 23% in 2001 to 11% at the end of 2008.

Our projections predict a value around 12%.

The projections also reveal a scenario depicting prevention fatigue - mirrors complacency.
Results-Conclusion:

- A positive response to an intervention program, leading to a fast decline in the contact rate.
- This is then followed by a sustained slow decline of the contact rate.
- Leads to a relapse of risky sexual behavior and this has been observed recently.
- An individual’s behavior depends on prevalence.
Results-Conclusion:

- Treatment of AIDS, especially in CHBC, has reduced stigma and encouraged testing [?].
- In CHBC, the fight against HIV/AIDS is led by victims of the disease and they have made a significant impact in reducing stigma and caring for their fellow sufferers.
- The success of HIV treatment programs, depends on identification and screening of asymptomatic HIV infectives and an effective monitoring of the hospitalized cases.
Work in progress:

- We are in the progress of evaluating the benefits of CHBC and the cost effectiveness of HIV/AIDS interventions discussed in this paper.

- This is being necessitated by the need to strike the right balance between prevention, treatment and care, all of which are essential components of a comprehensive fight against HIV/AIDS.

- Policy makers need to know the benefits and costs of the intervention.